Section 5.7 Practice Exercises

1. division; quotient; remainder

3. $(7a^2+a-6)+(2a^2+5a+11)$
$=7a^2+a-6+2a^2+5a+11$
$=9a^2+6a+5$

5. $8b^2(2b^2-5b+12)=16b^4-40b^3+96b^2$

7. $(2w^3+5)^2=(2w^3)^2+2(2w^3)(5)+5^2$
$=4w^6+20w^3+25$

9. $\left(\frac{7}{8}w-1\right)\left(\frac{7}{8}w+1\right)=\left(\frac{7}{8}w\right)^2-1^2$
$=\frac{49}{64}w^2-1$

11. $(2x+1)(5x-3)=10x^2-6x+5x-3$
$=10x^2-x-3$

13. (a) $\frac{15t^3+18t^2}{3t}=\frac{15t^3}{3t}+\frac{18t^2}{3t}=5t^2+6t$

(b) $3t(5t^2+6t)=15t^3+18t^2$

15. $(6a^2+4a-14)\div 2=\frac{6a^2}{2}+\frac{4a}{2}-\frac{14}{2}$
$=3a^2+2a-7$

17. $\frac{-5x^2-20x+5}{-5}=\frac{-5x^2}{-5}-\frac{20x}{-5}+\frac{5}{-5}$
$=x^2+4x-1$

19. $\frac{3p^3-p^2}{p}=\frac{3p^3}{p}-\frac{p^2}{p}=3p^2-p$

21. $(4m^2+8m)\div 4m^2=\frac{4m^2}{4m^2}+\frac{8m}{4m^2}$
$=1+\frac{2}{m}$

23. $\frac{14y^4-7y^3+21y^2}{-7y^2}$
$=\frac{14y^4}{-7y^2}-\frac{7y^3}{-7y^2}+\frac{21y^2}{-7y^2}$
$=-2y^2+y-3$

25. $(4x^3-24x^2-x+8)\div(4x)$
$=\frac{4x^3}{4x}-\frac{24x^2}{4x}-\frac{x}{4x}+\frac{8}{4x}$
$=x^2-6x-\frac{1}{4}+\frac{2}{x}$

27. $\frac{-a^3b^2+a^2b^2-ab^3}{-a^2b^2}$
$=\frac{-a^3b^2}{-a^2b^2}+\frac{a^2b^2}{-a^2b^2}-\frac{ab^3}{-a^2b^2}$
$=a-1+\frac{b}{a}$

29. $(6t^4-2t^3+3t^2-t+4)\div(2t^3)$

$$= \frac{6t^4}{2t^3} - \frac{2t^3}{2t^3} + \frac{3t^2}{2t^3} - \frac{t}{2t^3} + \frac{4}{2t^3}$$
$$= 3t - 1 + \frac{3}{2t} - \frac{1}{2t^2} + \frac{2}{t^3}$$

31. (a)
$$\begin{array}{r} z+2 \\ z+5\overline{)\ z^2+7z+11} \\ \underline{-(z^2+5z)} \\ 2z+11 \\ \underline{-(2z+10)} \\ 1 \end{array}$$

$$z+2+\frac{1}{z+5}$$

(b) $(z+5)(z+2)+1$
$$= z^2+2z+5z+10+1$$
$$= z^2+7z+11$$

33.
$$\begin{array}{r} t+3 \\ t+1\overline{)\ t^2+4t+5} \\ \underline{-(t^2+\ t)} \\ 3t+5 \\ \underline{-(3t+3)} \\ 2 \end{array}$$

$$t+3+\frac{2}{t+1}$$

35.
$$\begin{array}{r} 7b+4 \\ b-1\overline{)\ 7b^2-3b-4} \\ \underline{-(7b^2-7b)} \\ 4b-4 \\ \underline{-(4b-4)} \end{array}$$

$7b+4$

37.
$$\begin{array}{r} k-6 \\ 5k+1\overline{)\ 5k^2-29k-6} \\ \underline{-(5k^2\ \ +k)} \\ -30k-6 \\ \underline{-(-30k-6)} \end{array}$$

$k-6$

39.
$$\begin{array}{r} 2p^2+3p-4 \\ 2p+3\overline{)\ 4p^3+12p^2+p-12} \\ \underline{-(4p^3+6p^2)} \\ 6p^2+p \\ \underline{-(6p^2+9p)} \\ -8p-12 \\ \underline{-(-8p-12)} \end{array}$$

$2p^2+3p-4$

41. Arrange both the dividend and divisor in descending order.
$$\begin{array}{r} k-2 \\ k+1\overline{)\ k^2-k-6} \\ \underline{-(k^2+k)} \\ -2k-6 \\ \underline{-(-2k-2)} \\ -4 \end{array}$$

$$k-2+\frac{-4}{k+1}$$

43.
$$\begin{array}{r} 2x^2-x+6 \\ 2x-3\overline{)\ 4x^3-8x^2+15x-16} \\ \underline{-(4x^3-6x^2)} \\ -2x^2+15x \\ \underline{-(-2x^2+\ 3x)} \\ 12x-16 \\ \underline{-(12x-18)} \\ 2 \end{array}$$

$$2x^2-x+6+\frac{2}{2x-3}$$

45.
$$\begin{array}{r} y^2+2y+1 \\ 3y-1\overline{)\ 3y^3+5y^2+\ y+1} \\ \underline{-(3y^3-y^2)} \\ 6y^2+\ \ y \end{array}$$

$$\underline{-(6y^2-2y)}$$
$$3y+1$$
$$\underline{-(3y-1)}$$
$$2$$

$$y^2+2y+1+\frac{2}{3y-1}$$

47. Arrange the dividend in descending order. The term $0a$ is a placeholder for the missing term.

$$\begin{array}{r} a-3 \\ a+3\overline{)\ a^2+0a+9} \\ \underline{-(a^2+3a)} \\ -3a+9 \\ \underline{-(-3a-9)} \\ 18 \end{array}$$

$$a-3+\frac{18}{a+3}$$

49. The term $0x^2$ is a placeholder for the missing term in the dividend.

$$\begin{array}{r} 4x^2+8x+13 \\ x-2\overline{)\ 4x^3+0x^2-3x-26} \\ \underline{-(4x^3-8x^2)} \\ 8x^2-3x \\ \underline{-(8x^2-16x)} \\ 13x-26 \\ \underline{-(13x-26)} \end{array}$$

$$4x^2+8x+13$$

51. The term $0w$ is a placeholder for the missing term in the divisor.

$$\begin{array}{r} w^2+5w-2 \\ w^2+0w-3\overline{)w^4+5w^3-5w^2-15w+7} \\ \underline{-(w^4+0w^3-3w^2)} \\ 5w^3-2w^2-15w \\ \underline{-(5w^3+0w^2-15w)} \\ -2w^2+0w+7 \\ \underline{-(-2w^2+0w+6)} \\ 1 \end{array}$$

$$w^2+5w-2+\frac{1}{w^2-3}$$

53.
$$\begin{array}{r} n^2+n-6 \\ 2n^2+3n-2\overline{)\ 2n^4+5n^3-11n^2-20n+12} \\ \underline{-(2n^4+3n^3-2n^2)} \\ 2n^3-9n^2-20n \\ \underline{-(2n^3+3n^2-2n)} \\ -12n^2-18n+12 \\ \underline{-(-12n^2-18n+12)} \end{array}$$

$$n^2+n-6$$

55.
$$\begin{array}{r} 3y^2-3 \\ y^2+1\overline{)\ 3y^4+0y^3+0y^2+2y+3} \\ \underline{-(3y^4+0y^3+3y^2)} \\ -3y^2+2y+3 \\ \underline{-(-3y^2+0y-3)} \\ 2y+6 \end{array}$$

$$3y^2-3+\frac{2y+6}{y^2+1}$$

57.
$$\begin{array}{r} x-1 \\ 5x^2+5x+1\overline{)\ 5x^3+0x^2-4x-9} \\ \underline{-(5x^3+5x^2+x)} \\ -5x^2-5x-9 \\ \underline{-(-5x^2-5x-1)} \\ -8 \end{array}$$

$$x-1+\frac{-8}{5x^2+5x+1}$$

59. To check, multiply the divisor $(x-2)$ by the quotient (x^2+4).

$(x-2)(x^2+4)=x^3-2x^2+4x-8$ which does not equal x^3-8.

61. Monomial division;

$$\frac{9a^3}{3a}+\frac{12a^2}{3a}=3a^2+4a$$

63. Long division;

$$\begin{array}{r} p+2 \\ p^2-p-2\overline{\left) \begin{array}{r} p^3+p^2-4p-4 \\ \underline{-(p^3-p^2-2p)} \\ 2p^2-2p-4 \\ \underline{-(2p^2-2p-4)} \\ 0 \end{array}\right.} \end{array}$$

$p+2$

65. Long division; the terms $0t^3$ and $0t$ are placeholders for the missing terms in the dividend.

$$\begin{array}{r} t^3-2t^2+5t-10 \\ t+2\overline{\left) \begin{array}{r} t^4+0t^3+t^2+0t-16 \\ \underline{-(t^4+2t^3)} \\ -2t^3+t^2 \\ \underline{-(-2t^3-4t^2)} \\ 5t^2+0t \\ \underline{-(5t^2+10t)} \\ -10t-16 \\ \underline{-(-10t-20)} \\ 4 \end{array}\right.} \end{array}$$

$$t^3-2t^2+5t-10+\frac{4}{t+2}$$

67. Long division; the terms $0w^3$ and $0w$ are placeholders for the missing terms in the dividend and $0w$ in the divisor.

$$\begin{array}{r} w^2+3 \\ w^2+0w-2\overline{\left) \begin{array}{r} w^4+0w^3+w^2+0w-5 \\ \underline{-(w^4+0w^3-2w^2)} \\ 3w^2+0w-5 \\ \underline{-(3w^2+0w-6)} \\ 1 \end{array}\right.} \end{array}$$

$$w^2+3+\frac{1}{w^2-2}$$

69. Long division; the terms $0n^2$ and $0n$ are placeholders for the missing terms in the dividend.

$$\begin{array}{r} n^2+4n+16 \\ n-4\overline{\left) \begin{array}{r} n^3+0n^2+0n-64 \\ \underline{-(n^3-4n^2)} \\ 4n^2+0n \\ \underline{-(4n^2-16n)} \\ 16n-64 \\ \underline{-(16n-64)} \\ 0 \end{array}\right.} \end{array}$$

$n^2+4n+16$

71. Monomial division;

$$\frac{9r^3}{-3r^2}+\frac{-12r^2}{-3r^2}+\frac{9}{-3r^2}$$
$$=-3r+4-\frac{3}{r^2}$$

73. Insert placeholders for missing terms in the dividend.

$$\begin{array}{r} x+1 \\ x-1\overline{\left) \begin{array}{r} x^2+0x-1 \\ \underline{-(x^2-x)} \\ x-1 \\ \underline{-(x-1)} \\ 0 \end{array}\right.} \end{array}$$

$$\frac{x^2-1}{x-1}=x+1$$

75. Insert placeholders for missing terms in the dividend.

$$
\begin{array}{r}
x^3+x^2+x+1 \\
x-1\overline{\left)\begin{array}{r} x^4+0x^3+0x^2+0x-1 \end{array}\right.} \\
\underline{-(x^4-x^3)} \\
x^3+0x^2 \\
\underline{-(x^3-x^2)} \\
x^2+0x \\
\underline{-(x^2-x)} \\
x\ -1 \\
\underline{-(x-1)} \\
0
\end{array}
$$

$$\frac{x^4-1}{x-1}=x^3+x^2+x+1$$

77. Insert placeholders for missing terms in the dividend.

$$
\begin{array}{r}
x+1 \\
x-1\overline{\left)\begin{array}{r} x^2+0x+0 \end{array}\right.} \\
\underline{-(x^2-x)} \\
x+0 \\
\underline{-(x-1)} \\
1
\end{array}
$$

$$\frac{x^2}{x-1}=x+1+\frac{1}{x-1}$$

79. Insert placeholders for missing terms in the dividend.

$$
\begin{array}{r}
x^3+x^2+x+1 \\
x-1\overline{\left)\begin{array}{r} x^4+0x^3+0x^2+0x+0 \end{array}\right.} \\
\underline{-(x^4-x^3)} \\
x^3+0x^2 \\
\underline{-(x^3-x^2)} \\
x^2+0x \\
\underline{-(x^2-x)} \\
x+0 \\
\underline{-(x-1)} \\
1
\end{array}
$$

$$\frac{x^4}{x-1}=x^3+x^2+x+1+\frac{1}{x-1}$$

Problem Recognition Exercises

1. (a) $6x^2+2x^2=8x^2$

(b) $\left(6x^2\right)\left(2x^2\right)=12x^4$

3. (a) $(4x+y)^2=(4x)^2+2(4x)(y)+(y)^2$

$=16x^2+8xy+y^2$

(b) $(4xy)^2=16x^2y^2$

5. (a) $(2x+3)+(4x-2)=6x+1$

(b) $(2x+3)(4x-2)$

$=2x(4x)+2x(-2)+3(4x)+3(-2)$

$=8x^2-4x+12x-6$

$=8x^2+8x-6$

7. (a) $(3z+2)^2=(3z)^2+2(3z)(2)+(2)^2$

$=9z^2+12z+4$

(b) $(3z+2)(3z-2)$

$=(3z)^2-(2)^2$

$=9z^2-4$

9. (a) $(2x-4)\left(x^2-2x+3\right)$

$=2x\left(x^2\right)+2x(-2x)+2x(3)$

$-4\left(x^2\right)-4(-2x)-4(+3)$

$=2x^3-4x^2+6x-4x^2+8x-12$

$=2x^3-8x^2+14x-12$

(b) $(2x-4)+\left(x^2-2x+3\right)$

$= x^2+2x-2x-4+3 = x^2-1$

11. (a) $x+x=2x$

(b) $x\cdot x = x^2$

13. $(4xy)^2 = 4^2x^2y^2 = 16x^2y^2$

15. $(-2x^4-6x^3+8x^2)\div(2x^2)$

$= \dfrac{-2x^4}{2x^2}-\dfrac{6x^3}{2x^2}+\dfrac{8x^2}{2x^2}$

$= -x^2-3x+4$

17. $(m^3-4m^2-6)-(3m^2+7m)$

$+(-m^3-9m+6)$

$= m^3-4m^2-6-3m^2-7m$

$-m^3-9m+6$

$= -7m^2-16m$

19. Insert a placeholder for the missing term in the dividend.

$$\begin{array}{r} 8x^2+16x+34 \\ x-2\overline{)\ 8x^3+0x^2+2x+6} \\ \underline{-(8x^3-16x^2)} \\ 16x^2+2x \\ \underline{-(16x^2-32x)} \\ 34x+6 \\ \underline{-(34x-68)} \\ 74 \end{array}$$

$8x^2+16x+34+\dfrac{74}{x-2}$

21. $(2x-y)(3x^2+4xy-y^2)$

$= 6x^3+8x^2y-2xy^2-3x^2y-4xy^2+y^3$

$= 6x^3+5x^2y-6xy^2+y^3$

23. $(x+y^2)(x^2-xy^2+y^4)$

$= x^3-x^2y^2+xy^4+x^2y^2-xy^4+y^6$

$= x^3+y^6$

25. $(a^2+2b)-(a^2-2b)$

$= a^2+2b-a^2+2b = 4b$

27. $(a^3+2b)(a^3-2b)$

$= (a^3)^2-(2b)^2$

$= a^6-4b^2$

29.
$$\begin{array}{r} 4p+4 \\ 2p-1\overline{)\ 8p^2+4p-6} \\ \underline{-(8p^2-4p)} \\ 8p-6 \\ \underline{-(8p-4)} \\ -2 \end{array}$$

$4p+4+\dfrac{-2}{2p-1}$

31. $\dfrac{12x^3y^7}{3xy^5} = \dfrac{12}{3}x^{3-1}y^{7-5}$

$= 4x^2y^2$

33. $\left(\dfrac{3}{7}x-\dfrac{1}{2}\right)\left(\dfrac{3}{7}x+\dfrac{1}{2}\right)$

$= \left(\dfrac{3}{7}x\right)^2-\left(\dfrac{1}{2}\right)^2$

$= \dfrac{9}{49}x^2-\dfrac{1}{4}$

35. $\left(\frac{1}{9}x^3+\frac{2}{3}x^2+\frac{1}{6}x-3\right)-\left(\frac{4}{3}x^3+\frac{1}{9}x^2+\frac{2}{3}x+1\right)$

$= \dfrac{1}{9}x^3+\dfrac{2}{3}x^2+\dfrac{1}{6}x-3-\dfrac{4}{3}x^3-\dfrac{1}{9}x^2-\dfrac{2}{3}x-1$

$= \dfrac{1}{9}x^3-\dfrac{4}{3}x^3+\dfrac{2}{3}x^2-\dfrac{1}{9}x^2$

$+\dfrac{1}{6}x-\dfrac{2}{3}x-3-1$

$= \dfrac{1}{9}x^3-\dfrac{12}{9}x^3+\dfrac{6}{9}x^2-\dfrac{1}{9}x^2$

$+\dfrac{1}{6}x-\dfrac{4}{6}x-3-1$

$= -\dfrac{11}{9}x^3+\dfrac{5}{9}x^2-\dfrac{3}{6}x-4$

$= -\dfrac{11}{9}x^3+\dfrac{5}{9}x^2-\dfrac{1}{2}x-4$

37. $(0.05x^2-0.16x-0.75)$

$+(1.25x^2-0.14x+0.25)$

$= 0.05x^2 + 1.25x^2 - 0.16x - 0.14x - 0.75 + 0.25$

$= 1.3x^2 - 0.3x - 0.5$

39. $(3x^2y)(-2xy)$

$= -6x^3y^2$