Section 7.1 Practice Exercises

1. **a.** exponent

b. 1

c. $\left(\frac{1}{b}\right)^n$ or $\frac{1}{b^n}$

d. scientific notation

3. $ab^3 = a \cdot b \cdot b \cdot b$

$$\begin{aligned}(ab)^3 &= (ab)\cdot(ab)\cdot(ab)\\ &= a \cdot a \cdot a \cdot b \cdot b \cdot b\\ &= a^3 \cdot b^3\end{aligned}$$

5. For example: $(5x)^2 = 5^2 x^2$

$(xy)^3 = x^3 y^3$

7. For example: $\frac{x^5}{x^2} = x^3$

$\frac{8^4}{8^2} = 8^2$

9. For example: $6^0 = 1$

$x^0 = 1 \ (x \neq 0)$

11. $\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3$

13. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

15. $-5^{-2} = -\frac{1}{5^2} = -\frac{1}{25}$

17. $(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$

19. $\left(-\frac{1}{4}\right)^{-3} = \left(-\frac{4}{1}\right)^3 = (-4)^3 = -64$

21. $\left(-\frac{3}{2}\right)^{-4} = \left(-\frac{2}{3}\right)^4 = \frac{(-2)^4}{3^4} = \frac{16}{81}$

23. $-\left(\frac{2}{5}\right)^{-3} = -\left(\frac{5}{2}\right)^3 = -\frac{5^3}{2^3} = -\frac{125}{8}$

25. $(10ab)^0 = 1$

27. $10ab^0 = 10a \cdot 1 = 10a$

29. $y^3 \cdot y^5 = y^{3+5} = y^8$

31. $\frac{13^8}{13^6} = 13^{8-6} = 13^2 = 169$

33. $\left(y^2\right)^4 = y^{2\cdot 4} = y^8$

35. $\left(3x^2\right)^4 = 3^4\left(x^2\right)^4 = 3^4 x^{2\cdot 4} = 81x^8$

37. $p^{-3} = \dfrac{1}{p^3}$

39. $7^{10}\cdot 7^{-13} = 7^{10+(-13)} = 7^{-3} = \dfrac{1}{7^3}$

$= \dfrac{1}{343}$

41. $\dfrac{w^3}{w^5} = w^{3-5} = w^{-2} = \dfrac{1}{w^2}$

43. $a^{-2}a^{-5} = a^{-2+(-5)} = a^{-7} = \dfrac{1}{a^7}$

45. $\dfrac{r}{r^{-1}} = r^{1-(-1)} = r^2$

47. $\dfrac{z^{-6}}{z^{-2}} = z^{-6-(-2)} = z^{-4} = \dfrac{1}{z^4}$

49. $\dfrac{a^3}{b^{-2}} = a^3\cdot\dfrac{1}{b^{-2}} = a^3b^2$

51. $\left(6xyz^2\right)^0 = 1$

53. $2^4 + 2^{-2} = 2^4 + \dfrac{1}{2^2}$

$= 16 + \dfrac{1}{4}$

$= 16\frac{1}{4}$ or $\dfrac{65}{4}$

55. $1^{-2} + 5^{-2} = \dfrac{1}{1^2} + \dfrac{1}{5^2}$

$= \dfrac{1}{1} + \dfrac{1}{25}$

$= 1\frac{1}{25}$ or $\dfrac{26}{25}$

57. $\left(\dfrac{2}{3}\right)^{-2} - \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{3}\right)^0 = \left(\dfrac{3}{2}\right)^2 - \dfrac{1}{4} + 1$

$= \dfrac{9}{4} - \dfrac{1}{4} + \dfrac{4}{4}$

$= \dfrac{12}{4} = 3$

59. $\left(\dfrac{4}{5}\right)^{-1} + \left(\dfrac{3}{2}\right)^2 - \left(\dfrac{2}{7}\right)^0 = \dfrac{5}{4} + \dfrac{9}{4} - 1$

$= \dfrac{5}{4} + \dfrac{9}{4} - \dfrac{4}{4}$

$= \dfrac{10}{4} = \dfrac{5}{2}$

61. $\dfrac{p^2q}{p^5q^{-1}} = p^{2-5}q^{1-(-1)}$

$= p^{-3}q^2$

$= \dfrac{1}{p^3}\cdot q^2 = \dfrac{q^2}{p^3}$

63. $\dfrac{-48ab^{10}}{32a^4b^3} = -\dfrac{48}{32}a^{1-4}b^{10-3} = -\dfrac{3}{2}a^{-3}b^7$

$= -\dfrac{3}{2}\cdot\dfrac{1}{a^3}\cdot b^7 = -\dfrac{3b^7}{2a^3}$

65.
$$\begin{aligned}&\left(-3x^{-4}y^5z^2\right)^{-4}\\&=(-3)^{-4}\left(x^{-4}\right)^{-4}\left(y^5\right)^{-4}\left(z^2\right)^{-4}\\&=\left(-\frac{1}{3}\right)^4 x^{16}y^{-20}z^{-8}\\&=\frac{1}{81}\cdot x^{16}\cdot\frac{1}{y^{20}}\cdot\frac{1}{z^8}=\frac{x^{16}}{81y^{20}z^8}\end{aligned}$$

67.
$$\begin{aligned}\left(4m^{-2}n\right)\left(-m^6n^{-3}\right)&=-4m^{-2+6}n^{1+(-3)}\\&=-4m^4n^{-2}\\&=-4m^4\cdot\frac{1}{n^2}\\&=-\frac{4m^4}{n^2}\end{aligned}$$

69.
$$\begin{aligned}&\left(p^{-2}q\right)^3\left(2pq^4\right)^2\\&=\left(p^{-2}\right)^3q^3\cdot2^2p^2\left(q^4\right)^2\\&=p^{-6}q^3\cdot4p^2q^8\\&=4p^{-6+2}q^{3+8}\\&=4p^{-4}q^{11}\\&=4\cdot\frac{1}{p^4}\cdot q^{11}=\frac{4q^{11}}{p^4}\end{aligned}$$

71.
$$\begin{aligned}\left(\frac{x^2}{y}\right)^3\left(5x^2y\right)&=\frac{x^6}{y^3}\left(5x^2y\right)\\&=5x^{6+2}y^{1-3}\\&=5x^8y^{-2}\\&=5x^8\frac{1}{y^2}\\&=\frac{5x^8}{y^2}\end{aligned}$$

73.
$$\begin{aligned}\frac{\left(-8a^2b^2\right)^4}{\left(16a^3b^7\right)^2}&=\frac{(-8)^4\left(a^2\right)^4\left(b^2\right)^4}{(16)^2\left(a^3\right)^2\left(b^7\right)^2}\\&=\frac{4096a^8b^8}{256a^6b^{14}}\\&=16a^{8-6}b^{8-14}\\&=16a^2b^{-6}\\&=16a^2\cdot\frac{1}{b^6}\\&=\frac{16a^2}{b^6}\end{aligned}$$

75.
$$\begin{aligned}&\left(\frac{-2x^6y^{-5}}{3x^{-2}y^4}\right)^{-3}\\&=\left(-\frac{2}{3}x^{6-(-2)}y^{-5-4}\right)^{-3}\\&=\left(-\frac{2}{3}x^8y^{-9}\right)^{-3}\\&=\left(-\frac{2}{3}\right)^{-3}\left(x^8\right)^{-3}\left(y^{-9}\right)^{-3}\\&=\left(-\frac{3}{2}\right)^3x^{-24}y^{27}\\&=-\frac{27}{8}\cdot\frac{1}{x^{24}}\cdot y^{27}\\&=-\frac{27y^{27}}{8x^{24}}\end{aligned}$$

77. $\left(\dfrac{2x^{-3}y^{0}}{4x^{6}y^{-5}}\right)^{-2} = \left(\dfrac{1}{2}x^{-3-6}y^{0-(-5)}\right)^{-2}$

$= \left(\dfrac{1}{2}x^{-9}y^{5}\right)^{-2}$

$= \left(\dfrac{1}{2}\right)^{-2}\left(x^{-9}\right)^{-2}\left(y^{5}\right)^{-2}$

$= (2)^{2}x^{18}y^{-10}$

$= 4x^{18}\cdot\dfrac{1}{y^{10}}$

$= \dfrac{4x^{18}}{y^{10}}$

79. $3xy^{5}\left(\dfrac{2x^{4}y}{6x^{5}y^{3}}\right)^{-2}$

$= 3xy^{5}\left(\dfrac{1}{3}x^{4-5}y^{1-3}\right)^{-2}$

$= 3xy^{5}\left(\dfrac{1}{3}x^{-1}y^{-2}\right)^{-2}$

$= 3xy^{5}\left(\dfrac{1}{3}\right)^{-2}\left(x^{-1}\right)^{-2}\left(y^{-2}\right)^{-2}$

$= 3xy^{5}(3)^{2}x^{2}y^{4}$

$= 3\cdot 9x^{1+2}y^{5+4}$

$= 27x^{3}y^{9}$

81. **a.** $\$8,000,000,000 = \$8\times 10^{9}$

b. $3,000,000 = 3\times 10^{6}$ DVDs

c. $14,000,000,000,000 = 1.4\times 10^{13}$ eV

d. $0.000\,000\,000\,000\,000\,0001602$

$= 1.602\times 10^{-19}$ J

83. **a.** $2\times 10^{11} = 200,000,000,000$

b. $4\times 10^{-6} = 0.000004$

c. $1.082\times 10^{11} = 108,200,000,000$

85. $35\times 10^{4} = 3.5\times 10^{1}\times 10^{4}$

$= 3.5\times 10^{5}$

87. 7.0×10^{0} Proper

89. 9×10^{1} Proper

91. $\left(6.5\times 10^{3}\right)\left(5.2\times 10^{-8}\right) = 33.8\times 10^{3+(-8)}$

$= 3.38\times 10^{1}\times 10^{-5}$

$= 3.38\times 10^{-4}$

93. $(0.0000024)(6,700,000,000)$

$= \left(2.4\times 10^{-6}\right)\left(6.7\times 10^{9}\right)$

$= 16.08\times 10^{-6+9}$

$= 1.608\times 10^{1}\times 10^{3} = 1.608\times 10^{4}$

95. $\left(8.5\times 10^{-2}\right)\div\left(2.5\times 10^{-15}\right)$

$= 3.4\times 10^{-2-(-15)}$

$= 3.4\times 10^{13}$

97. $(900000000) \div (360000)$

$= (9 \times 10^8) \div (3.6 \times 10^5)$

$= 2.5 \times 10^{8-5}$

$= 2.5 \times 10^3$

99. $2 \cdot (6.02 \times 10^{23}) = 12.04 \times 10^{23}$

$= 1.204 \times 10^1 \times 10^{23}$

$= 1.204 \times 10^{24}$ hydrogen atoms

$1 \cdot (6.02 \times 10^{23}) = 6.02 \times 10^{23}$ oxygen atoms

101. $2{,}200{,}000 \div 110$

$= (2.2 \times 10^6) \div (1.1 \times 10^2)$

$= 2 \times 10^4$ or $20{,}000$ people per mi^2

103. $(\$3.5 \times 10^9)(15) = \$52.5 \times 10^9$

$= \$5.25 \times 10^{10}$

105. **a.** $45 \cdot 12 = 540$ months

b. $\$20(540) = \$10{,}800$

c. $A = \$20\left[\left(1 + \dfrac{0.06}{12}\right)^{540} - 1\right]\left(1 + \dfrac{12}{0.06}\right) = \$55{,}395.45$

107. $y^{a-5} y^{a+7} = y^{a-5+a+7} = y^{2a+2}$

109. $\dfrac{x^{3a-3}}{x^{a+1}} = x^{(3a-3)-(a+1)}$

$= x^{3a-3-a-1} = x^{2a-4}$

111. $\dfrac{x^{2a-2} y^{a+3}}{x^{a+4} y^{a-3}} = x^{(2a-2)-(a+4)} y^{(a+3)-(a-3)} = x^{2a-2-a-4} y^{a+3-a+3} = x^{a-6} y^6$

Section 7.2 Practice Exercises

1. **a.** polynomial

b. coefficient; n

c. 1; 1

d. one

e. binomial

f. trinomial

g. leading; leading coefficient

h. greatest

i. zero

j. exponents

k. polynomial

3. $(2ac^{-2})(5a^{-1}c^4) = 10a^{1+(-1)} c^{-2+4}$

$= 10a^0 c^2 = 10c^2$

5. $(3.4 \times 10^5)(5.0 \times 10^{-2}) = 17 \times 10^3$

$= 1.7 \times 10^4$

7. $-6a^3 + a^2 - a$

leading coefficient: –6

degree: 3

9. $3x^4 + 6x^2 - x - 1$

leading coefficient: 3

degree: 4

11. $-t^2 + 100$

leading coefficient: –1

degree: 2

13. For example: $3x^5$

15. For example: $x^2 + 2x + 1$

17. For example: $6x^4 - x^2$

19. $\left(-4m^2 + 4m\right) + \left(5m^2 + 6m\right)$

$= -4m^2 + 5m^2 + 4m + 6m$

$= m^2 + 10m$

21. $\left(3x^4 - x^3 - x^2\right) + \left(3x^3 - 7x^2 + 2x\right)$

$= 3x^4 + \left(-x^3\right) + 3x^3 + \left(-x^2\right) + \left(-7x^2\right) + 2x$

$= 3x^4 + 2x^3 - 8x^2 + 2x$

23. $\left(\frac{1}{2}w^3 + \frac{2}{9}w^2 - 1.8w\right) + \left(\frac{3}{2}w^3 - \frac{1}{9}w^2 + 2.7w\right)$

$= \frac{1}{2}w^3 + \frac{3}{2}w^3 + \frac{2}{9}w^2 - \frac{1}{9}w^2 - 1.8w + 2.7w$

$= 2w^3 + \frac{1}{9}w^2 + 0.9w$

25. $\left(9x^2y - 5xy + 1\right) + \left(8x^2y + xy - 15\right)$

$= 9x^2y + 8x^2y - 5xy + xy + 1 - 15$

$= 17x^2y - 4xy - 14$

27. $\left(-7a + 6a^2 + 1\right) + \left(-8 - 4a - 2a^2\right)$

$= 6a^2 - 2a^2 - 7a - 4a + 1 - 8$

$= 4a^2 - 11a - 7$

29.

$$\begin{array}{r} 12x^3 \qquad +6x - 8 \\ +\left(-3x^3 - 5x^2 - 4x \quad\right) \\ \hline 9x^3 - 5x^2 + 2x - 8 \end{array}$$

31. $-\left(-30y^3\right) = 30y^3$

33. $-\left(4p^3 + 2p - 12\right) = -4p^3 - 2p + 12$

35. $-\left(-11ab^2 + a^2b\right) = 11ab^2 - a^2b$

37. $\left(13z^5 - z^2\right) - \left(7z^5 + 5z^2\right)$

$= \left(13z^5 - z^2\right) + \left(-7z^5 - 5z^2\right)$

$= 13z^5 - 7z^5 - z^2 - 5z^2$

$= 6z^5 - 6z^2$

39. $\left(-3x^3+3x^2-x+6\right)-\left(1-x-x^2-x^3\right)$

$=\left(-3x^3+3x^2-x+6\right)+\left(-1+x+x^2+x^3\right)$

$=\left(-3x^3+3x^2-x+6\right)+\left(x^3+x^2+x-1\right)$

$=-3x^3+x^3+3x^2+x^2-x+x+6-1$

$=-2x^3+4x^2+5$

41. $\left(-3xy^3+3x^2y-x+6\right)-\left(-xy^3-xy-x+1\right)$

$=\left(-3xy^3+3x^2y-x+6\right)+\left(xy^3+xy+x-1\right)$

$=-3xy^3+xy^3+3x^2y+xy-x+x+6-1$

$=-2xy^3+3x^2y+xy+5$

43.
$$\begin{array}{rl} & 4t^3-6t^2 \quad -18 \\ \underline{-\left(3t^3+7t^2+9t-5\right)} & \rightarrow \underline{+\left(-3t^3-7t^2-9t+5\right)} \\ & t^3-13t^2-9t-13 \end{array}$$

45. $\left(\frac{1}{5}a^2-\frac{1}{2}ab+\frac{1}{10}b^2+3\right)-\left(-\frac{3}{10}a^2+\frac{2}{5}ab-\frac{1}{2}b^2-5\right)$

$=\left(\frac{1}{5}a^2-\frac{1}{2}ab+\frac{1}{10}b^2+3\right)+\left(\frac{3}{10}a^2-\frac{2}{5}ab+\frac{1}{2}b^2+5\right)$

$=\frac{1}{5}a^2+\frac{3}{10}a^2-\frac{1}{2}ab-\frac{2}{5}ab+\frac{1}{10}b^2+\frac{1}{2}b^2+3+5$

$=\frac{2}{10}a^2+\frac{3}{10}a^2-\frac{5}{10}ab-\frac{4}{10}ab+\frac{1}{10}b^2+\frac{5}{10}b^2+3+5$

$=\frac{1}{2}a^2-\frac{9}{10}ab+\frac{3}{5}b^2+8$

47. $\left(8x^2+x-15\right)-\left(9x^2-5x+1\right)$

$=\left(8x^2+x-15\right)+\left(-9x^2+5x-1\right)$

$=8x^2-9x^2+x+5x-15-1$

$=-x^2+6x-16$

49. $\left(3x^5-2x^3+4\right)-\left(x^4+2x^3-7\right)$

$=\left(3x^5-2x^3+4\right)+\left(-x^4-2x^3+7\right)$

$=3x^5-x^4-2x^3-2x^3+4+7$

$=3x^5-x^4-4x^3+11$

51. $\left(8y^2-4y^3\right)-\left(3y^2-8y^3\right)$

$=\left(8y^2-4y^3\right)+\left(-3y^2+8y^3\right)$

$=-4y^3+8y^3+8y^2-3y^2$

$=4y^3+5y^2$

53. $\left(-2r-6r^4\right)+\left(-r^4-9r\right)$

$=-6r^4-r^4-2r-9r$

$=-7r^4-11r$

55. $\left(5xy+13x^2+3y\right)-\left(4x^2-8y\right)$

$=\left(5xy+13x^2+3y\right)+\left(-4x^2+8y\right)$

$=13x^2-4x^2+5xy+3y+8y$

$=9x^2+5xy+11y$

57. $\left(11ab-23b^2\right)+\left(7ab-19b^2\right)$

$=11ab+7ab-23b^2-19b^2$

$=18ab-42b^2$

59. $\left[2p-\left(3p+5\right)\right]+\left(4p-6\right)+2$

$=\left[2p-3p-5\right]+\left(4p-6\right)+2$

$=-p-5+4p-6+2$

$=-p+4p-5-6+2$

$=3p-9$

61. $5-\left[2m^2-\left(4m^2+1\right)\right]$

$=5-\left[2m^2-4m^2-1\right]$

$=5-\left[-2m^2-1\right]$

$=5+2m^2+1=2m^2+6$

63. $\left(6x^3-5\right)-\left(-3x^3+2x\right)-\left(2x^3-6x\right)$

$=6x^3-5+3x^3-2x-2x^3+6x$

$=7x^3+4x-5$

65. $\left(-ab+5a^2b\right)-\left[7ab^2-2ab-\left(7a^2b+2ab^2\right)\right]=-ab+5a^2b-\left[7ab^2-2ab-7a^2b-2ab^2\right]$

$=-ab+5a^2b-\left[5ab^2-2ab-7a^2b\right]$

$=-ab+5a^2b-5ab^2+2ab+7a^2b$

$=12a^2b+ab-5ab^2$

67. $\left(8x^3-x^2+3\right)-\left[5x^2+x-\left(4x^3+x-2\right)\right]$

$=\left(8x^3-x^2+3\right)-\left[5x^2+x-4x^3-x+2\right]$

$=\left(8x^3-x^2+3\right)-\left(-4x^3+5x^2+2\right)$

$=8x^3-x^2+3+4x^3-5x^2-2$

$=8x^3+4x^3-x^2-5x^2+3-2$

$=12x^3-6x^2+1$

69.

$12a^2b-4ab^2-ab \rightarrow \quad 12a^2b-4ab^2-ab$

$\underline{-\left(4a^2b+\ ab^2-5ab\right)} \rightarrow \underline{+\left(-4a^2b-\ ab^2+5ab\right)}$

$8a^2b-5ab^2+4ab$

71.

$-5x^4 \quad -11x^2 \quad +6 \rightarrow \quad -5x^4 \quad -11x^2 \quad +6$

$\underline{-\left(-5x^4+3x^3+5x^2-10x+5\right)} \rightarrow \underline{+\left(5x^4-3x^3-5x^2+10x-5\right)}$

$-3x^3-16x^2+10x+1$

73.
$$\begin{array}{r} -2.2p^5 - 9.1p^4 \quad + 5.3p^2 - 7.9p \\ +\left(\quad -6.4p^4 - 8.5p^3 - 10.3p^2 \quad \right) \\ \hline -2.2p^5 - 15.5p^4 - 8.5p^3 - \ 5p^2 - 7.9p \end{array}$$

75.
$$\begin{aligned} P &= \left(2x^3 + 6x\right) + \left(4x^3 - 5x\right) + \left(6x^3 + x\right) \\ &= 2x^3 + 6x + 4x^3 - 5x + 6x^3 + x \\ &= 12x^3 + 2x \end{aligned}$$

77. $h(x) = \frac{2}{3}x^2 - 5$

It is a polynomial function. The degree is 2.

79. $p(x) = 8x^3 + 2x^2 - \frac{3}{x}$

It is not a polynomial function. The term $-\frac{3}{x} = -3x^{-1}$ and –1 is not a whole number.

81. $g(x) = -7$

It is a polynomial function. The degree is 0.

83. $M(x) = |x| + 5x$

It is not a polynomial function. The term $|x|$ is not of the form ax^n.

85. **a.**
$$\begin{aligned} P(x) &= -x^4 + 2x - 5 \\ P(2) &= -(2)^4 + 2(2) - 5 \\ &= -16 + 4 - 5 \\ &= -17 \end{aligned}$$

b.
$$\begin{aligned} P(-1) &= -(-1)^4 + 2(-1) - 5 \\ &= -1 - 2 - 5 \\ &= -8 \end{aligned}$$

c.
$$\begin{aligned} P(0) &= -(0)^4 + 2(0) - 5 \\ &= 0 + 0 - 5 \\ &= -5 \end{aligned}$$

d.
$$\begin{aligned} P(1) &= -(1)^4 + 2(1) - 5 \\ &= -1 + 2 - 5 \\ &= -4 \end{aligned}$$

87. **a.**
$$\begin{aligned} H(x) &= \tfrac{1}{2}x^3 - x + \tfrac{1}{4} \\ H(0) &= \frac{1}{2}(0)^3 - (0) + \frac{1}{4} \\ &= 0 - 0 + \frac{1}{4} = \frac{1}{4} \end{aligned}$$

b.
$$\begin{aligned} H(2) &= \frac{1}{2}(2)^3 - (2) + \frac{1}{4} \\ &= 4 - 2 + \frac{1}{4} = 2 + \frac{1}{4} \\ &= \frac{9}{4} \end{aligned}$$

c.
$$\begin{aligned} H(-2) &= \frac{1}{2}(-2)^3 - (-2) + \frac{1}{4} \\ &= -4 + 2 + \frac{1}{4} \\ &= -2 + \frac{1}{4} = -\frac{7}{4} \end{aligned}$$

d.
$$\begin{aligned} H(-1) &= \frac{1}{2}(-1)^3 - (-1) + \frac{1}{4} \\ &= -\frac{1}{2} + 1 + \frac{1}{4} = \frac{3}{4} \end{aligned}$$

89. Let x = the width of the garden

$x + 3$ = the length of the garden

$$P(x) = 2x + 2(x+3)$$
$$= 2x + 2x + 6$$
$$= 4x + 6$$

91. **a.**
$$P(x) = R(x) - C(x)$$
$$= (12x) - (5.40x + 99)$$
$$= 12x - 5.40x - 99$$
$$= 6.6x - 99$$

b.
$$P(50) = 6.6(50) - 99$$
$$= 330 - 99$$
$$= 231$$

The profit will be \$231.

93. **a.**
$$D(x) = 5.2x^2 + 40.4x + 1636$$
$$D(0) = 5.2(0)^2 + 40.4(0) + 1636$$
$$= 0 + 0 + 1636 = 1636$$

$D(0) = 1636$ means that at the beginning of the study, (year 0)the annual dormitory charge was \$1636.

$$D(18) = 5.2(18)^2 + 40.4(18) + 1636$$
$$= 1684.8 + 727.2 + 1636 = 404$$

In 2008, the annual dormitory charge was \$4048.

b.
$$D(25) = 5.2(25)^2 + 40.4(25) + 1636$$
$$= 3250 + 1010 + 1636 = 5896$$

The annual dormitory charge will be \$5896.

95. **a.**
$$W(t) = 143t + 6580$$
$$W(0) = 143(0) + 6580$$
$$= 6580$$
$$W(5) = 143(5) + 6580$$
$$= 715 + 6580$$
$$= 7295$$
$$W(10) = 143(10) + 6580$$
$$= 1430 + 6580$$
$$= 8010$$

b. $W(10) = 8010$ means that in Year 10, 8010 thousand (8,010,000) women were due in child support.

97. **a.**
$$x(t) = 25t$$
$$y(t) = -16t^2 + 43.3t$$
$$x(0) = 25(0) = 0$$
$$y(0) = -16(0)^2 + 43.3(0) = 0 + 0 = 0$$

(0, 0); at $t = 0$ sec, the position of the rocket is at the origin.

b.
$$x(1) = 25(1) = 25$$
$$y(1) = -16(1)^2 + 43.3(1)$$
$$= -16 + 43.3 = 27.3$$

(25, 27.3) At $t = 1$ sec, the position of the rocket is (25, 27.3).

c. $x(2) = 25(2)$
$= 50$

$y(2) = -16(2)^2 + 43.3(2)$
$= -64 + 86.6$
$= 22.6$

(50, 22.6) At $t = 2$ sec, the position of the rocket is (50, 22.6).

Section 7.3 Practice Exercises

1. **a.** distributive

b. $4x - 7$

c. squares; $a^2 - b^2$

d. perfect; $a^2 + 2ab + b^2$

3.
$$\begin{aligned}(-2-3x)-\left[5-\left(6x^2+4x+1\right)\right] &= -2-3x-\left[5-6x^2-4x-1\right] \\ &= -2-3x-\left[-6x^2-4x+4\right] \\ &= -2-3x+6x^2+4x-4 \\ &= 6x^2+x-6\end{aligned}$$

5. **a.** $g(x) = x^4 - x^2 - 3$

$g(-1) = (-1)^4 - (-1)^2 - 3$
$= 1 - 1 - 3$
$= -3$

b. $g(2) = (2)^4 - (2)^2 - 3 = 16 - 4 - 3 = 9$

c. $g(0) = (0)^4 - (0)^2 - 3 = 0 - 0 - 3 = -3$

7.
$$\begin{aligned}\left(7x^4y\right)\left(-6xy^5\right) &= 7(-6)\left(x^4 \cdot x\right)\left(y \cdot y^5\right) \\ &= -42x^5y^6\end{aligned}$$

9. $\left(2.2a^6b^4c^7\right)\left(5ab^4c^3\right) = 11a^7b^8c^{10}$

11. $\frac{1}{5}(2a-3) = \frac{1}{5}(2a) + \frac{1}{5}(-3) = \frac{2}{5}a - \frac{3}{5}$

13.
$$\begin{aligned}&2m^3n^2\left(m^2n^3 - 3m^2 + 4n\right) \\ &= 2m^3n^2\left(m^2n^3\right) - 2m^3n^2\left(3m^2\right) + 2m^3n^2(4n) \\ &= 2m^5n^5 - 6m^4n^4 + 8m^3n^3\end{aligned}$$

15.
$$\begin{aligned}6xy^2\left(\frac{1}{2}x - \frac{2}{3}xy\right) &= 6xy^2\left(\frac{1}{2}x\right) - 6xy^2\left(\frac{2}{3}xy\right) \\ &= 3x^2y^2 - 4x^2y^3\end{aligned}$$

17. $(x+y)(x-2y)$

$$\begin{aligned}&= x(x) - x(2y) + y(x) - y(2y)\\ &= x^2 - 2xy + xy - 2y^2\\ &= x^2 - xy - 2y^2\end{aligned}$$

19. $(6x-1)(5+2x)$

$$\begin{aligned}&= 6x(5) + 6x(2x) - 1(5) - 1(2x)\\ &= 30x + 12x^2 - 5 - 2x\\ &= 12x^2 + 28x - 5\end{aligned}$$

21. $(y^2-12)(2y^2+3)$

$$\begin{aligned}&= y^2(2y^2) + y^2(3) - 12(2y^2) - 12(3)\\ &= 2y^4 + 3y^2 - 24y^2 - 36\\ &= 2y^4 - 21y^2 - 36\end{aligned}$$

23. $(5s+3t)(5s-2t)$

$$\begin{aligned}&= 5s(5s) - 5s(2t) + 3t(5s) - 3t(2t)\\ &= 25s^2 - 10st + 15st - 6t^2\\ &= 25s^2 + 5st - 6t^2\end{aligned}$$

25. $(n^2+10)(5n+3)$

$$\begin{aligned}&= n^2(5n) + n^2(3) + 10(5n) + 10(3)\\ &= 5n^3 + 3n^2 + 50n + 30\end{aligned}$$

27. $(1.3a-4b)(2.5a+7b)$

$$\begin{aligned}&= 1.3a(2.5a) + 1.3a(7b) - 4b(2.5a) - 4b(7b)\\ &= 3.25a^2 + 9.1ab - 10ab - 28b^2\\ &= 3.25a^2 - 0.9ab - 28b^2\end{aligned}$$

29. $(2x+y)(3x^2+2xy+y^2)$

$$\begin{aligned}&= 2x(3x^2) + 2x(2xy) + 2x(y^2) + y(3x^2) + y(2xy) + y(y^2)\\ &= 6x^3 + 4x^2y + 2xy^2 + 3x^2y + 2xy^2 + y^3\\ &= 6x^3 + 7x^2y + 4xy^2 + y^3\end{aligned}$$

31. $(x-7)(x^2+7x+49)$

$$\begin{aligned}&= x(x^2) + x(7x) + x(49) - 7(x^2) - 7(7x) - 7(49)\\ &= x^3 + 7x^2 + 49x - 7x^2 - 49x - 343\\ &= x^3 - 343\end{aligned}$$

33. $(4a-b)(a^3-4a^2b+ab^2-b^3)$

$$\begin{aligned}&= 4a(a^3) - 4a(4a^2b) + 4a(ab^2) - 4a(b^3) - b(a^3) + b(4a^2b) - b(ab^2) + b(b^3)\\ &= 4a^4 - 16a^3b + 4a^2b^2 - 4ab^3 - a^3b + 4a^2b^2 - ab^3 + b^4\\ &= 4a^4 - 17a^3b + 8a^2b^2 - 5ab^3 + b^4\end{aligned}$$

35. $\left(\frac{1}{2}a-2b+c\right)\left(a+6b-c\right)$

$$=\frac{1}{2}a\left(a\right)+\frac{1}{2}a\left(6b\right)-\frac{1}{2}a\left(c\right)-2b\left(a\right)-2b\left(6b\right)+2b\left(c\right)+c\left(a\right)+c\left(6b\right)-c\left(c\right)$$
$$=\frac{1}{2}a^2+3ab-\frac{1}{2}ac-2ab-12b^2+2bc+ac+6bc-c^2$$
$$=\frac{1}{2}a^2+ab+\frac{1}{2}ac-12b^2+8bc-c^2$$

37. $\left(-x^2+2x+1\right)\left(3x-5\right)=-x^2\left(3x\right)+x^2\left(5\right)+2x\left(3x\right)-2x\left(5\right)+1\left(3x\right)-1\left(5\right)$

$$=-3x^3+5x^2+6x^2-10x+3x-5$$
$$=-3x^3+11x^2-7x-5$$

39. $\left(\frac{1}{5}y-10\right)\left(\frac{1}{2}y-15\right)$

$$=\frac{1}{5}y\left(\frac{1}{2}y\right)+\frac{1}{5}y\left(-15\right)-10\left(\frac{1}{2}y\right)-10\left(-15\right)$$
$$=\frac{1}{10}y^2-3y-5y+150=\frac{1}{10}y^2-8y+150$$

41. $\left(a-8\right)\left(a+8\right)=a^2-8^2$

$$=a^2-64$$

43. $\left(3p+1\right)\left(3p-1\right)=\left(3p\right)^2-1^2=9p^2-1$

45. $\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)=x^2-\left(\frac{1}{3}\right)^2$

$$=x^2-\frac{1}{9}$$

47. $\left(3h-k\right)\left(3h+k\right)=\left(3h\right)^2-k^2$

$$=9h^2-k^2$$

49. $\left(3h-k\right)^2=\left(3h\right)^2-2\left(3h\right)\left(k\right)+k^2$

$$=9h^2-6hk+k^2$$

51. $\left(t-7\right)^2=t^2-2\left(t\right)\left(7\right)+7^2$

$$=t^2-14t+49$$

53. $\left(u+3v\right)^2=u^2+2\left(u\right)\left(3v\right)+\left(3v\right)^2$

$$=u^2+6uv+9v^2$$

55. $\left(h+\frac{1}{6}k\right)^2=h^2+2\left(h\right)\left(\frac{1}{6}k\right)+\left(\frac{1}{6}k\right)^2$

$$=h^2+\frac{1}{3}hk+\frac{1}{36}k^2$$

57. $\left(2z^2-w^3\right)\left(2z^2+w^3\right)=\left(2z^2\right)^2-\left(w^3\right)^2$

$$=4z^4-w^6$$

59. $\left(5x^2-3y\right)^2=\left(5x^2\right)^2-2\left(5x^2\right)\left(3y\right)+\left(3y\right)^2=25x^4-30x^2y+9y^2$

61. **a.** When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-5x+4)(5x+4)$
$=-25x^2-20x+20x+16$
$=16-25x^2$

Since $(-5x+4)(5x+4)=16-25x^2$ is a difference of squares, the binomials are conjugates.

b. When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-5x+4)(5x-4)$
$=-25x^2+20x+20x+16$
$=-25x^2+40x+16$

Since $(-5x+4)(5x-4)=-25x^2+40x+16$ is not a difference of squares, the binomials are not conjugates.

63. **a.** $\left(A-B\right)\left(A+B\right)=A^2-B^2$

b. $\left[\left(x+y\right)-B\right]\left[\left(x+y\right)+B\right]$
$=\left(x+y\right)^2-B^2$
$=x^2+2xy+y^2-B^2$

Both are examples of multiplying conjugates to get a difference of squares.

65. $\left[\left(w+v\right)-2\right]\left[\left(w+v\right)+2\right]=\left(w+v\right)^2-2^2$
$=w^2+2wv+v^2-4$

67. $\left[2-\left(x+y\right)\right]\left[2+\left(x+y\right)\right]=2^2-\left(x+y\right)^2$
$=4-\left(x^2+2xy+y^2\right)$
$=4-x^2-2xy-y^2$

69. $\left[\left(3a-4\right)+b\right]\left[\left(3a-4\right)-b\right]$
$=\left(3a-4\right)^2-b^2$
$=\left(3a\right)^2-2\left(3a\right)\left(4\right)+4^2-b^2$
$=9a^2-24a+16-b^2$

71. Write $\left(x+y\right)^3$ as $\left(x+y\right)^2\left(x+y\right)$. Square the binomial and then use the distributive property to multiply the resulting trinomial by the remaining factor of $x+y$.

73. $\begin{aligned}(2x+y)^3 &= (2x+y)^2(2x+y)\\ &= (4x^2+4xy+y^2)(2x+y)\\ &= 4x^2(2x)+4x^2(y)+4xy(2x)+4xy(y)+y^2(2x)+y^2(y)\\ &= 8x^3+4x^2y+8x^2y+4xy^2+2xy^2+y^3\\ &= 8x^3+12x^2y+6xy^2+y^3\end{aligned}$

75. $\begin{aligned}(4a-b)^3 &= (4a-b)^2(4a-b)\\ &= (16a^2-8ab+b^2)(4a-b)\\ &= 16a^2(4a)-16a^2(b)-8ab(4a)+8ab(b)+b^2(4a)-b^2(b)\\ &= 64a^3-16a^2b-32a^2b+8ab^2+4ab^2-b^3\\ &= 64a^3-48a^2b+12ab^2-b^3\end{aligned}$

77. Multiply the first two binomials and simplify.

Then multiply the resulting trinomial and the third binomial, using the distributive property.

79. $\begin{aligned}&2a^2(a+5)(3a+1)\\ &= 2a^2\left[a(3a)+a(1)+5(3a)+5(1)\right]\\ &= 2a^2\left[3a^2+a+15a+5\right]\\ &= 2a^2\left(3a^2+16a+5\right)\\ &= 2a^2\left(3a^2\right)+2a^2(16a)+2a^2(5)\\ &= 6a^4+32a^3+10a^2\end{aligned}$

81. $\begin{aligned}&(x+3)(x-3)(x+5)\\ &= (x^2-9)(x+5)\\ &= x^2(x)+x^2(5)-9(x)-9(5)\\ &= x^3+5x^2-9x-45\end{aligned}$

83. $\begin{aligned}&128p^6+54q^3\\ &= 2\left(64p^6+27q^3\right)\\ &= 2\left[\left(4p^2\right)^3+(3q)^3\right]\\ &= 2\left(4p^2+3q\right)\left(16p^4-12p^2q+9q^2\right)\end{aligned}$

85. $\begin{aligned}(y+1)^2-(2y+3)^2 &= \left(y^2+2y+1\right)-\left(4y^2+12y+9\right)\\ &= y^2+2y+1-4y^2-12y-9\\ &= -3y^2-10y-8\end{aligned}$

87. $(r+t)^2$

89. x^2-y^3

91. The sum of the cube of p and the square of q.

93. The product of x and the square of y.

95. Let x = the width of the walk

$2x + 20$ = length of garden and walk

$2x + 15$ = width of garden and walk

$$\begin{aligned} A(x) &= (2x+20)(2x+15) \\ &= 2x(2x)+2x(15)+20(2x)+20(15) \\ &= 4x^2+30x+40x+300 \\ &= 4x^2+70x+300 \end{aligned}$$

97. a. Let x = the length of a side of the square

$8 - 2x$ = length and width of base

x = the height of the box

$$\begin{aligned} V(x) &= (8-2x)(8-2x)x \\ &= \left(64-32x+4x^2\right)x \\ &= 4x^3-32x^2+64x \end{aligned}$$

b. $$\begin{aligned} V(1) &= 4(1)^3-32(1)^2+64(1) \\ &= 4-32+64 \\ &= 36 \text{ in}^3 \end{aligned}$$

99. $$\begin{aligned} (x-2)^2 &= x^2-2(x)(2)+2^2 \\ &= x^2-4x+4 \end{aligned}$$

101. $$\begin{aligned} (x-2)(x+2) &= x^2-2^2 \\ &= x^2-4 \end{aligned}$$

103. $$\begin{aligned} \frac{1}{2}(2x-6)(x+3) &= (x-3)(x+3) \\ &= x^2-3^2 \\ &= x^2-9 \end{aligned}$$

105. $$\begin{aligned} x(3x)(3x+10) &= 3x^2(3x+10) \\ &= 3x^2(3x)+3x^2(10) \\ &= 9x^3+30x^2 \end{aligned}$$

107. $$\begin{aligned} \frac{\left[(x+h)^2-3(x+h)-5\right]-\left(x^2-3x-5\right)}{h} &= \frac{x^2+2xh+h^2-3x-3h-5-x^2+3x+5}{h} \\ &= \frac{x^2-x^2+2xh+h^2-3x+3x-3h-5+5}{h} \\ &= \frac{2xh+h^2-3h}{h} \\ &= \frac{h(2x+h-3)}{h} \\ &= 2x+h-3 \end{aligned}$$

109. Multiply $(x+2)^2(x+2)^2$ by squaring the binomials.

Then multiply the resulting trinomials using the distributive property.

111. $(5x-6)$

Check:

$$(2x-3)(5x-6)$$
$$= 2x(5x)-2x(6)-3(5x)+3(6)$$
$$= 10x^2-12x-15x+18$$
$$= 10x^2-27x+18$$

113. $(2y-1)$

Check:

$$(4y+3)(2y-1)$$
$$= 4y(2y)-4y(1)+3(2y)-3(1)$$
$$= 8y^2-4y+6y-3$$
$$= 8y^2+2y-3$$

Section 7.4 Practice Exercises

1. **a.** division; quotient; remainder

b. Synthetic

3. **a.** $(a-10b)-(5a+b) = a-10b-5a-b$
$= -4a-11b$

b. $(a-10b)(5a+b)$
$= a(5a)+a(b)-10b(5a)-10b(b)$
$= 5a^2+ab-50ab-10b^2$
$= 5a^2-49ab-10b^2$

5. **a.** $(x^2-x)+(6x^2+x+2)$
$= x^2+6x^2-x+x+2$
$= 7x^2+2$

b. $(x^2-x)(6x^2+x+2)$
$= x^2(6x^2)+x^2(x)+x^2(2)$
$-x(6x^2)-x(x)-x(2)$
$= 6x^4+x^3+2x^2-6x^3-x^2-2x$
$= 6x^4-5x^3+x^2-2x$

7. For example:

$$(5y+1)^2 = (5y)^2+2(5y)(1)+1^2$$
$$= 25y^2+10y+1$$

9. $\dfrac{16t^4-4t^2+20t}{-4t} = \dfrac{16t^4}{-4t}-\dfrac{4t^2}{-4t}+\dfrac{20t}{-4t}$

$$= -4t^3+t-5$$

11. $\left(36y+24y^2+6y^3\right)\div\left(3y\right)$

$$=\frac{36y}{3y}+\frac{24y^2}{3y}+\frac{6y^3}{3y}$$

$$=12+8y+2y^2$$

13. $\left(4x^3y+12x^2y^2-4xy^3\right)\div\left(4xy\right)$

$$=\frac{4x^3y}{4xy}+\frac{12x^2y^2}{4xy}-\frac{4xy^3}{4xy}$$

$$=x^2+3xy-y^2$$

15. $\left(-8y^4-12y^3+32y^2\right)\div\left(-4y^2\right)$

$$=\frac{-8y^4}{-4y^2}-\frac{12y^3}{-4y^2}+\frac{32y^2}{-4y^2}$$

$$=2y^2+3y-8$$

17. $\left(3p^4-6p^3+2p^2-p\right)\div\left(-6p\right)$

$$=\frac{3p^4}{-6p}-\frac{6p^3}{-6p}+\frac{2p^2}{-6p}-\frac{p}{-6p}$$

$$=-\frac{1}{2}p^3+p^2-\frac{1}{3}p+\frac{1}{6}$$

19. $\left(a^3+5a^2+a-5\right)\div\left(a\right)$

$$=\frac{a^3}{a}+\frac{5a^2}{a}+\frac{a}{a}-\frac{5}{a}$$

$$=a^2+5a+1-\frac{5}{a}$$

21. $\dfrac{6s^3t^5-8s^2t^4+10st^2}{-2st^4}$

$$=\frac{6s^3t^5}{-2st^4}-\frac{8s^2t^4}{-2st^4}+\frac{10st^2}{-2st^4}$$

$$=-3s^2t+4s-\frac{5}{t^2}$$

23. $\left(8p^4q^7-9p^5q^6-11p^3q-4\right)\div\left(p^2q\right)$

$$=\frac{8p^4q^7}{p^2q}-\frac{9p^5q^6}{p^2q}-\frac{11p^3q}{p^2q}-\frac{4}{p^2q}$$

$$=8p^2q^6-9p^3q^5-11p-\frac{4}{p^2q}$$

25. a.

$$
\begin{array}{r}
2x^2-3x-1 \\
x-2\overline{\big)\ 2x^3-7x^2+5x-1} \\
\underline{-\left(2x^3-4x^2\right)} \\
-3x^2+5x \\
\underline{-\left(-3x^2+6x\right)} \\
-x-1 \\
\underline{-\left(-x+2\right)} \\
-3
\end{array}
$$

Divisor: $(x-2)$ Quotient: $(2x^2-3x-1)$

Remainder: (–3)

b. Multiply the quotient and divisor; then add the remainder.

The result should equal the dividend.

27.

$$
\begin{array}{r}
x+7 \\
x+4\overline{\big)\ x^2+11x+19} \\
\underline{-\left(x^2+4x\right)} \\
7x+19 \\
\underline{-\left(7x+28\right)} \\
-9
\end{array}
$$

Solution: $x+7-\dfrac{9}{x+4}$

Check:

$$\left(x+4\right)\left(x+7\right)+\left(-9\right)=x^2+11x+28-9$$
$$=x^2+11x+19$$

29.

$$
\begin{array}{r}
3y^2+2y+2 \\
y-3\overline{\big)\ 3y^3-7y^2-4y+3} \\
\underline{-\left(3y^3-9y^2\right)} \\
2y^2-4y \\
\underline{-\left(2y^2-6y\right)} \\
2y+3 \\
\underline{-\left(2y-6\right)} \\
9
\end{array}
$$

Solution: $3y^2+2y+2+\dfrac{9}{y-3}$

Check:

$$\left(y-3\right)\left(3y^2+2y+2\right)+\left(9\right)$$
$$=3y^3+2y^2+2y-9y^2-6y-6+9$$
$$=3y^3-7y^2-4y+3$$

31.

$$\begin{array}{r} -4a+11 \\ 3a-11\overline{)\ -12a^2+77a-121} \\ \underline{-\left(-12a^2+44a\right)} \\ 33a-121 \\ \underline{-\left(33a-121\right)} \\ 0 \end{array}$$

Solution: $-4a+11$

Check:

$$\begin{aligned}&\left(3a-11\right)\left(-4a+11\right)+\left(0\right)\\ &\quad=-12a^2+33a+44a-121\\ &\quad=-12a^2+77a-121\end{aligned}$$

33.

$$\begin{array}{r} 6y-5 \\ 3y+4\overline{)\ 18y^2+9y-20} \\ \underline{-\left(18y^2+24y\right)} \\ -15y-20 \\ \underline{-\left(-15y-20\right)} \\ 0 \end{array}$$

Solution: $6y-5$

Check:

$$\begin{aligned}&\left(3y+4\right)\left(6y-5\right)+\left(0\right)\\ &\quad=18y^2-15y+24y-20\\ &\quad=18y^2+9y-20\end{aligned}$$

35.

$$\begin{array}{r} 6x^2+4x+5 \\ 3x-2\overline{)\ 18x^3\quad +7x+12} \\ \underline{-\left(18x^3-12x^2\right)} \\ 12x^2+7x \\ \underline{-\left(12x^2-8x\right)} \\ 15x+12 \\ \underline{-\left(15x-10\right)} \\ 22 \end{array}$$

Solution: $6x^2+4x+5+\dfrac{22}{3x-2}$

Check:

$$\begin{aligned}&\left(3x-2\right)\left(6x^2+4x+5\right)+\left(22\right)\\ &=18x^3+12x^2+15x-12x^2-8x-10+22\\ &=18x^3+7x+12\end{aligned}$$

37.

$$\begin{array}{r} 4a^2-2a+1 \\ 2a+1\overline{)\ 8a^3\quad +1} \\ \underline{-\left(8a^3+4a^2\right)} \\ -4a^2 \\ \underline{-\left(-4a^2-2a\right)} \\ 2a+1 \\ \underline{-\left(2a+1\right)} \\ 0 \end{array}$$

Solution: $4a^2-2a+1$

Check:

$$\begin{aligned}&\left(2a+1\right)\left(4a^2-2a+1\right)+\left(0\right)\\ &\quad=8a^3-4a^2+2a+4a^2-2a+1\\ &\quad=8a^3+1\end{aligned}$$

39.

$$\begin{array}{r} x^2-2x+2 \\ x^2+x-1\overline{\left)\begin{array}{r} x^4-x^3-x^2+4x-2 \\ -\left(x^4+x^3-x^2\right) \\ \hline -2x^3\quad +4x \\ -\left(-2x^3-2x^2+2x\right) \\ \hline 2x^2+2x-2 \\ -\left(2x^2+2x-2\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: x^2-2x+2

Check:

$$\begin{aligned}\left(x^2+x-1\right)\left(x^2-2x+2\right)+(0) &= x^4-2x^3+2x^2+x^3-2x^2+2x \\ &\quad -x^2+2x-2 \\ &= x^4-x^3-x^2+4x-2\end{aligned}$$

41.

$$\begin{array}{r} x^2+2x+5 \\ x^2-5\overline{\left)\begin{array}{r} x^4+2x^3\quad -10x-25 \\ -\left(x^4\quad -5x^2\right) \\ \hline 2x^3+5x^2-10x \\ -\left(2x^3\quad -10x\right) \\ \hline 5x^2\quad -25 \\ -\left(5x^2\quad -25\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: x^2+2x+5

Check:

$$\begin{aligned}\left(x^2-5\right)\left(x^2+2x+5\right)+(0) &= x^4+2x^3+5x^2-5x^2-10x-25 \\ &= x^4+2x^3-10x-25\end{aligned}$$

43.

$$\begin{array}{r} x^2-1 \\ x^2-2\overline{\left)\begin{array}{r} x^4-3x^2+10 \\ -\left(x^4-2x^2\right) \\ \hline -x^2+10 \\ -\left(-x^2+\ 2\right) \\ \hline 8 \end{array}\right.} \end{array}$$

Solution: $x^2-1+\dfrac{8}{x^2-2}$

Check: $\left(x^2-2\right)\left(x^2-1\right)+(8)$

$$\begin{aligned} &= x^4-x^2-2x^2+2+8 \\ &= x^4-3x^2+10\end{aligned}$$

45.

$$\begin{array}{r} n^3+2n^2+4n+8 \\ n-2\overline{\left)\begin{array}{r} n^4\quad\quad\quad -16 \\ -\left(n^4-2n^3\right) \\ \hline 2n^3 \\ -\left(2n^3-4n^2\right) \\ \hline 4n^2 \\ -\left(4n^2-8n\right) \\ \hline 8n-16 \\ -\left(8n-16\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: n^3+2n^2+4n+8

Check:

$$(n-2)\left(n^3+2n^2+4n+8\right)+(0)$$
$$= n^4+2n^3+4n^2+8n-2n^3-4n^2$$
$$-8n-16$$
$$= n^4-16$$

47. The divisor must be of the form $x-r$.

49. No, the divisor is not of the form $x-r$.

51. **a.** Divisor: $x-5$

b. Quotient: $x^2+3x+11$

c. Remainder: 58

53.

8 |	1	−2	−48
		8	48
	1	6	|0

Quotient: $x+6$

Check:

$$(x-8)(x+6)+(0) = x^2+6x-8x-48$$
$$= x^2-2x-48$$

55.

−1 |	1	−3	−4
		−1	4
	1	−4	|0

Quotient: $t-4$

Check:

$$(t+1)(t-4)+(0) = t^2-4t+t-4$$
$$= t^2-3t-4$$

57.

1 |	5	5	1
		5	10
	5	10	|11

Quotient: $5y+10+\dfrac{11}{y-1}$

Check:

$$(y-1)(5y+10)+(11)$$
$$= 5y^2+10y-5y-10+11$$
$$= 5y^2+5y+1$$

59.

−3 |	3	7	−4	3
		−9	6	−6
	3	−2	2	|−3

Quotient: $3y^2-2y+2+\dfrac{-3}{y+3}$

61.

2 |	1	−3	0	4
		2	−2	−4
	1	−1	−2	|0

Quotient: x^2-x-2

Check:

$$(y+3)\left(3y^2-2y+2\right)+(-3)$$
$$=3y^3-2y^2+2y+9y^2-6y+6-3$$
$$=3y^3+7y^2-4y+3$$

63.

2 |	1	0	0	0	0	−32
		2	4	8	16	32
	1	2	4	8	16	| 0

Quotient: $a^4+2a^3+4a^2+8a+16$

Check:

$$(a-2)\left(a^4+2a^3+4a^2+8a+16\right)+(0)$$
$$=a^5+2a^4+4a^3+8a^2+16a$$
$$-2a^4-4a^3-8a^2-16a-32$$
$$=a^5-32$$

65.

6 |	1	0	0	−216	
		6	36	216	
	1	6	36	| 0	

Quotient: $x^2+6x+36$

Check:

$$(x-6)\left(x^2+6x+36\right)+(0)$$
$$=x^3+6x^2+36x-6x^2-36x-216$$
$$=x^3-216$$

67.

$-\frac{2}{3}$ |	6	7	−1	3
		−4	−2	2
	6	3	−3	| 5

Quotient: $6t^2+3t-3+\dfrac{5}{t+\frac{2}{3}}$

Check:

$$\left(t+\frac{2}{3}\right)\left[(6t^2+3t-3)+\frac{5}{t+\frac{2}{3}}\right]$$
$$=\left(t+\frac{2}{3}\right)(6t^2+3t-3)+\left(t+\frac{2}{3}\right)\left(\frac{5}{t+\frac{2}{3}}\right)$$
$$=6t^3+3t^2-3t+4t^2+2t-2+5$$
$$=6t^3+7t^2-t+3$$

69.

$\frac{1}{2}$ |	4	0	−1	6	−3
		2	1	0	3
	4	2	0	6	| 0

Quotient: $4w^3+2w^2+6$

Check:

$$\left(w-\frac{1}{2}\right)\left(4w^3+2w^2+6\right)+(0)$$
$$=4w^4+2w^3+6w-2w^3-w^2-3$$
$$=4w^4-w^2+6w-3$$

71.
$$\begin{array}{r|rrrr} -4 & -1 & -8 & -3 & -2 \\ & & 4 & 16 & -52 \\ \hline & -1 & -4 & 13 & -54 \end{array}$$

Quotient: $-x^2-4x+13+\dfrac{-54}{x+4}$

73. $\left(22x^2-11x+33\right)\div\left(11x\right)$

$$=\frac{22x^2}{11x}-\frac{11x}{11x}+\frac{33}{11x}=2x-1+\frac{3}{x}$$

75.
$$\begin{array}{r} 4y-3 \\ 3y^2-2y+5\overline{)\ 12y^3-17y^2+30y-10} \\ \underline{-\left(12y^3-8y^2+20y\right)} \\ -9y^2+10y-10 \\ \underline{-\left(-9y^2+6y-15\right)} \\ 4y+5 \end{array}$$

Quotient: $4y-3+\dfrac{4y+5}{3y^2-2y+5}$

77.
$$\begin{array}{r} 2x^2+3x-1 \\ 2x^2+1\overline{)\ 4x^4+6x^3\quad +3x-1} \\ \underline{-\left(4x^4\quad +2x^2\right)} \\ 6x^3-2x^2+3x \\ \underline{-\left(6x^3\quad +3x\right)} \\ -2x^2\quad -1 \\ \underline{-\left(-2x^2\quad -1\right)} \\ 0 \end{array}$$

Quotient: $2x^2+3x-1$

79. $\left(16k^{11}-32k^{10}+8k^8-40k^4\right)\div\left(8k^8\right)$

$$=\frac{16k^{11}}{8k^8}-\frac{32k^{10}}{8k^8}+\frac{8k^8}{8k^8}-\frac{40k^4}{8k^8}$$
$$=2k^3-4k^2+1-\frac{5}{k^4}$$

81. $\left(5x^3+9x^2+10x\right)\div\left(5x^2\right)$

$$=\frac{5x^3}{5x^2}+\frac{9x^2}{5x^2}+\frac{10x}{5x^2}$$
$$=x+\frac{9}{5}+\frac{2}{x}$$

83. **a.** $P\left(-4\right)=4\left(-4\right)^3+10\left(-4\right)^2-8\left(-4\right)-20$

$$=4\left(-64\right)+10\left(16\right)+32-20$$
$$=-256+160+32-20$$
$$=-84$$

b.
$$\begin{array}{r|rrrr} -4 & 4 & 10 & -8 & -20 \\ & & -16 & 24 & -64 \\ \hline & 4 & -6 & 16 & -84 \end{array}$$

Quotient: $4x^2-6x+16+\dfrac{-84}{x+4}$

c. The values are the same.

85. $P\left(r\right)$ equals the remainder of $P\left(x\right)\div\left(x-r\right)$.

87. **a.**

$$\begin{array}{r|rrr} -1 & 8 & 13 & 5 \\ & & -8 & -5 \\ \hline & 8 & 5 & 0 \end{array}$$

Quotient: $8x+5$

b. Yes Yes

Problem Recognition Exercises

1. **a.** $(3x+1)^2 = (3x)^2 + 2(3x)(1) + 1^2$
$= 9x^2 + 6x + 1$

b $(3x+1)(3x-1) = (3x)^2 - 1^2$
$= 9x^2 - 1$

c. $(3x+1) - (3x-1) = 3x + 1 - 3x + 1$
$= 2$

3. **a.** $\dfrac{4x^2+8x-10}{2x} = \dfrac{4x^2}{2x} + \dfrac{8x}{2x} - \dfrac{10}{2x}$
$= 2x + 4 - \dfrac{5}{x}$

b.

$$\begin{array}{r} 2x+5 \\ 2x-1\overline{)\ 4x^2+8x-10} \\ \underline{-(4x^2-2x)} \\ 10x-10 \\ \underline{-(10x-5)} \\ -5 \end{array}$$

Solution: $2x+5+\dfrac{-5}{2x-1}$

c.

$$\begin{array}{r|rrr} 1 & 4 & 8 & -10 \\ & & 4 & 12 \\ \hline & 4 & 12 & 2 \end{array}$$

Quotient: $4x+12+\dfrac{2}{x-1}$

5. **a.** $(p-5)(p+5) - (p^2+5)$
$= p^2 - 25 - p^2 - 5$
$= -30$

b. $(p-5)(p+5) - (p+5)^2$
$= p^2 - 25 - p^2 - 10p - 25$
$= -10p - 50$

c. $(p-5)(p+5) - (p^2-25)$
$= p^2 - 25 - p^2 + 25 = 0$

7. $(5t^2 - 6t + 2) - (3t^2 - 7t + 3) = 5t^2 - 6t + 2 - 3t^2 + 7t - 3$
$= 2t^2 + t - 1$

9. $(6z+5)(6z-5) = (6z)^2 - 5^2$
$= 36z^2 - 25$

11. $(3b-4)(2b-1)$
$= 3b(2b) - 3b(1) - 4(2b) + 4(1)$
$= 6b^2 - 3b - 8b + 4$
$= 6b^2 - 11b + 4$

13. $(t^3 - 4t^2 + t - 9) + (t + 12) - (2t^2 - 6t)$
$= t^3 - 4t^2 + t - 9 + t + 12 - 2t^2 + 6t$
$= t^3 - 6t^2 + 8t + 3$

15. $(k+4)^2 + (-4k+9)$
$= k^2 + 2(k)(4) + 4^2 - 4k + 9$
$= k^2 + 8k + 16 - 4k + 9$
$= k^2 + 4k + 25$

17. $-2t(t^2 + 6t - 3) + t(3t+2)(3t-2)$
$= -2t^3 - 12t^2 + 6t + t(9t^2 - 4)$
$= -2t^3 - 12t^2 + 6t + 9t^3 - 4t$
$= 7t^3 - 12t^2 + 2t$

19. $\left(\frac{1}{4}p^3 - \frac{1}{6}p^2 + 5\right) - \left(-\frac{2}{3}p^3 + \frac{1}{3}p^2 - \frac{1}{5}p\right)$
$= \frac{3}{12}p^3 - \frac{1}{6}p^2 + 5 + \frac{8}{12}p^3 - \frac{2}{6}p^2 + \frac{1}{5}p$
$= \frac{11}{12}p^3 - \frac{1}{2}p^2 + \frac{1}{5}p + 5$

21. $(6a^2 - 4b)^2 = (6a^2)^2 - 2(6a^2)(4b) + (4b)^2$
$= 36a^4 - 48a^2b + 16b^2$

23. $(m-3)^2 - 2(m+8)$
$= m^2 - 6m + 9 - 2m - 16$
$= m^2 - 8m - 7$

25. $(m^2 - 6m + 7)(2m^2 + 4m - 3) = m^2(2m^2 + 4m - 3) - 6m(2m^2 + 4m - 3) + 7(2m^2 + 4m - 3)$
$= 2m^4 + 4m^3 - 3m^2 - 12m^3 - 24m^2 + 18m + 14m^2 + 28m - 21$
$= 2m^4 - 8m^3 - 13m^2 + 46m - 21$

27. $[5 - (a+b)]^2 = 5^2 - 2(5)(a+b) + (a+b)^2$
$= 25 - 10a - 10b + a^2 + 2ab + b^2$

29. $(x+y)^2 - (x-y)^2$
$= x^2 + 2xy + y^2 - (x^2 - 2xy + y^2)$
$= x^2 + 2xy + y^2 - x^2 + 2xy - y^2$
$= 4xy$

31. $\left(-\frac{1}{2}x + \frac{1}{3}\right)\left(\frac{1}{4}x - \frac{1}{2}\right) = -\frac{1}{8}x^2 + \frac{1}{4}x + \frac{1}{12}x - \frac{1}{6} = -\frac{1}{8}x^2 + \frac{1}{3}x - \frac{1}{6}$