Section 4.1 Practice Exercises

1. **a.** exponent

b. 1

c. $\left(\frac{1}{b}\right)^n$ or $\frac{1}{b^n}$

d. scientific notation

3. $ab^3 = a \cdot b \cdot b \cdot b$

$$\begin{aligned}(ab)^3 &= (ab)\cdot(ab)\cdot(ab)\\ &= a \cdot a \cdot a \cdot b \cdot b \cdot b\\ &= a^3 \cdot b^3\end{aligned}$$

5. For example: $(5x)^2 = 5^2 x^2$

$(xy)^3 = x^3 y^3$

7. For example: $\frac{x^5}{x^2} = x^3$

$\frac{8^4}{8^2} = 8^2$

9. For example: $6^0 = 1$

$x^0 = 1 \ (x \neq 0)$

11. $\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3$

13. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

15. $-5^{-2} = -\frac{1}{5^2} = -\frac{1}{25}$

17. $(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$

19. $\left(-\frac{1}{4}\right)^{-3} = \left(-\frac{4}{1}\right)^3 = (-4)^3 = -64$

21. $\left(-\frac{3}{2}\right)^{-4} = \left(-\frac{2}{3}\right)^4 = \frac{(-2)^4}{3^4} = \frac{16}{81}$

23. $-\left(\frac{2}{5}\right)^{-3} = -\left(\frac{5}{2}\right)^3 = -\frac{5^3}{2^3} = -\frac{125}{8}$

25. $(10ab)^0 = 1$

27. $10ab^0 = 10a \cdot 1 = 10a$

29. $y^3 \cdot y^5 = y^{3+5} = y^8$

31. $\frac{13^8}{13^6} = 13^{8-6} = 13^2 = 169$

33. $\left(y^2\right)^4 = y^{2\cdot 4} = y^8$

35. $\left(3x^2\right)^4 = 3^4\left(x^2\right)^4 = 3^4 x^{2\cdot 4} = 81x^8$

37. $p^{-3} = \dfrac{1}{p^3}$

39. $7^{10}\cdot 7^{-13} = 7^{10+(-13)} = 7^{-3} = \dfrac{1}{7^3}$

$= \dfrac{1}{343}$

41. $\dfrac{w^3}{w^5} = w^{3-5} = w^{-2} = \dfrac{1}{w^2}$

43. $a^{-2}a^{-5} = a^{-2+(-5)} = a^{-7} = \dfrac{1}{a^7}$

45. $\dfrac{r}{r^{-1}} = r^{1-(-1)} = r^2$

47. $\dfrac{z^{-6}}{z^{-2}} = z^{-6-(-2)} = z^{-4} = \dfrac{1}{z^4}$

49. $\dfrac{a^3}{b^{-2}} = a^3\cdot\dfrac{1}{b^{-2}} = a^3b^2$

51. $\left(6xyz^2\right)^0 = 1$

53. $2^4 + 2^{-2} = 2^4 + \dfrac{1}{2^2}$

$= 16 + \dfrac{1}{4}$

$= 16\frac{1}{4}$ or $\dfrac{65}{4}$

55. $1^{-2} + 5^{-2} = \dfrac{1}{1^2} + \dfrac{1}{5^2}$

$= \dfrac{1}{1} + \dfrac{1}{25}$

$= 1\frac{1}{25}$ or $\dfrac{26}{25}$

57. $\left(\dfrac{2}{3}\right)^{-2} - \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{3}\right)^0 = \left(\dfrac{3}{2}\right)^2 - \dfrac{1}{4} + 1$

$= \dfrac{9}{4} - \dfrac{1}{4} + \dfrac{4}{4}$

$= \dfrac{12}{4} = 3$

59. $\left(\dfrac{4}{5}\right)^{-1} + \left(\dfrac{3}{2}\right)^2 - \left(\dfrac{2}{7}\right)^0 = \dfrac{5}{4} + \dfrac{9}{4} - 1$

$= \dfrac{5}{4} + \dfrac{9}{4} - \dfrac{4}{4}$

$= \dfrac{10}{4} = \dfrac{5}{2}$

61. $\dfrac{p^2q}{p^5q^{-1}} = p^{2-5}q^{1-(-1)}$

$= p^{-3}q^2$

$= \dfrac{1}{p^3}\cdot q^2 = \dfrac{q^2}{p^3}$

63. $\dfrac{-48ab^{10}}{32a^4b^3} = -\dfrac{48}{32}a^{1-4}b^{10-3} = -\dfrac{3}{2}a^{-3}b^7$

$= -\dfrac{3}{2}\cdot\dfrac{1}{a^3}\cdot b^7 = -\dfrac{3b^7}{2a^3}$

65. $\left(-3x^{-4}y^5z^2\right)^{-4}$

$=\left(-3\right)^{-4}\left(x^{-4}\right)^{-4}\left(y^5\right)^{-4}\left(z^2\right)^{-4}$

$=\left(-\frac{1}{3}\right)^4 x^{16}y^{-20}z^{-8}$

$=\frac{1}{81}\cdot x^{16}\cdot\frac{1}{y^{20}}\cdot\frac{1}{z^8}=\frac{x^{16}}{81y^{20}z^8}$

67. $\left(4m^{-2}n\right)\left(-m^6n^{-3}\right)=-4m^{-2+6}n^{1+(-3)}$

$=-4m^4n^{-2}$

$=-4m^4\cdot\frac{1}{n^2}$

$=-\frac{4m^4}{n^2}$

69. $\left(p^{-2}q\right)^3\left(2pq^4\right)^2$

$=\left(p^{-2}\right)^3q^3\cdot2^2p^2\left(q^4\right)^2$

$=p^{-6}q^3\cdot4p^2q^8$

$=4p^{-6+2}q^{3+8}$

$=4p^{-4}q^{11}$

$=4\cdot\frac{1}{p^4}\cdot q^{11}=\frac{4q^{11}}{p^4}$

71. $\left(\frac{x^2}{y}\right)^3\left(5x^2y\right)=\frac{x^6}{y^3}\left(5x^2y\right)$

$=5x^{6+2}y^{1-3}$

$=5x^8y^{-2}$

$=5x^8\frac{1}{y^2}$

$=\frac{5x^8}{y^2}$

73. $\frac{\left(-8a^2b^2\right)^4}{\left(16a^3b^7\right)^2}=\frac{\left(-8\right)^4\left(a^2\right)^4\left(b^2\right)^4}{\left(16\right)^2\left(a^3\right)^2\left(b^7\right)^2}$

$=\frac{4096a^8b^8}{256a^6b^{14}}$

$=16a^{8-6}b^{8-14}$

$=16a^2b^{-6}$

$=16a^2\cdot\frac{1}{b^6}$

$=\frac{16a^2}{b^6}$

75. $\left(\frac{-2x^6y^{-5}}{3x^{-2}y^4}\right)^{-3}$

$=\left(-\frac{2}{3}x^{6-(-2)}y^{-5-4}\right)^{-3}$

$=\left(-\frac{2}{3}x^8y^{-9}\right)^{-3}$

$=\left(-\frac{2}{3}\right)^{-3}\left(x^8\right)^{-3}\left(y^{-9}\right)^{-3}$

$=\left(-\frac{3}{2}\right)^3x^{-24}y^{27}$

$=-\frac{27}{8}\cdot\frac{1}{x^{24}}\cdot y^{27}$

$=-\frac{27y^{27}}{8x^{24}}$

77. $\left(\dfrac{2x^{-3}y^{0}}{4x^{6}y^{-5}}\right)^{-2} = \left(\dfrac{1}{2}x^{-3-6}y^{0-(-5)}\right)^{-2}$

$= \left(\dfrac{1}{2}x^{-9}y^{5}\right)^{-2}$

$= \left(\dfrac{1}{2}\right)^{-2}\left(x^{-9}\right)^{-2}\left(y^{5}\right)^{-2}$

$= (2)^{2}x^{18}y^{-10}$

$= 4x^{18}\cdot\dfrac{1}{y^{10}}$

$= \dfrac{4x^{18}}{y^{10}}$

79. $3xy^{5}\left(\dfrac{2x^{4}y}{6x^{5}y^{3}}\right)^{-2}$

$= 3xy^{5}\left(\dfrac{1}{3}x^{4-5}y^{1-3}\right)^{-2}$

$= 3xy^{5}\left(\dfrac{1}{3}x^{-1}y^{-2}\right)^{-2}$

$= 3xy^{5}\left(\dfrac{1}{3}\right)^{-2}\left(x^{-1}\right)^{-2}\left(y^{-2}\right)^{-2}$

$= 3xy^{5}(3)^{2}x^{2}y^{4}$

$= 3\cdot 9x^{1+2}y^{5+4}$

$= 27x^{3}y^{9}$

81. **a.** $\$8,000,000,000 = \$8\times 10^{9}$

b. $3,000,000 = 3\times 10^{6}$ DVDs

c. $14,000,000,000,000 = 1.4\times 10^{13}$ eV

d. $0.000\,000\,000\,000\,000\,0001602$

$= 1.602\times 10^{-19}$ J

83. **a.** $2\times 10^{11} = 200,000,000,000$

b. $4\times 10^{-6} = 0.000004$

c. $1.082\times 10^{11} = 108,200,000,000$

85. $35\times 10^{4} = 3.5\times 10^{1}\times 10^{4}$

$= 3.5\times 10^{5}$

87. 7.0×10^{0} Proper

89. 9×10^{1} Proper

91. $\left(6.5\times 10^{3}\right)\left(5.2\times 10^{-8}\right) = 33.8\times 10^{3+(-8)}$

$= 3.38\times 10^{1}\times 10^{-5}$

$= 3.38\times 10^{-4}$

93. $(0.0000024)(6,700,000,000)$

$= \left(2.4\times 10^{-6}\right)\left(6.7\times 10^{9}\right)$

$= 16.08\times 10^{-6+9}$

$= 1.608\times 10^{1}\times 10^{3} = 1.608\times 10^{4}$

95. $\left(8.5\times 10^{-2}\right)\div\left(2.5\times 10^{-15}\right)$

$= 3.4\times 10^{-2-(-15)}$

$= 3.4\times 10^{13}$

97. $(900000000)\div(360000)$
$=\left(9\times10^{8}\right)\div\left(3.6\times10^{5}\right)$
$=2.5\times10^{8-5}$
$=2.5\times10^{3}$

99. $2\cdot\left(6.02\times10^{23}\right)=12.04\times10^{23}$
$=1.204\times10^{1}\times10^{23}$
$=1.204\times10^{24}$ hydrogen atoms
$1\cdot\left(6.02\times10^{23}\right)=6.02\times10^{23}$ oxygen atoms

101. $2{,}200{,}000\div110$
$=\left(2.2\times10^{6}\right)\div\left(1.1\times10^{2}\right)$
$=2\times10^{4}$ or $20{,}000$ people per mi^2

103. $\left(\$3.5\times10^{9}\right)(15)=\$52.5\times10^{9}$
$=\$5.25\times10^{10}$

105. **a.** $45\cdot12=540$ months

b. $\$20(540)=\$10{,}800$

c. $A=\$20\left[\left(1+\dfrac{0.06}{12}\right)^{540}-1\right]\left(1+\dfrac{12}{0.06}\right)=\$55{,}395.45$

107. $y^{a-5}y^{a+7}=y^{a-5+a+7}=y^{2a+2}$

109. $\dfrac{x^{3a-3}}{x^{a+1}}=x^{(3a-3)-(a+1)}$
$=x^{3a-3-a-1}=x^{2a-4}$

111. $\dfrac{x^{2a-2}y^{a+3}}{x^{a+4}y^{a-3}}=x^{(2a-2)-(a+4)}y^{(a+3)-(a-3)}=x^{2a-2-a-4}y^{a+3-a+3}=x^{a-6}y^{6}$

Section 4.2 Practice Exercises

1. **a.** polynomial
b. coefficient; n
c. 1; 1
d. one
e. binomial
f. trinomial
g. leading; leading coefficient
h. greatest
i. zero
j. exponents
k. polynomial

3. $\left(2ac^{-2}\right)\left(5a^{-1}c^{4}\right)=10a^{1+(-1)}c^{-2+4}$
$=10a^{0}c^{2}=10c^{2}$

5. $\left(3.4\times10^{5}\right)\left(5.0\times10^{-2}\right)=17\times10^{3}$
$=1.7\times10^{4}$

7. $-6a^3 + a^2 - a$

leading coefficient: –6

degree: 3

9. $3x^4 + 6x^2 - x - 1$

leading coefficient: 3

degree: 4

11. $-t^2 + 100$

leading coefficient: –1

degree: 2

13. For example: $3x^5$

15. For example: $x^2 + 2x + 1$

17. For example: $6x^4 - x^2$

19. $\left(-4m^2 + 4m\right) + \left(5m^2 + 6m\right)$

$= -4m^2 + 5m^2 + 4m + 6m$

$= m^2 + 10m$

21. $\left(3x^4 - x^3 - x^2\right) + \left(3x^3 - 7x^2 + 2x\right)$

$= 3x^4 + \left(-x^3\right) + 3x^3 + \left(-x^2\right) + \left(-7x^2\right) + 2x$

$= 3x^4 + 2x^3 - 8x^2 + 2x$

23. $\left(\frac{1}{2}w^3 + \frac{2}{9}w^2 - 1.8w\right) + \left(\frac{3}{2}w^3 - \frac{1}{9}w^2 + 2.7w\right)$

$= \frac{1}{2}w^3 + \frac{3}{2}w^3 + \frac{2}{9}w^2 - \frac{1}{9}w^2 - 1.8w + 2.7w$

$= 2w^3 + \frac{1}{9}w^2 + 0.9w$

25. $\left(9x^2y - 5xy + 1\right) + \left(8x^2y + xy - 15\right)$

$= 9x^2y + 8x^2y - 5xy + xy + 1 - 15$

$= 17x^2y - 4xy - 14$

27. $\left(-7a + 6a^2 + 1\right) + \left(-8 - 4a - 2a^2\right)$

$= 6a^2 - 2a^2 - 7a - 4a + 1 - 8$

$= 4a^2 - 11a - 7$

29.

$$\begin{array}{r} 12x^3 \qquad +6x - 8 \\ +\left(-3x^3 - 5x^2 - 4x \quad\right) \\ \hline 9x^3 - 5x^2 + 2x - 8 \end{array}$$

31. $-\left(-30y^3\right) = 30y^3$

33. $-\left(4p^3 + 2p - 12\right) = -4p^3 - 2p + 12$

35. $-\left(-11ab^2 + a^2b\right) = 11ab^2 - a^2b$

37. $\left(13z^5 - z^2\right) - \left(7z^5 + 5z^2\right)$

$= \left(13z^5 - z^2\right) + \left(-7z^5 - 5z^2\right)$

$= 13z^5 - 7z^5 - z^2 - 5z^2$

$= 6z^5 - 6z^2$

39. $\left(-3x^3+3x^2-x+6\right)-\left(1-x-x^2-x^3\right)$

$=\left(-3x^3+3x^2-x+6\right)+\left(-1+x+x^2+x^3\right)$

$=\left(-3x^3+3x^2-x+6\right)+\left(x^3+x^2+x-1\right)$

$=-3x^3+x^3+3x^2+x^2-x+x+6-1$

$=-2x^3+4x^2+5$

41. $\left(-3xy^3+3x^2y-x+6\right)-\left(-xy^3-xy-x+1\right)$

$=\left(-3xy^3+3x^2y-x+6\right)+\left(xy^3+xy+x-1\right)$

$=-3xy^3+xy^3+3x^2y+xy-x+x+6-1$

$=-2xy^3+3x^2y+xy+5$

43.
$$\begin{array}{rl} & 4t^3-6t^2 \quad -18 \\ \underline{-\left(3t^3+7t^2+9t-5\right)} & \rightarrow \underline{+\left(-3t^3-7t^2-9t+5\right)} \\ & t^3-13t^2-9t-13 \end{array}$$

45. $\left(\frac{1}{5}a^2-\frac{1}{2}ab+\frac{1}{10}b^2+3\right)-\left(-\frac{3}{10}a^2+\frac{2}{5}ab-\frac{1}{2}b^2-5\right)$

$=\left(\frac{1}{5}a^2-\frac{1}{2}ab+\frac{1}{10}b^2+3\right)+\left(\frac{3}{10}a^2-\frac{2}{5}ab+\frac{1}{2}b^2+5\right)$

$=\frac{1}{5}a^2+\frac{3}{10}a^2-\frac{1}{2}ab-\frac{2}{5}ab+\frac{1}{10}b^2+\frac{1}{2}b^2+3+5$

$=\frac{2}{10}a^2+\frac{3}{10}a^2-\frac{5}{10}ab-\frac{4}{10}ab+\frac{1}{10}b^2+\frac{5}{10}b^2+3+5$

$=\frac{1}{2}a^2-\frac{9}{10}ab+\frac{3}{5}b^2+8$

47. $\left(8x^2+x-15\right)-\left(9x^2-5x+1\right)$

$=\left(8x^2+x-15\right)+\left(-9x^2+5x-1\right)$

$=8x^2-9x^2+x+5x-15-1$

$=-x^2+6x-16$

49. $\left(3x^5-2x^3+4\right)-\left(x^4+2x^3-7\right)$

$=\left(3x^5-2x^3+4\right)+\left(-x^4-2x^3+7\right)$

$=3x^5-x^4-2x^3-2x^3+4+7$

$=3x^5-x^4-4x^3+11$

51. $\left(8y^2-4y^3\right)-\left(3y^2-8y^3\right)$

$=\left(8y^2-4y^3\right)+\left(-3y^2+8y^3\right)$

$=-4y^3+8y^3+8y^2-3y^2$

$=4y^3+5y^2$

53. $\left(-2r-6r^4\right)+\left(-r^4-9r\right)$

$=-6r^4-r^4-2r-9r$

$=-7r^4-11r$

55. $\left(5xy+13x^2+3y\right)-\left(4x^2-8y\right)$

$=\left(5xy+13x^2+3y\right)+\left(-4x^2+8y\right)$

$=13x^2-4x^2+5xy+3y+8y$

$=9x^2+5xy+11y$

57. $\left(11ab-23b^2\right)+\left(7ab-19b^2\right)$

$=11ab+7ab-23b^2-19b^2$

$=18ab-42b^2$

59. $\left[2p-\left(3p+5\right)\right]+\left(4p-6\right)+2$

$=\left[2p-3p-5\right]+\left(4p-6\right)+2$

$=-p-5+4p-6+2$

$=-p+4p-5-6+2$

$=3p-9$

61. $5-\left[2m^2-\left(4m^2+1\right)\right]$

$=5-\left[2m^2-4m^2-1\right]$

$=5-\left[-2m^2-1\right]$

$=5+2m^2+1=2m^2+6$

63. $\left(6x^3-5\right)-\left(-3x^3+2x\right)-\left(2x^3-6x\right)$

$=6x^3-5+3x^3-2x-2x^3+6x$

$=7x^3+4x-5$

65. $\left(-ab+5a^2b\right)-\left[7ab^2-2ab-\left(7a^2b+2ab^2\right)\right]=-ab+5a^2b-\left[7ab^2-2ab-7a^2b-2ab^2\right]$

$=-ab+5a^2b-\left[5ab^2-2ab-7a^2b\right]$

$=-ab+5a^2b-5ab^2+2ab+7a^2b$

$=12a^2b+ab-5ab^2$

67. $\left(8x^3-x^2+3\right)-\left[5x^2+x-\left(4x^3+x-2\right)\right]$

$=\left(8x^3-x^2+3\right)-\left[5x^2+x-4x^3-x+2\right]$

$=\left(8x^3-x^2+3\right)-\left(-4x^3+5x^2+2\right)$

$=8x^3-x^2+3+4x^3-5x^2-2$

$=8x^3+4x^3-x^2-5x^2+3-2$

$=12x^3-6x^2+1$

69.

$12a^2b-4ab^2-ab \rightarrow \quad 12a^2b-4ab^2-ab$

$\underline{-\left(4a^2b+\ ab^2-5ab\right)} \rightarrow \underline{+\left(-4a^2b-\ ab^2+5ab\right)}$

$8a^2b-5ab^2+4ab$

71.

$-5x^4 \quad -11x^2 \quad +6 \rightarrow \quad -5x^4 \quad -11x^2 \quad +6$

$\underline{-\left(-5x^4+3x^3+5x^2-10x+5\right)} \rightarrow \underline{+\left(5x^4-3x^3-5x^2+10x-5\right)}$

$-3x^3-16x^2+10x+1$

73.
$$\begin{array}{r} -2.2p^5 - 9.1p^4 \quad + 5.3p^2 - 7.9p \\ +\left(\quad -6.4p^4 - 8.5p^3 - 10.3p^2 \quad \right) \\ \hline -2.2p^5 - 15.5p^4 - 8.5p^3 - \ 5p^2 - 7.9p \end{array}$$

75.
$$\begin{aligned} P &= \left(2x^3 + 6x\right) + \left(4x^3 - 5x\right) + \left(6x^3 + x\right) \\ &= 2x^3 + 6x + 4x^3 - 5x + 6x^3 + x \\ &= 12x^3 + 2x \end{aligned}$$

77. $h(x) = \frac{2}{3}x^2 - 5$

It is a polynomial function. The degree is 2.

79. $p(x) = 8x^3 + 2x^2 - \frac{3}{x}$

It is not a polynomial function. The term $-\frac{3}{x} = -3x^{-1}$ and –1 is not a whole number.

81. $g(x) = -7$

It is a polynomial function. The degree is 0.

83. $M(x) = |x| + 5x$

It is not a polynomial function. The term $|x|$ is not of the form ax^n.

85. **a.**
$$\begin{aligned} P(x) &= -x^4 + 2x - 5 \\ P(2) &= -(2)^4 + 2(2) - 5 \\ &= -16 + 4 - 5 \\ &= -17 \end{aligned}$$

b.
$$\begin{aligned} P(-1) &= -(-1)^4 + 2(-1) - 5 \\ &= -1 - 2 - 5 \\ &= -8 \end{aligned}$$

c.
$$\begin{aligned} P(0) &= -(0)^4 + 2(0) - 5 \\ &= 0 + 0 - 5 \\ &= -5 \end{aligned}$$

d.
$$\begin{aligned} P(1) &= -(1)^4 + 2(1) - 5 \\ &= -1 + 2 - 5 \\ &= -4 \end{aligned}$$

87. **a.**
$$\begin{aligned} H(x) &= \tfrac{1}{2}x^3 - x + \tfrac{1}{4} \\ H(0) &= \frac{1}{2}(0)^3 - (0) + \frac{1}{4} \\ &= 0 - 0 + \frac{1}{4} = \frac{1}{4} \end{aligned}$$

b.
$$\begin{aligned} H(2) &= \frac{1}{2}(2)^3 - (2) + \frac{1}{4} \\ &= 4 - 2 + \frac{1}{4} = 2 + \frac{1}{4} \\ &= \frac{9}{4} \end{aligned}$$

c.
$$\begin{aligned} H(-2) &= \frac{1}{2}(-2)^3 - (-2) + \frac{1}{4} \\ &= -4 + 2 + \frac{1}{4} \\ &= -2 + \frac{1}{4} = -\frac{7}{4} \end{aligned}$$

d.
$$\begin{aligned} H(-1) &= \frac{1}{2}(-1)^3 - (-1) + \frac{1}{4} \\ &= -\frac{1}{2} + 1 + \frac{1}{4} = \frac{3}{4} \end{aligned}$$

89. Let x = the width of the garden

$x + 3$ = the length of the garden

$$P(x) = 2x + 2(x+3)$$
$$= 2x + 2x + 6$$
$$= 4x + 6$$

91. **a.**
$$P(x) = R(x) - C(x)$$
$$= (12x) - (5.40x + 99)$$
$$= 12x - 5.40x - 99$$
$$= 6.6x - 99$$

b.
$$P(50) = 6.6(50) - 99$$
$$= 330 - 99$$
$$= 231$$

The profit will be \$231.

93. **a.**
$$D(x) = 5.2x^2 + 40.4x + 1636$$
$$D(0) = 5.2(0)^2 + 40.4(0) + 1636$$
$$= 0 + 0 + 1636 = 1636$$

$D(0) = 1636$ means that at the beginning of the study, (year 0)the annual dormitory charge was \$1636.

$$D(18) = 5.2(18)^2 + 40.4(18) + 1636$$
$$= 1684.8 + 727.2 + 1636 = 404$$

In 2008, the annual dormitory charge was \$4048.

b.
$$D(25) = 5.2(25)^2 + 40.4(25) + 1636$$
$$= 3250 + 1010 + 1636 = 5896$$

The annual dormitory charge will be \$5896.

95. **a.**
$$W(t) = 143t + 6580$$
$$W(0) = 143(0) + 6580$$
$$= 6580$$
$$W(5) = 143(5) + 6580$$
$$= 715 + 6580$$
$$= 7295$$
$$W(10) = 143(10) + 6580$$
$$= 1430 + 6580$$
$$= 8010$$

b. $W(10) = 8010$ means that in Year 10, 8010 thousand (8,010,000) women were due in child support.

97. **a.**
$$x(t) = 25t$$
$$y(t) = -16t^2 + 43.3t$$
$$x(0) = 25(0) = 0$$
$$y(0) = -16(0)^2 + 43.3(0) = 0 + 0 = 0$$

(0, 0); at $t = 0$ sec, the position of the rocket is at the origin.

b.
$$x(1) = 25(1) = 25$$
$$y(1) = -16(1)^2 + 43.3(1)$$
$$= -16 + 43.3 = 27.3$$

(25, 27.3) At $t = 1$ sec, the position of the rocket is (25, 27.3).

c. $x(2) = 25(2)$
$= 50$

$y(2) = -16(2)^2 + 43.3(2)$
$= -64 + 86.6$
$= 22.6$

(50, 22.6) At $t = 2$ sec, the position of the rocket is (50, 22.6).

Section 4.3 Practice Exercises

1. **a.** distributive

b. $4x - 7$

c. squares; $a^2 - b^2$

d. perfect; $a^2 + 2ab + b^2$

3. $(-2-3x) - \left[5 - \left(6x^2 + 4x + 1\right)\right] = -2 - 3x - \left[5 - 6x^2 - 4x - 1\right]$
$= -2 - 3x - \left[-6x^2 - 4x + 4\right]$
$= -2 - 3x + 6x^2 + 4x - 4$
$= 6x^2 + x - 6$

5. **a.** $g(x) = x^4 - x^2 - 3$

$g(-1) = (-1)^4 - (-1)^2 - 3$
$= 1 - 1 - 3$
$= -3$

b. $g(2) = (2)^4 - (2)^2 - 3 = 16 - 4 - 3 = 9$

c. $g(0) = (0)^4 - (0)^2 - 3 = 0 - 0 - 3 = -3$

7. $\left(7x^4y\right)\left(-6xy^5\right) = 7(-6)\left(x^4 \cdot x\right)\left(y \cdot y^5\right)$
$= -42x^5y^6$

9. $\left(2.2a^6b^4c^7\right)\left(5ab^4c^3\right) = 11a^7b^8c^{10}$

11. $\frac{1}{5}(2a-3) = \frac{1}{5}(2a) + \frac{1}{5}(-3) = \frac{2}{5}a - \frac{3}{5}$

13. $2m^3n^2\left(m^2n^3 - 3m^2 + 4n\right)$
$= 2m^3n^2\left(m^2n^3\right) - 2m^3n^2\left(3m^2\right) + 2m^3n^2(4n)$
$= 2m^5n^5 - 6m^4n^4 + 8m^3n^3$

15. $6xy^2\left(\frac{1}{2}x - \frac{2}{3}xy\right) = 6xy^2\left(\frac{1}{2}x\right) - 6xy^2\left(\frac{2}{3}xy\right)$
$= 3x^2y^2 - 4x^2y^3$

17. $(x+y)(x-2y)$

$$\begin{aligned}&= x(x) - x(2y) + y(x) - y(2y)\\ &= x^2 - 2xy + xy - 2y^2\\ &= x^2 - xy - 2y^2\end{aligned}$$

19. $(6x-1)(5+2x)$

$$\begin{aligned}&= 6x(5) + 6x(2x) - 1(5) - 1(2x)\\ &= 30x + 12x^2 - 5 - 2x\\ &= 12x^2 + 28x - 5\end{aligned}$$

21. $(y^2-12)(2y^2+3)$

$$\begin{aligned}&= y^2(2y^2) + y^2(3) - 12(2y^2) - 12(3)\\ &= 2y^4 + 3y^2 - 24y^2 - 36\\ &= 2y^4 - 21y^2 - 36\end{aligned}$$

23. $(5s+3t)(5s-2t)$

$$\begin{aligned}&= 5s(5s) - 5s(2t) + 3t(5s) - 3t(2t)\\ &= 25s^2 - 10st + 15st - 6t^2\\ &= 25s^2 + 5st - 6t^2\end{aligned}$$

25. $(n^2+10)(5n+3)$

$$\begin{aligned}&= n^2(5n) + n^2(3) + 10(5n) + 10(3)\\ &= 5n^3 + 3n^2 + 50n + 30\end{aligned}$$

27. $(1.3a-4b)(2.5a+7b)$

$$\begin{aligned}&= 1.3a(2.5a) + 1.3a(7b) - 4b(2.5a) - 4b(7b)\\ &= 3.25a^2 + 9.1ab - 10ab - 28b^2\\ &= 3.25a^2 - 0.9ab - 28b^2\end{aligned}$$

29. $(2x+y)(3x^2+2xy+y^2)$

$$\begin{aligned}&= 2x(3x^2) + 2x(2xy) + 2x(y^2) + y(3x^2) + y(2xy) + y(y^2)\\ &= 6x^3 + 4x^2y + 2xy^2 + 3x^2y + 2xy^2 + y^3\\ &= 6x^3 + 7x^2y + 4xy^2 + y^3\end{aligned}$$

31. $(x-7)(x^2+7x+49)$

$$\begin{aligned}&= x(x^2) + x(7x) + x(49) - 7(x^2) - 7(7x) - 7(49)\\ &= x^3 + 7x^2 + 49x - 7x^2 - 49x - 343\\ &= x^3 - 343\end{aligned}$$

33. $(4a-b)(a^3-4a^2b+ab^2-b^3)$

$$\begin{aligned}&= 4a(a^3) - 4a(4a^2b) + 4a(ab^2) - 4a(b^3) - b(a^3) + b(4a^2b) - b(ab^2) + b(b^3)\\ &= 4a^4 - 16a^3b + 4a^2b^2 - 4ab^3 - a^3b + 4a^2b^2 - ab^3 + b^4\\ &= 4a^4 - 17a^3b + 8a^2b^2 - 5ab^3 + b^4\end{aligned}$$

35. $\left(\frac{1}{2}a-2b+c\right)\left(a+6b-c\right)$

$$=\frac{1}{2}a\left(a\right)+\frac{1}{2}a\left(6b\right)-\frac{1}{2}a\left(c\right)-2b\left(a\right)-2b\left(6b\right)+2b\left(c\right)+c\left(a\right)+c\left(6b\right)-c\left(c\right)$$
$$=\frac{1}{2}a^2+3ab-\frac{1}{2}ac-2ab-12b^2+2bc+ac+6bc-c^2$$
$$=\frac{1}{2}a^2+ab+\frac{1}{2}ac-12b^2+8bc-c^2$$

37. $\left(-x^2+2x+1\right)\left(3x-5\right)=-x^2\left(3x\right)+x^2\left(5\right)+2x\left(3x\right)-2x\left(5\right)+1\left(3x\right)-1\left(5\right)$

$$=-3x^3+5x^2+6x^2-10x+3x-5$$
$$=-3x^3+11x^2-7x-5$$

39. $\left(\frac{1}{5}y-10\right)\left(\frac{1}{2}y-15\right)$

$$=\frac{1}{5}y\left(\frac{1}{2}y\right)+\frac{1}{5}y\left(-15\right)-10\left(\frac{1}{2}y\right)-10\left(-15\right)$$
$$=\frac{1}{10}y^2-3y-5y+150=\frac{1}{10}y^2-8y+150$$

41. $\left(a-8\right)\left(a+8\right)=a^2-8^2$

$$=a^2-64$$

43. $\left(3p+1\right)\left(3p-1\right)=\left(3p\right)^2-1^2=9p^2-1$

45. $\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)=x^2-\left(\frac{1}{3}\right)^2$

$$=x^2-\frac{1}{9}$$

47. $\left(3h-k\right)\left(3h+k\right)=\left(3h\right)^2-k^2$

$$=9h^2-k^2$$

49. $\left(3h-k\right)^2=\left(3h\right)^2-2\left(3h\right)\left(k\right)+k^2$

$$=9h^2-6hk+k^2$$

51. $\left(t-7\right)^2=t^2-2\left(t\right)\left(7\right)+7^2$

$$=t^2-14t+49$$

53. $\left(u+3v\right)^2=u^2+2\left(u\right)\left(3v\right)+\left(3v\right)^2$

$$=u^2+6uv+9v^2$$

55. $\left(h+\frac{1}{6}k\right)^2=h^2+2\left(h\right)\left(\frac{1}{6}k\right)+\left(\frac{1}{6}k\right)^2$

$$=h^2+\frac{1}{3}hk+\frac{1}{36}k^2$$

57. $\left(2z^2-w^3\right)\left(2z^2+w^3\right)=\left(2z^2\right)^2-\left(w^3\right)^2$

$$=4z^4-w^6$$

59. $\left(5x^2-3y\right)^2=\left(5x^2\right)^2-2\left(5x^2\right)\left(3y\right)+\left(3y\right)^2=25x^4-30x^2y+9y^2$

61. **a.** When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-5x+4)(5x+4)$
$=-25x^2-20x+20x+16$
$=16-25x^2$

Since $(-5x+4)(5x+4)=16-25x^2$ is a difference of squares, the binomials are conjugates.

b. When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-5x+4)(5x-4)$
$=-25x^2+20x+20x+16$
$=-25x^2+40x+16$

Since $(-5x+4)(5x-4)=-25x^2+40x+16$ is not a difference of squares, the binomials are not conjugates.

63. **a.** $\left(A-B\right)\left(A+B\right)=A^2-B^2$

b. $\left[\left(x+y\right)-B\right]\left[\left(x+y\right)+B\right]$
$=\left(x+y\right)^2-B^2$
$=x^2+2xy+y^2-B^2$

Both are examples of multiplying conjugates to get a difference of squares.

65. $\left[\left(w+v\right)-2\right]\left[\left(w+v\right)+2\right]=\left(w+v\right)^2-2^2$
$=w^2+2wv+v^2-4$

67. $\left[2-\left(x+y\right)\right]\left[2+\left(x+y\right)\right]=2^2-\left(x+y\right)^2$
$=4-\left(x^2+2xy+y^2\right)$
$=4-x^2-2xy-y^2$

69. $\left[\left(3a-4\right)+b\right]\left[\left(3a-4\right)-b\right]$
$=\left(3a-4\right)^2-b^2$
$=\left(3a\right)^2-2\left(3a\right)\left(4\right)+4^2-b^2$
$=9a^2-24a+16-b^2$

71. Write $\left(x+y\right)^3$ as $\left(x+y\right)^2\left(x+y\right)$. Square the binomial and then use the distributive property to multiply the resulting trinomial by the remaining factor of $x+y$.

73. $\begin{aligned}(2x+y)^3 &= (2x+y)^2(2x+y)\\ &= (4x^2+4xy+y^2)(2x+y)\\ &= 4x^2(2x)+4x^2(y)+4xy(2x)+4xy(y)+y^2(2x)+y^2(y)\\ &= 8x^3+4x^2y+8x^2y+4xy^2+2xy^2+y^3\\ &= 8x^3+12x^2y+6xy^2+y^3\end{aligned}$

75. $\begin{aligned}(4a-b)^3 &= (4a-b)^2(4a-b)\\ &= (16a^2-8ab+b^2)(4a-b)\\ &= 16a^2(4a)-16a^2(b)-8ab(4a)+8ab(b)+b^2(4a)-b^2(b)\\ &= 64a^3-16a^2b-32a^2b+8ab^2+4ab^2-b^3\\ &= 64a^3-48a^2b+12ab^2-b^3\end{aligned}$

77. Multiply the first two binomials and simplify.

Then multiply the resulting trinomial and the third binomial, using the distributive property.

79. $\begin{aligned}&2a^2(a+5)(3a+1)\\ &= 2a^2\left[a(3a)+a(1)+5(3a)+5(1)\right]\\ &= 2a^2\left[3a^2+a+15a+5\right]\\ &= 2a^2\left(3a^2+16a+5\right)\\ &= 2a^2\left(3a^2\right)+2a^2(16a)+2a^2(5)\\ &= 6a^4+32a^3+10a^2\end{aligned}$

81. $\begin{aligned}&(x+3)(x-3)(x+5)\\ &= (x^2-9)(x+5)\\ &= x^2(x)+x^2(5)-9(x)-9(5)\\ &= x^3+5x^2-9x-45\end{aligned}$

83. $\begin{aligned}&128p^6+54q^3\\ &= 2\left(64p^6+27q^3\right)\\ &= 2\left[\left(4p^2\right)^3+(3q)^3\right]\\ &= 2\left(4p^2+3q\right)\left(16p^4-12p^2q+9q^2\right)\end{aligned}$

85. $\begin{aligned}(y+1)^2-(2y+3)^2 &= \left(y^2+2y+1\right)-\left(4y^2+12y+9\right)\\ &= y^2+2y+1-4y^2-12y-9\\ &= -3y^2-10y-8\end{aligned}$

87. $(r+t)^2$

89. x^2-y^3

91. The sum of the cube of p and the square of q.

93. The product of x and the square of y.

95. Let x = the width of the walk

$2x + 20$ = length of garden and walk

$2x + 15$ = width of garden and walk

$$\begin{aligned} A(x) &= (2x+20)(2x+15) \\ &= 2x(2x)+2x(15)+20(2x)+20(15) \\ &= 4x^2+30x+40x+300 \\ &= 4x^2+70x+300 \end{aligned}$$

97. a. Let x = the length of a side of the square

$8 - 2x$ = length and width of base

x = the height of the box

$$\begin{aligned} V(x) &= (8-2x)(8-2x)x \\ &= \left(64-32x+4x^2\right)x \\ &= 4x^3-32x^2+64x \end{aligned}$$

b. $$\begin{aligned} V(1) &= 4(1)^3-32(1)^2+64(1) \\ &= 4-32+64 \\ &= 36 \text{ in}^3 \end{aligned}$$

99. $$\begin{aligned} (x-2)^2 &= x^2-2(x)(2)+2^2 \\ &= x^2-4x+4 \end{aligned}$$

101. $$\begin{aligned} (x-2)(x+2) &= x^2-2^2 \\ &= x^2-4 \end{aligned}$$

103. $$\begin{aligned} \frac{1}{2}(2x-6)(x+3) &= (x-3)(x+3) \\ &= x^2-3^2 \\ &= x^2-9 \end{aligned}$$

105. $$\begin{aligned} x(3x)(3x+10) &= 3x^2(3x+10) \\ &= 3x^2(3x)+3x^2(10) \\ &= 9x^3+30x^2 \end{aligned}$$

107. $$\begin{aligned} \frac{\left[(x+h)^2-3(x+h)-5\right]-\left(x^2-3x-5\right)}{h} &= \frac{x^2+2xh+h^2-3x-3h-5-x^2+3x+5}{h} \\ &= \frac{x^2-x^2+2xh+h^2-3x+3x-3h-5+5}{h} \\ &= \frac{2xh+h^2-3h}{h} \\ &= \frac{h(2x+h-3)}{h} \\ &= 2x+h-3 \end{aligned}$$

109. Multiply $(x+2)^2(x+2)^2$ by squaring the binomials.

Then multiply the resulting trinomials using the distributive property.

111. $(5x-6)$

Check:

$$\begin{aligned}&(2x-3)(5x-6)\\&=2x(5x)-2x(6)-3(5x)+3(6)\\&=10x^2-12x-15x+18\\&=10x^2-27x+18\end{aligned}$$

113. $(2y-1)$

Check:

$$\begin{aligned}&(4y+3)(2y-1)\\&=4y(2y)-4y(1)+3(2y)-3(1)\\&=8y^2-4y+6y-3\\&=8y^2+2y-3\end{aligned}$$

Section 4.4 Practice Exercises

1. a. division; quotient; remainder

b. Synthetic

3. a. $(a-10b)-(5a+b)=a-10b-5a-b$
$=-4a-11b$

b. $(a-10b)(5a+b)$
$=a(5a)+a(b)-10b(5a)-10b(b)$
$=5a^2+ab-50ab-10b^2$
$=5a^2-49ab-10b^2$

5. a. $(x^2-x)+(6x^2+x+2)$
$=x^2+6x^2-x+x+2$
$=7x^2+2$

b. $(x^2-x)(6x^2+x+2)$
$=x^2(6x^2)+x^2(x)+x^2(2)$
$-x(6x^2)-x(x)-x(2)$
$=6x^4+x^3+2x^2-6x^3-x^2-2x$
$=6x^4-5x^3+x^2-2x$

7. For example:

$$(5y+1)^2=(5y)^2+2(5y)(1)+1^2$$
$$=25y^2+10y+1$$

9. $\dfrac{16t^4-4t^2+20t}{-4t}=\dfrac{16t^4}{-4t}-\dfrac{4t^2}{-4t}+\dfrac{20t}{-4t}$

$=-4t^3+t-5$

11. $\left(36y+24y^2+6y^3\right)\div\left(3y\right)$

$$=\frac{36y}{3y}+\frac{24y^2}{3y}+\frac{6y^3}{3y}$$

$$=12+8y+2y^2$$

13. $\left(4x^3y+12x^2y^2-4xy^3\right)\div\left(4xy\right)$

$$=\frac{4x^3y}{4xy}+\frac{12x^2y^2}{4xy}-\frac{4xy^3}{4xy}$$

$$=x^2+3xy-y^2$$

15. $\left(-8y^4-12y^3+32y^2\right)\div\left(-4y^2\right)$

$$=\frac{-8y^4}{-4y^2}-\frac{12y^3}{-4y^2}+\frac{32y^2}{-4y^2}$$

$$=2y^2+3y-8$$

17. $\left(3p^4-6p^3+2p^2-p\right)\div\left(-6p\right)$

$$=\frac{3p^4}{-6p}-\frac{6p^3}{-6p}+\frac{2p^2}{-6p}-\frac{p}{-6p}$$

$$=-\frac{1}{2}p^3+p^2-\frac{1}{3}p+\frac{1}{6}$$

19. $\left(a^3+5a^2+a-5\right)\div\left(a\right)$

$$=\frac{a^3}{a}+\frac{5a^2}{a}+\frac{a}{a}-\frac{5}{a}$$

$$=a^2+5a+1-\frac{5}{a}$$

21. $\dfrac{6s^3t^5-8s^2t^4+10st^2}{-2st^4}$

$$=\frac{6s^3t^5}{-2st^4}-\frac{8s^2t^4}{-2st^4}+\frac{10st^2}{-2st^4}$$

$$=-3s^2t+4s-\frac{5}{t^2}$$

23. $\left(8p^4q^7-9p^5q^6-11p^3q-4\right)\div\left(p^2q\right)$

$$=\frac{8p^4q^7}{p^2q}-\frac{9p^5q^6}{p^2q}-\frac{11p^3q}{p^2q}-\frac{4}{p^2q}$$

$$=8p^2q^6-9p^3q^5-11p-\frac{4}{p^2q}$$

25. **a.**

$$x-2\overline{)\,2x^3-7x^2+5x-1}$$

$$\begin{array}{r} 2x^2-3x-1 \\ -\left(2x^3-4x^2\right) \\ \hline -3x^2+5x \\ -\left(-3x^2+6x\right) \\ \hline -x-1 \\ -\left(-x+2\right) \\ \hline -3 \end{array}$$

Divisor: $(x-2)$ Quotient: $(2x^2-3x-1)$

Remainder: (–3)

b. Multiply the quotient and divisor; then add the remainder.

The result should equal the dividend.

27.

$$x+4\overline{)\,x^2+11x+19}$$

$$\begin{array}{r} x+7 \\ -\left(x^2+4x\right) \\ \hline 7x+19 \\ -\left(7x+28\right) \\ \hline -9 \end{array}$$

Solution: $x+7-\dfrac{9}{x+4}$

Check:

$$\left(x+4\right)\left(x+7\right)+\left(-9\right)=x^2+11x+28-9$$
$$=x^2+11x+19$$

29.

$$y-3\overline{)\,3y^3-7y^2-4y+3}$$

$$\begin{array}{r} 3y^2+2y+2 \\ -\left(3y^3-9y^2\right) \\ \hline 2y^2-4y \\ -\left(2y^2-6y\right) \\ \hline 2y+3 \\ -\left(2y-6\right) \\ \hline 9 \end{array}$$

Solution: $3y^2+2y+2+\dfrac{9}{y-3}$

Check:

$$\left(y-3\right)\left(3y^2+2y+2\right)+\left(9\right)$$
$$=3y^3+2y^2+2y-9y^2-6y-6+9$$
$$=3y^3-7y^2-4y+3$$

31.

$$\begin{array}{r} -4a+11 \\ 3a-11\overline{)\ -12a^2+77a-121} \\ \underline{-\left(-12a^2+44a\right)} \\ 33a-121 \\ \underline{-\left(33a-121\right)} \\ 0 \end{array}$$

Solution: $-4a+11$

Check:

$$\begin{aligned}&\left(3a-11\right)\left(-4a+11\right)+\left(0\right)\\ &\quad=-12a^2+33a+44a-121\\ &\quad=-12a^2+77a-121\end{aligned}$$

33.

$$\begin{array}{r} 6y-5 \\ 3y+4\overline{)\ 18y^2+9y-20} \\ \underline{-\left(18y^2+24y\right)} \\ -15y-20 \\ \underline{-\left(-15y-20\right)} \\ 0 \end{array}$$

Solution: $6y-5$

Check:

$$\begin{aligned}&\left(3y+4\right)\left(6y-5\right)+\left(0\right)\\ &\quad=18y^2-15y+24y-20\\ &\quad=18y^2+9y-20\end{aligned}$$

35.

$$\begin{array}{r} 6x^2+4x+5 \\ 3x-2\overline{)\ 18x^3\quad +7x+12} \\ \underline{-\left(18x^3-12x^2\right)} \\ 12x^2+7x \\ \underline{-\left(12x^2-8x\right)} \\ 15x+12 \\ \underline{-\left(15x-10\right)} \\ 22 \end{array}$$

Solution: $6x^2+4x+5+\dfrac{22}{3x-2}$

Check:

$$\begin{aligned}&\left(3x-2\right)\left(6x^2+4x+5\right)+\left(22\right)\\ &=18x^3+12x^2+15x-12x^2-8x-10+22\\ &=18x^3+7x+12\end{aligned}$$

37.

$$\begin{array}{r} 4a^2-2a+1 \\ 2a+1\overline{)\ 8a^3\quad +1} \\ \underline{-\left(8a^3+4a^2\right)} \\ -4a^2 \\ \underline{-\left(-4a^2-2a\right)} \\ 2a+1 \\ \underline{-\left(2a+1\right)} \\ 0 \end{array}$$

Solution: $4a^2-2a+1$

Check:

$$\begin{aligned}&\left(2a+1\right)\left(4a^2-2a+1\right)+\left(0\right)\\ &\quad=8a^3-4a^2+2a+4a^2-2a+1\\ &\quad=8a^3+1\end{aligned}$$

39.

$$x^2+x-1\overline{)\,x^4-x^3-x^2+4x-2}$$

$$\begin{array}{r} x^2-2x+2 \\ x^2+x-1\overline{)\,x^4-x^3-x^2+4x-2} \\ \underline{-\left(x^4+x^3-x^2\right)} \\ -2x^3+4x \\ \underline{-\left(-2x^3-2x^2+2x\right)} \\ 2x^2+2x-2 \\ \underline{-\left(2x^2+2x-2\right)} \\ 0 \end{array}$$

Solution: x^2-2x+2

Check:

$$\begin{aligned}\left(x^2+x-1\right)\left(x^2-2x+2\right)+(0) &= x^4-2x^3+2x^2+x^3-2x^2+2x \\ &\quad -x^2+2x-2 \\ &= x^4-x^3-x^2+4x-2\end{aligned}$$

41.

$$\begin{array}{r} x^2+2x+5 \\ x^2-5\overline{)\,x^4+2x^3\quad -10x-25} \\ \underline{-\left(x^4\quad -5x^2\right)} \\ 2x^3+5x^2-10x \\ \underline{-\left(2x^3\quad -10x\right)} \\ 5x^2\quad -25 \\ \underline{-\left(5x^2\quad -25\right)} \\ 0 \end{array}$$

Solution: x^2+2x+5

Check:

$$\begin{aligned}\left(x^2-5\right)\left(x^2+2x+5\right)+(0) &= x^4+2x^3+5x^2-5x^2-10x-25 \\ &= x^4+2x^3-10x-25\end{aligned}$$

43.

$$\begin{array}{r} x^2-1 \\ x^2-2\overline{)\,x^4-3x^2+10} \\ \underline{-\left(x^4-2x^2\right)} \\ -x^2+10 \\ \underline{-\left(-x^2+2\right)} \\ 8 \end{array}$$

Solution: $x^2-1+\dfrac{8}{x^2-2}$

Check: $\left(x^2-2\right)\left(x^2-1\right)+(8)$

$$\begin{aligned} &= x^4-x^2-2x^2+2+8 \\ &= x^4-3x^2+10\end{aligned}$$

45.

$$\begin{array}{r} n^3+2n^2+4n+8 \\ n-2\overline{)\,n^4\qquad\quad -16} \\ \underline{-\left(n^4-2n^3\right)} \\ 2n^3 \\ \underline{-\left(2n^3-4n^2\right)} \\ 4n^2 \\ \underline{-\left(4n^2-8n\right)} \\ 8n-16 \\ \underline{-\left(8n-16\right)} \\ 0 \end{array}$$

Solution: n^3+2n^2+4n+8

Check:

$$(n-2)\left(n^3+2n^2+4n+8\right)+(0)$$
$$= n^4+2n^3+4n^2+8n-2n^3-4n^2$$
$$-8n-16$$
$$= n^4-16$$

47. The divisor must be of the form $x-r$.

49. No, the divisor is not of the form $x-r$.

51. **a.** Divisor: $x-5$

b. Quotient: $x^2+3x+11$

c. Remainder: 58

53.

8 |	1	−2	−48
		8	48
	1	6	|0

Quotient: $x+6$

Check:

$$(x-8)(x+6)+(0) = x^2+6x-8x-48$$
$$= x^2-2x-48$$

55.

−1 |	1	−3	−4
		−1	4
	1	−4	|0

Quotient: $t-4$

Check:

$$(t+1)(t-4)+(0) = t^2-4t+t-4$$
$$= t^2-3t-4$$

57.

1 |	5	5	1
		5	10
	5	10	|11

Quotient: $5y+10+\dfrac{11}{y-1}$

Check:

$$(y-1)(5y+10)+(11)$$
$$= 5y^2+10y-5y-10+11$$
$$= 5y^2+5y+1$$

59.

−3 |	3	7	−4	3
		−9	6	−6
	3	−2	2	|−3

Quotient: $3y^2-2y+2+\dfrac{-3}{y+3}$

61.

2 |	1	−3	0	4
		2	−2	−4
	1	−1	−2	|0

Quotient: x^2-x-2

Check:

$$(y+3)\left(3y^2-2y+2\right)+(-3)$$
$$=3y^3-2y^2+2y+9y^2-6y+6-3$$
$$=3y^3+7y^2-4y+3$$

63.

$$\begin{array}{r|cccccc} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 0 \end{array}$$

Quotient: $a^4+2a^3+4a^2+8a+16$

Check:

$$(a-2)\left(a^4+2a^3+4a^2+8a+16\right)+(0)$$
$$=a^5+2a^4+4a^3+8a^2+16a$$
$$-2a^4-4a^3-8a^2-16a-32$$
$$=a^5-32$$

67.

$$\begin{array}{r|cccc} -\frac{2}{3} & 6 & 7 & -1 & 3 \\ & & -4 & -2 & 2 \\ \hline & 6 & 3 & -3 & 5 \end{array}$$

Quotient: $6t^2+3t-3+\dfrac{5}{t+\frac{2}{3}}$

Check:

$$\left(t+\frac{2}{3}\right)\left[(6t^2+3t-3)+\frac{5}{t+\frac{2}{3}}\right]$$
$$=\left(t+\frac{2}{3}\right)(6t^2+3t-3)+\left(t+\frac{2}{3}\right)\left(\frac{5}{t+\frac{2}{3}}\right)$$
$$=6t^3+3t^2-3t+4t^2+2t-2+5$$
$$=6t^3+7t^2-t+3$$

Check:

$$(x-2)\left(x^2-x-2\right)+(0)$$
$$=x^3-x^2-2x-2x^2+2x+4$$
$$=x^3-3x^2+4$$

65.

$$\begin{array}{r|cccc} 6 & 1 & 0 & 0 & -216 \\ & & 6 & 36 & 216 \\ \hline & 1 & 6 & 36 & 0 \end{array}$$

Quotient: $x^2+6x+36$

Check:

$$(x-6)\left(x^2+6x+36\right)+(0)$$
$$=x^3+6x^2+36x-6x^2-36x-216$$
$$=x^3-216$$

69.

$$\begin{array}{r|ccccc} \frac{1}{2} & 4 & 0 & -1 & 6 & -3 \\ & & 2 & 1 & 0 & 3 \\ \hline & 4 & 2 & 0 & 6 & 0 \end{array}$$

Quotient: $4w^3+2w^2+6$

Check:

$$\left(w-\frac{1}{2}\right)\left(4w^3+2w^2+6\right)+(0)$$
$$=4w^4+2w^3+6w-2w^3-w^2-3$$
$$=4w^4-w^2+6w-3$$

71.
$$\begin{array}{r|rrrr} -4 & -1 & -8 & -3 & -2 \\ & & 4 & 16 & -52 \\ \hline & -1 & -4 & 13 & -54 \end{array}$$

Quotient: $-x^2-4x+13+\dfrac{-54}{x+4}$

73. $\left(22x^2-11x+33\right)\div\left(11x\right)$

$$=\frac{22x^2}{11x}-\frac{11x}{11x}+\frac{33}{11x}=2x-1+\frac{3}{x}$$

75.
$$\begin{array}{r} 4y-3 \\ 3y^2-2y+5\overline{)\ 12y^3-17y^2+30y-10} \\ \underline{-\left(12y^3-8y^2+20y\right)} \\ -9y^2+10y-10 \\ \underline{-\left(-9y^2+6y-15\right)} \\ 4y+5 \end{array}$$

Quotient: $4y-3+\dfrac{4y+5}{3y^2-2y+5}$

77.
$$\begin{array}{r} 2x^2+3x-1 \\ 2x^2+1\overline{)\ 4x^4+6x^3\quad +3x-1} \\ \underline{-\left(4x^4\quad +2x^2\right)} \\ 6x^3-2x^2+3x \\ \underline{-\left(6x^3\quad +3x\right)} \\ -2x^2\quad -1 \\ \underline{-\left(-2x^2\quad -1\right)} \\ 0 \end{array}$$

Quotient: $2x^2+3x-1$

79. $\left(16k^{11}-32k^{10}+8k^8-40k^4\right)\div\left(8k^8\right)$

$$=\frac{16k^{11}}{8k^8}-\frac{32k^{10}}{8k^8}+\frac{8k^8}{8k^8}-\frac{40k^4}{8k^8}$$
$$=2k^3-4k^2+1-\frac{5}{k^4}$$

81. $\left(5x^3+9x^2+10x\right)\div\left(5x^2\right)$

$$=\frac{5x^3}{5x^2}+\frac{9x^2}{5x^2}+\frac{10x}{5x^2}$$
$$=x+\frac{9}{5}+\frac{2}{x}$$

83. **a.** $P\left(-4\right)=4\left(-4\right)^3+10\left(-4\right)^2-8\left(-4\right)-20$

$$=4\left(-64\right)+10\left(16\right)+32-20$$
$$=-256+160+32-20$$
$$=-84$$

b.
$$\begin{array}{r|rrrr} -4 & 4 & 10 & -8 & -20 \\ & & -16 & 24 & -64 \\ \hline & 4 & -6 & 16 & -84 \end{array}$$

Quotient: $4x^2-6x+16+\dfrac{-84}{x+4}$

c. The values are the same.

85. $P\left(r\right)$ equals the remainder of $P\left(x\right)\div\left(x-r\right)$.

87. **a.**
$$\begin{array}{r|rrr} -1 & 8 & 13 & 5 \\ & & -8 & -5 \\ \hline & 8 & 5 & 0 \end{array}$$
Quotient: $8x+5$

b. Yes Yes

Problem Recognition Exercises

1. **a.** $(3x+1)^2 = (3x)^2 + 2(3x)(1) + 1^2$
$= 9x^2 + 6x + 1$

b $(3x+1)(3x-1) = (3x)^2 - 1^2$
$= 9x^2 - 1$

c. $(3x+1) - (3x-1) = 3x + 1 - 3x + 1$
$= 2$

3. **a.** $\dfrac{4x^2+8x-10}{2x} = \dfrac{4x^2}{2x} + \dfrac{8x}{2x} - \dfrac{10}{2x}$
$= 2x + 4 - \dfrac{5}{x}$

b.
$$\begin{array}{r} 2x+5 \\ 2x-1 \overline{)\ 4x^2+8x-10} \\ \underline{-(4x^2-2x)} \\ 10x-10 \\ \underline{-(10x-5)} \\ -5 \end{array}$$
Solution: $2x+5+\dfrac{-5}{2x-1}$

c.
$$\begin{array}{r|rrr} 1 & 4 & 8 & -10 \\ & & 4 & 12 \\ \hline & 4 & 12 & 2 \end{array}$$
Quotient: $4x+12+\dfrac{2}{x-1}$

5. **a.** $(p-5)(p+5) - (p^2+5)$
$= p^2 - 25 - p^2 - 5$
$= -30$

b. $(p-5)(p+5) - (p+5)^2$
$= p^2 - 25 - p^2 - 10p - 25$
$= -10p - 50$

c. $(p-5)(p+5) - (p^2-25)$
$= p^2 - 25 - p^2 + 25 = 0$

7. $(5t^2 - 6t + 2) - (3t^2 - 7t + 3) = 5t^2 - 6t + 2 - 3t^2 + 7t - 3$
$= 2t^2 + t - 1$

9. $(6z+5)(6z-5) = (6z)^2 - 5^2$
$= 36z^2 - 25$

11. $(3b-4)(2b-1)$
$= 3b(2b) - 3b(1) - 4(2b) + 4(1)$
$= 6b^2 - 3b - 8b + 4$
$= 6b^2 - 11b + 4$

13. $(t^3 - 4t^2 + t - 9) + (t + 12) - (2t^2 - 6t)$
$= t^3 - 4t^2 + t - 9 + t + 12 - 2t^2 + 6t$
$= t^3 - 6t^2 + 8t + 3$

15. $(k+4)^2 + (-4k+9)$
$= k^2 + 2(k)(4) + 4^2 - 4k + 9$
$= k^2 + 8k + 16 - 4k + 9$
$= k^2 + 4k + 25$

17. $-2t(t^2 + 6t - 3) + t(3t+2)(3t-2)$
$= -2t^3 - 12t^2 + 6t + t(9t^2 - 4)$
$= -2t^3 - 12t^2 + 6t + 9t^3 - 4t$
$= 7t^3 - 12t^2 + 2t$

19. $\left(\frac{1}{4}p^3 - \frac{1}{6}p^2 + 5\right) - \left(-\frac{2}{3}p^3 + \frac{1}{3}p^2 - \frac{1}{5}p\right)$
$= \frac{3}{12}p^3 - \frac{1}{6}p^2 + 5 + \frac{8}{12}p^3 - \frac{2}{6}p^2 + \frac{1}{5}p$
$= \frac{11}{12}p^3 - \frac{1}{2}p^2 + \frac{1}{5}p + 5$

21. $(6a^2 - 4b)^2 = (6a^2)^2 - 2(6a^2)(4b) + (4b)^2$
$= 36a^4 - 48a^2b + 16b^2$

23. $(m-3)^2 - 2(m+8)$
$= m^2 - 6m + 9 - 2m - 16$
$= m^2 - 8m - 7$

25. $(m^2 - 6m + 7)(2m^2 + 4m - 3) = m^2(2m^2 + 4m - 3) - 6m(2m^2 + 4m - 3) + 7(2m^2 + 4m - 3)$
$= 2m^4 + 4m^3 - 3m^2 - 12m^3 - 24m^2 + 18m + 14m^2 + 28m - 21$
$= 2m^4 - 8m^3 - 13m^2 + 46m - 21$

27. $[5 - (a+b)]^2 = 5^2 - 2(5)(a+b) + (a+b)^2$
$= 25 - 10a - 10b + a^2 + 2ab + b^2$

29. $(x+y)^2 - (x-y)^2$
$= x^2 + 2xy + y^2 - (x^2 - 2xy + y^2)$
$= x^2 + 2xy + y^2 - x^2 + 2xy - y^2$
$= 4xy$

31. $\left(-\frac{1}{2}x + \frac{1}{3}\right)\left(\frac{1}{4}x - \frac{1}{2}\right) = -\frac{1}{8}x^2 + \frac{1}{4}x + \frac{1}{12}x - \frac{1}{6} = -\frac{1}{8}x^2 + \frac{1}{3}x - \frac{1}{6}$

Section 4.5 Practice Exercises

1. **a.** product
b. greatest common factor
c. greatest common factor
d. grouping

3.
$$\begin{aligned}&\left(7t^4+5t^3-9t\right)-\left(-2t^4+6t^2-3t\right)\\&=7t^4+5t^3-9t+2t^4-6t^2+3t\\&=9t^4+5t^3-6t^2-6t\end{aligned}$$

5.
$$\begin{aligned}&\left(5y^2-3\right)\left(y^2+y+2\right)\\&=5y^4+5y^3+10y^2-3y^2-3y-6\\&=5y^4+5y^3+7y^2-3y-6\end{aligned}$$

7.
$$\begin{aligned}\frac{6v^3-12v^2+2v}{-2v}&=\frac{6v^3}{-2v}-\frac{12v^2}{-2v}+\frac{2v}{-2v}\\&=-3v^2+6v-1\end{aligned}$$

9.
$$\begin{aligned}3x+12&=3\left(x\right)+3\left(4\right)\\&=3\left(x+4\right)\end{aligned}$$

11. $6z^2+4z=2z\left(3z\right)+2z\left(2\right)=2z\left(3z+2\right)$

13. $4p^6-4p=4p\left(p^5\right)-4p\left(1\right)=4p\left(p^5-1\right)$

15.
$$\begin{aligned}12x^4-36x^2&=12x^2\left(x^2\right)-12x^2\left(3\right)\\&=12x^2\left(x^2-3\right)\end{aligned}$$

17.
$$\begin{aligned}9st^2+27t&=9t\left(st\right)+9t\left(3\right)\\&=9t\left(st+3\right)\end{aligned}$$

19.
$$\begin{aligned}&9a^4b^3+27a^3b^4-18a^2b^5\\&=9a^2b^3\left(a^2\right)+9a^2b^3\left(3ab\right)-9a^2b^3\left(2b^2\right)\\&=9a^2b^3\left(a^2+3ab-2b^2\right)\end{aligned}$$

21.
$$\begin{aligned}&10x^2y+15xy^2-5xy\\&=5xy\left(2x\right)+5xy\left(3y\right)-5xy\left(1\right)\\&=5xy\left(2x+3y-1\right)\end{aligned}$$

23.
$$\begin{aligned}&13b^2-11a^2b-12ab\\&=b\left(13b\right)-b\left(11a^2\right)-b\left(12a\right)\\&=b\left(13b-11a^2-12a\right)\end{aligned}$$

25. $-x^2-10x+7=-1\left(x^2+10x-7\right)$

27.
$$\begin{aligned}&-12x^3y-6x^2y-3xy\\&=-3xy\left(4x^2\right)-3xy\left(2x\right)-3xy\left(1\right)\\&=-3xy\left(4x^2+2x+1\right)\end{aligned}$$

29.
$$\begin{aligned}&-2t^3+11t^2-3t\\&=-t\left(2t^2\right)-t\left(-11t\right)-t\left(3\right)\\&=-t\left(2t^2-11t+3\right)\end{aligned}$$

31. $2a(3z-2b)-5(3z-2b)$
$=(3z-2b)(2a-5)$

33. $2x^2(2x-3)+(2x-3)=(2x-3)(2x^2+1)$

35. $y(2x+1)^2-3(2x+1)^2=(2x+1)^2(y-3)$

37. $3y(x-2)^2+6(x-2)^2$
$=3\left[y(x-2)^2+2(x-2)^2\right]$
$=3(x-2)^2(y+2)$

39. For example: $3x^3+6x^2+12x^4$

41. For example: $6(c+d)+y(c+d)$

43. **a.** $2ax-ay+6bx-3by$
$=a(2x-y)+3b(2x-y)$
$=(2x-y)(a+3b)$

b. $10w^2-5w-6bw+3b$
$=5w(2w-1)-3b(2w-1)$
$=(2w-1)(5w-3b)$

c. In part (b), $-3b$ was factored out so that the signs in the last two terms were changed. The resulting binomial factor matches the binomial factor in the first two terms.

45. $y^3+4y^2+3y+12=y^2(y+4)+3(y+4)$
$=(y+4)(y^2+3)$

47. $6p-42+pq-7q=6(p-7)+q(p-7)$
$=(p-7)(6+q)$

49. $2mx+2nx+3my+3ny$
$=2x(m+n)+3y(m+n)$
$=(m+n)(2x+3y)$

51. $10ax-15ay-8bx+12by$
$=5a(2x-3y)-4b(2x-3y)$
$=(2x-3y)(5a-4b)$

53. $x^3-x^2-3x+3=x^2(x-1)-3(x-1)$
$=(x-1)(x^2-3)$

55. $6p^2q+18pq-30p^2-90p$

$=6p\left[pq+3q-5p-15\right]$

$=6p\left[q\left(p+3\right)-5\left(p+3\right)\right]$

$=6p\left(p+3\right)\left(q-5\right)$

57. $100x^3-300x^2+200x-600$

$=100\left[x^3-3x^2+2x-6\right]$

$=100\left[x^2\left(x-3\right)+2\left(x-3\right)\right]$

$=100\left(x-3\right)\left(x^2+2\right)$

59. $6ax-by+2bx-3ay$

$=6ax+2bx-3ay-by$

$=2x\left(3a+b\right)-y\left(3a+b\right)$

$=\left(3a+b\right)\left(2x-y\right)$

61. $4a-3b-ab+12$

$=4a-ab+12-3b$

$=a\left(4-b\right)+3\left(4-b\right)$

$=\left(4-b\right)\left(a+3\right)$

63. $7y^3-21y^2+5y-10$ cannot be factored.

65. It is not possible to get a common binomial factor regardless of the order of the terms.

67. $U=Av+Acw$

$U=A\left(v+cw\right)$

$\dfrac{U}{v+cw}=A$

69. $ay+bx=cy$

$bx=cy-ay$

$bx=y\left(c-a\right)$

$y=\dfrac{bx}{c-a}$ or $y=\dfrac{-bx}{a-c}$

71. $A=2w^2+w$

$A=w\left(2w+1\right)$

The length of the rectangle is $2w+1$.

73. $\left(a+3\right)^4+6\left(a+3\right)^5$

$=\left(a+3\right)^4\left[1+6\left(a+3\right)\right]$

$=\left(a+3\right)^4\left[1+6a+18\right]$

$=\left(a+3\right)^4\left(6a+19\right)$

75. $24\left(3x+5\right)^3-30\left(3x+5\right)^2$

$=6\left(3x+5\right)^2\left[4\left(3x+5\right)-5\right]$

$=6\left(3x+5\right)^2\left[12x+20-5\right]$

$=6\left(3x+5\right)^2\left(12x+15\right)$

$=6\left(3x+5\right)^2 3\left(4x+5\right)$

$=18\left(3x+5\right)^2\left(4x+5\right)$

77. $\left(t+4\right)^2-\left(t+4\right)$

$=\left(t+4\right)\left[\left(t+4\right)-1\right]$

$=\left(t+4\right)\left(t+3\right)$

79. $15w^2(2w-1)^3+5w^3(2w-1)^2=5w^2(2w-1)^2\left[3(2w-1)+w\right]$
$=5w^2(2w-1)^2\left[6w-3+w\right]$
$=5w^2(2w-1)^2(7w-3)$

Section 4.6 Practice Exercises

1. **a.** positive

b. opposite

c. $(2x+3)(x-4)=2x^2-8x+3x-12$
$=2x^2-5x-12$
$(x-4)(2x+3)=2x^2+3x-8x-12$
$=2x^2-5x-12$
Both are correct.

d. $6x^2-4x-10=2\left(3x^2-2x-5\right)$
$=2\left(3x^2+3x-5x-5\right)$
$=2\left[3x(x+1)-5(x+1)\right]$
$=2(3x-5)(x+1)$

e. $(a+b)^2$; $(a-b)^2$

3. $36c^2d^7e^{11}+12c^3d^5e^{15}-6c^2d^4e^7$
$=6c^2d^4e^7\left(6d^3e^4+2cde^8-1\right)$

5. $2x(3a-b)-(3a-b)=(3a-b)(2x-1)$

7. $wz^2+2wz-33az-66a$
$=wz(z+2)-33a(z+2)$
$=(z+2)(wz-33a)$

9. $b^2-12b+32=b^2-4b-8b+32$
$=b(b-4)-8(b-4)$
$=(b-4)(b-8)$

11. $y^2+10y-24=y^2+12y-2y-24$
$=y(y+12)-2(y+12)$
$=(y+12)(y-2)$

13. $x^2+13x+30=x^2+10x+3x+30$
$=x(x+10)+3(x+10)$
$=(x+10)(x+3)$

15. $c^2-6c-16$
$=c^2-8c+2c-16$
$=c(c-8)+2(c-8)$
$=(c-8)(c+2)$

17. $2x^2-7x-15$
$=2x^2-10x+3x-15$
$=2x(x-5)+3(x-5)$
$=(x-5)(2x+3)$

19. $a+6a^2-5=6a^2+a-5$
$=6a^2+6a-5a-5$
$=6a(a+1)-5(a+1)$
$=(a+1)(6a-5)$

21. $s^2+st-6t^2=s^2+3st-2st-6t^2$
$=s(s+3t)-2t(s+3t)$
$=(s+3t)(s-2t)$

23. $3x^2-60x+108=3(x^2-20x+36)$
$=3(x^2-18x-2x+36)$
$=3[x(x-18)-2(x-18)]$
$=3(x-18)(x-2)$

25. $2c^2-2c-24=2(c^2-c-12)$
$=2(c^2-4c+3c-12)$
$=2[c(c-4)+3(c-4)]$
$=2(c-4)(c+3)$

27. $2x^2+8xy-10y^2=2(x^2+4xy-5y^2)$
$=2(x^2+5xy-xy-5y^2)$
$=2[x(x+5y)-y(x+5y)]$
$=2(x+5y)(x-y)$

29. $33t^2-18t+2$
Since there are not two factors of 66 whose sum is –18, the polynomial is prime.

31. $3x^2+14xy+15y^2=3x^2+9xy+5xy+15y^2$
$=3x(x+3y)+5y(x+3y)$
$=(x+3y)(3x+5y)$

33. $5u^3v-30u^2v^2+45uv^3=5uv(u^2-6uv+9v^2)$
$=5uv(u^2-3uv-3uv+9v^2)$
$=5uv[u(u-3v)-3v(u-3v)]$
$=5uv(u-3v)(u-3v)$
$=5uv(u-3v)^2$

35. $x^3-5x^2-14x=x(x^2-5x-14)$
$=x(x^2-7x+2x-14)$
$=x[x(x-7)+2(x-7)]$
$=x(x-7)(x+2)$

37. $-23z-5+10z^2=10z^2-23z-5$
$=10z^2-25z+2z-5$
$=5z(2z-5)+(2z-5)$
$=(2z-5)(5z+1)$

39. $b^2+2b+15$
Since there are not two factors of 15 whose sum is 2, the polynomial is prime.

41. $-2t^2+12t+80=-2(t^2-6t-40)$
$=-2(t^2-10t+4t-40)$
$=-2[t(t-10)+4(t-10)]$
$=-2(t-10)(t+4)$

43. $14a^2+13a-12=14a^2+21a-8a-12$
$=7a(2a+3)-4(2a+3)$
$=(2a+3)(7a-4)$

45. $6a^2b+22ab+12b=2b(3a^2+11a+6)$
$=2b(3a^2+9a+2a+6)$
$=2b[3a(a+3)+2(a+3)]$
$=2b(a+3)(3a+2)$

47. **a.** $(x+5)(x+5)=x^2+5x+5x+25$
$=x^2+10x+25$

b. $x^2+10x+25=(x+5)^2$

49. **a.** $(3x-2y)(3x-2y)$
$=9x^2-6xy-6xy+4y^2$
$=9x^2-12xy+4y^2$

b. $9x^2-12xy+4y^2=(3x-2y)^2$

51. $9x^2+(\_)+25=(3x)^2+2(3x)(5)+5^2$
$=9x^2+(\underline{30x})+25$

53. $64z^4+(\_)+t^2=(8z^2)^2+2(8z^2)(t)+t^2$
$=64z^4+(\underline{16z^2t})+t^2$

55. $y^2-8y+16=y^2-2(y)(4)+4^2$
$=(y-4)^2$

57. $64m^2+80m+25=(8m)^2+2(8m)(5)+5^2$
$=(8m+5)^2$

59. $w^2-5w+9=w^2-2(w)(3)+3^2$

Not a perfect square trinomial.

61. $9a^2-30ab+25b^2$
$=(3a)^2-2(3a)(5b)+(5b)^2$
$=(3a-5b)^2$

63. $16t^2-80tv+20v^2=4(4t^2-20tv+5v^2)$

Not a perfect square trinomial.

65. $5b^4-20b^2+20=5(b^4-4b^2+4)$
$=5((b^2)^2-2(b^2)(2)+2^2)$
$=5(b^2-2)^2$

67. **a.** $u^2-10u+25=u^2-2(u)(5)+5^2$
$=(u-5)^2$

69. **a.** $u^2+11u-26=u^2+13u-2u-26$
$=u(u+13)-2(u+13)$
$=(u+13)(u-2)$

b. $x^4 - 10x^2 + 25 = \left(x^2\right)^2 - 10x^2 + 25$

Let $u = x^2$

$u^2 - 10u + 25 = (u-5)^2$

$= \left(x^2 - 5\right)^2$

c. $(a+1)^2 - 10(a+1) + 25$

Let $u = a+1$

$u^2 - 10u + 25 = (u-5)^2$

$= \left((a+1) - 5\right)^2$

$= (a-4)^2$

b. $w^6 + 11w^3 - 26 = \left(w^3\right)^2 + 11w^3 - 26$

Let $u = w^3$

$u^2 + 11u - 26 = (u+13)(u-2)$

$= \left(w^3 + 13\right)\left(w^3 - 2\right)$

c. $(y-4)^2 + 11(y-4) - 26$

Let $u = y-4$

$u^2 + 11u - 26 = (u+13)(u-2)$

$= \left((y-4) + 13\right)\left((y-4) - 2\right)$

$= (y+9)(y-6)$

71. $(3x-1)^2 - (3x-1) - 6$

Let $u = 3x-1$

$u^2 - u - 6 = u^2 - 3u + 2u - 6$

$= u(u-3) + 2(u-3)$

$= (u-3)(u+2)$

$= \left((3x-1) - 3\right)\left((3x-1) + 2\right)$

$= (3x-4)(3x+1)$

73. $2(x-5)^2 + 9(x-5) + 4$

Let $u = x-5$

$2u^2 + 9u + 4 = 2u^2 + 8u + u + 4$

$= 2u(u+4) + (u+4)$

$= (u+4)(2u+1)$

$= \left((x-5) + 4\right)\left(2(x-5) + 1\right)$

$= (x-1)(2x - 10 + 1)$

$= (x-1)(2x-9)$

75. $3(y+4)^2 + 5(y+4) - 2$

Let $u = y+4$

$3u^2 + 5u - 2 = 3u^2 + 6u - u - 2$

$= 3u(u+2) - (u+2)$

$= (u+2)(3u-1)$

$= \left((y+4) + 2\right)\left(3(y+4) - 1\right)$

$= (y+6)(3y + 12 - 1)$

$= (y+6)(3y+11)$

77. $3y^6 + 11y^3 + 6$

Let $u = y^3$

$3u^2 + 11u + 6 = 3u^2 + 9u + 2u + 6$

$= 3u(u+3) + 2(u+3)$

$= (u+3)(3u+2)$

$= \left(y^3 + 3\right)\left(3y^3 + 2\right)$

79. $4p^4+5p^2+1$

Let $u=p^2$

$$4u^2+5u+1=4u^2+4u+u+1$$
$$=4u(u+1)+(u+1)$$
$$=(u+1)(4u+1)$$
$$=(p^2+1)(4p^2+1)$$

81. x^4+15x^2+36

Let $u=x^2$

$$u^2+15u+36=u^2+12u+3u+36$$
$$=u(u+12)+3(u+12)$$
$$=(u+12)(u+3)$$
$$=(x^2+12)(x^2+3)$$

83. The factorization $(2y-1)(2y-4)$ is not factored completely because the factor $2y-4$ has a greatest common factor of 2.

85. $w^4+12w^2+36=(w^2)^2+2(w^2)(6)+6^2$

$$=(w^2+6)^2$$

87. $81w^2+90w+25=(9w)^2+2(9w)(5)+5^2$

$$=(9w+5)^2$$

89. $3x(a+b)-6(a+b)=(a+b)(3x-6)$

$$=3(a+b)(x-2)$$

91. $12a^2bc^2+4ab^2c^2-6abc^3$

$$=2abc^2(6a+2b-3c)$$

93. $-20x^3+74x^2-60x=-2x(10x^2-37x+30)$

$$=-2x(10x^2-25x-12x+30)$$
$$=-2x[5x(2x-5)-6(2x-5)]$$
$$=-2x(2x-5)(5x-6)$$

95. $2y^2-9y-4$

Since there are not two factors of –8 whose sum is –9, the polynomial is prime.

97. $2(w^2-5)^2+(w^2-5)-15$

Let $u=w^2-5$

$$2u^2+u-15=2u^2+6u-5u-15=2u(u+3)-5(u+3)$$
$$=(u+3)(2u-5)=[(w^2-5)+3][2(w^2-5)-5]$$
$$=[w^2-5+3][2w^2-10-5]=(w^2-2)(2w^2-15)$$

99. $1-4d+3d^2 = 1-3d-d+3d^2$
$= (1-3d)-d(1-3d)$
$= (1-3d)(1-d)$ or $(3d-1)(d-1)$

101. $ax-5a^2+2bx-10ab$
$= a(x-5a)+2b(x-5a)$
$= (x-5a)(a+2b)$

103. $8z^2+24zw-224w^2 = 8(z^2+3zw-28w^2)$
$= 8(z^2+7zw-4zw-28w^2)$
$= 8[z(z+7w)-4w(z+7w)]$
$= 8(z+7w)(z-4w)$

105. $ay+ax-5cy-5cx = a(y+x)-5c(y+x)$
$= (y+x)(a-5c)$

107. $g(x) = 3x^2+14x+8$
$= 3x^2+12x+2x+8$
$= 3x(x+4)+2(x+4)$
$= (x+4)(3x+2)$

109. $n(t) = t^2+20t+100$
$= t^2+2(t)(10)+10^2$
$= (t+10)^2$

111. $Q(x) = x^4+6x^3+8x^2$
$= x^2(x^2+6x+8)$
$= x^2(x^2+4x+2x+8)$
$= x^2[x(x+4)+2(x+4)]$
$= x^2(x+4)(x+2)$

113. $k(a) = a^3-4a^2+2a-8$
$= a^2(a-4)+2(a-4)$
$= (a-4)(a^2+2)$

Section 4.7 Practice Exercises

1. **a.** difference; $(a+b)(a-b)$
b. sum
c. is not
d. square
e. sum; cubes
f. difference; cubes
g. $a-b$; a^2+ab+b^2
h. $a+b$; a^2-ab+b^2

3. $4x^2-20x+25 = (2x)^2-2(2x)(5)+5^2$
$= (2x-5)^2$

5. $10x+6xy+5+3y = 2x(5+3y)+(5+3y)$
$= (5+3y)(2x+1)$

7. $32p^2-28p-4=4\left(8p^2-7p-1\right)$
$=4\left(8p^2-8p+p-1\right)$
$=4\left[8p\left(p-1\right)+\left(p-1\right)\right]$
$=4\left(p-1\right)\left(8p+1\right)$

9. Look for a binomial of the form
a^2-b^2; $a^2-b^2=\left(a+b\right)\left(a-b\right)$

11. $x^2-9=x^2-3^2$
$=\left(x+3\right)\left(x-3\right)$

13. $16-49w^2=4^2-\left(7w\right)^2$
$=\left(4+7w\right)\left(4-7w\right)$

15. $8a^2-162b^2=2\left(4a^2-81b^2\right)$
$=2\left[\left(2a\right)^2-\left(9b\right)^2\right]$
$=2\left(2a+9b\right)\left(2a-9b\right)$

17. $25u^2+1$ Prime

19. $2a^4-32=2\left(a^4-16\right)$
$=2\left(a^2+4\right)\left(a^2-4\right)$
$=2\left(a^2+4\right)\left(a+2\right)\left(a-2\right)$

21. $49-k^6=7^2-\left(k^3\right)^2$
$=\left(7+k^3\right)\left(7-k^3\right)$

23. $x^3-x^2-16x+16=x^2\left(x-1\right)-16\left(x-1\right)$
$=\left(x-1\right)\left(x^2-16\right)$
$=\left(x-1\right)\left(x^2-4^2\right)$
$=\left(x-1\right)\left(x+4\right)\left(x-4\right)$

25. $4x^3+12x^2-x-3=4x^2\left(x+3\right)-\left(x+3\right)$
$=\left(x+3\right)\left(4x^2-1\right)$
$=\left(x+3\right)\left(\left(2x\right)^2-1^2\right)$
$=\left(x+3\right)\left(2x+1\right)\left(2x-1\right)$

27. $9y^3+7y^2-36y-28$
$=y^2\left(9y+7\right)-4\left(9y+7\right)$
$=\left(9y+7\right)\left(y^2-4\right)$
$=\left(9y+7\right)\left(y^2-2^2\right)$
$=\left(9y+7\right)\left(y+2\right)\left(y-2\right)$

29. $49x^2+28x+4-y^2=\left(49x^2+28x+4\right)-y^2$
$=\left(7x+2\right)^2-y^2$
$=\left(7x+2+y\right)\left(7x+2-y\right)$

31. w^2-9n^2+6n-1

$$\begin{aligned}&=w^2-\left(9n^2-6n+1\right)\\&=w^2-\left(3n-1\right)^2\\&=\left[w+\left(3n-1\right)\right]\left[w-\left(3n-1\right)\right]\\&=\left(w+3n-1\right)\left(w-3n+1\right)\end{aligned}$$

33. $p^4-10p^2+25-t^4$

$$\begin{aligned}&=\left(p^4-10p^2+25\right)-t^4\\&=\left(p^2-5\right)^2-\left(t^2\right)^2\\&=\left(p^2-5+t^2\right)\left(p^2-5-t^2\right)\end{aligned}$$

35. $9u^4-4v^4+20v^2-25$

$$\begin{aligned}&=9u^4-\left(4v^4-20v^2+25\right)\\&=\left(3u^2\right)^2-\left(2v^2-5\right)^2\\&=\left[3u^2+\left(2v^2-5\right)\right]\left[3u^2-\left(2v^2-5\right)\right]\\&=\left(3u^2+2v^2-5\right)\left(3u^2-2v^2+5\right)\end{aligned}$$

37. Look for a binomial of the form a^3+b^3;

$$a^3+b^3=\left(a+b\right)\left(a^2-ab+b^2\right)$$

39. $8x^3-1=\left(2x\right)^3-1^3$

$$\begin{aligned}&=\left(2x-1\right)\left[\left(2x\right)^2+\left(2x\right)\left(1\right)+1^2\right]\\&=\left(2x-1\right)\left(4x^2+2x+1\right)\end{aligned}$$

Check:

$$\begin{aligned}&\left(2x-1\right)\left(4x^2+2x+1\right)\\&\quad=8x^3+4x^2+2x-4x^2-2x-1\\&\quad=8x^3-1\end{aligned}$$

41. $125c^3+27=\left(5c\right)^3+3^3$

$$\begin{aligned}&=\left(5c+3\right)\left[\left(5c\right)^2-\left(5c\right)\left(3\right)+3^2\right]\\&=\left(5c+3\right)\left(25c^2-15c+9\right)\end{aligned}$$

43. $x^3-1000=x^3-10^3$

$$\begin{aligned}&=\left(x-10\right)\left[x^2+\left(x\right)\left(10\right)+10^2\right]\\&=\left(x-10\right)\left(x^2+10x+100\right)\end{aligned}$$

45. $64t^6+1=\left(4t^2\right)^3+1^3$

$$\begin{aligned}&=\left(4t^2+1\right)\left[\left(4t^2\right)^2-\left(4t^2\right)\left(1\right)+1^2\right]\\&=\left(4t^2+1\right)\left(16t^4-4t^2+1\right)\end{aligned}$$

47. $2000y^6+2x^3=2\left(1000y^6+x^3\right)$
$=2\left[\left(10y^2\right)^3+x^3\right]$
$=2\left(10y^2+x\right)\left[\left(10y^2\right)^2-\left(10y^2\right)(x)+x^2\right]$
$=2\left(10y^2+x\right)\left(100y^4-10y^2x+x^2\right)$

49. $16z^4-54z=2z\left(8z^3-27\right)$
$=2z\left[(2z)^3-3^3\right]$
$=2z(2z-3)\left[(2z)^2+(2z)(3)+3^2\right]$
$=2z(2z-3)\left(4z^2+6z+9\right)$

51. $p^{12}-125=\left(p^4\right)^3-5^3$
$=\left(p^4-5\right)\left[\left(p^4\right)^2+p^4(5)+5^2\right]$
$=\left(p^4-5\right)\left(p^8+5p^4+25\right)$

53. $36y^2-\frac{1}{25}=(6y)^2-\left(\frac{1}{5}\right)^2$
$=\left(6y+\frac{1}{5}\right)\left(6y-\frac{1}{5}\right)$

55. $18d^{12}-32=2\left(9d^{12}-16\right)$
$=2\left[\left(3d^6\right)^2-4^2\right]$
$=2\left(3d^6+4\right)\left(3d^6-4\right)$

57. $242v^2+32=2\left(121v^2+16\right)$

59. $4x^2-16=4\left(x^2-4\right)=4\left(x^2-2^2\right)$
$=4(x+2)(x-2)$

61. $25-49q^2=5^2-(7q)^2$
$=(5+7q)(5-7q)$

63. $(t+2s)^2-36=(t+2s)^2-6^2$
$=(t+2s+6)(t+2s-6)$

65. $27-t^3=3^3-t^3$
$=(3-t)\left[3^2+(3)(t)+t^2\right]$
$=(3-t)\left(9+3t+t^2\right)$

67. $27a^3+\frac{1}{8}=(3a)^3+\left(\frac{1}{2}\right)^3$
$=\left(3a+\frac{1}{2}\right)\left[(3a)^2-(3a)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2\right]$

69. $2m^3+16=2\left(m^3+8\right)=2\left(m^3+2^3\right)$
$=2(m+2)\left[m^2-(m)(2)+2^2\right]$
$=2(m+2)\left(m^2-2m+4\right)$

$$= \left(3a + \frac{1}{2}\right)\left(9a^2 - \frac{3}{2}a + \frac{1}{4}\right)$$

71. $x^4 - y^4 = \left(x^2\right)^2 - \left(y^2\right)^2$
$= \left(x^2 + y^2\right)\left(x^2 - y^2\right)$
$= \left(x^2 + y^2\right)(x + y)(x - y)$

73. $a^9 + b^9 = \left(a^3\right)^3 + \left(b^3\right)^3$
$= \left(a^3 + b^3\right)\left[\left(a^3\right)^2 - \left(a^3\right)\left(b^3\right) + \left(b^3\right)^2\right]$
$= \left(a^3 + b^3\right)\left(a^6 - a^3b^3 + b^6\right)$
$= (a + b)\left[a^2 - (a)(b) + b^2\right]\left(a^6 - a^3b^3 + b^6\right)$
$= (a + b)\left(a^2 - ab + b^2\right)\left(a^6 - a^3b^3 + b^6\right)$

75. $\frac{1}{8}p^3 - \frac{1}{125} = \left(\frac{1}{2}p\right)^3 - \left(\frac{1}{5}\right)^3$
$= \left(\frac{1}{2}p - \frac{1}{5}\right)\left[\left(\frac{1}{2}p\right)^2 + \left(\frac{1}{2}p\right)\left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)^2\right]$
$= \left(\frac{1}{2}p - \frac{1}{5}\right)\left(\frac{1}{4}p^2 + \frac{1}{10}p + \frac{1}{25}\right)$

77. $4w^2 + 25$ Prime

79. $\frac{1}{25}x^2 - \frac{1}{4}y^2 = \left(\frac{1}{5}x\right)^2 - \left(\frac{1}{2}y\right)^2$
$= \left(\frac{1}{5}x + \frac{1}{2}y\right)\left(\frac{1}{5}x - \frac{1}{2}y\right)$

81. $a^6 - b^6$
$= \left(a^3\right)^2 - \left(b^3\right)^2$
$= \left(a^3 + b^3\right)\left(a^3 - b^3\right)$
$= (a + b)\left(a^2 - ab + b^2\right)(a - b)\left(a^2 + ab + b^2\right)$

83. $64 - y^6 = 8^2 - \left(y^3\right)^2$
$= \left(8 + y^3\right)\left(8 - y^3\right)$
$= \left[2^3 + y^3\right]\left[2^3 - y^3\right]$
$= (2 + y)\left(4 - 2y + y^2\right)(2 - y)\left(4 + 2y + y^2\right)$

85. $h^6 + k^6 = \left(h^2\right)^3 + \left(k^2\right)^3$
$= \left(h^2 + k^2\right)\left(h^4 - h^2k^2 + k^4\right)$

87. $8x^6+125=\left(2x^2\right)^3+5^3$

$=\left(2x^2+5\right)\left[\left(2x^2\right)^2-\left(2x^2\right)(5)+5^2\right]$

$=\left(2x^2+5\right)\left(4x^4-10x^2+25\right)$

89. $(2x+3)(2x-3)=(2x)^2-3^2$

$=4x^2-9$

91. $\left(4a^2+6a+9\right)(2a-3)=(2a)^3-3^3$

$=8a^3-27$

93. $\left(4x^2+y\right)\left(16x^4-4x^2y+y^2\right)=\left(4x^2\right)^3+y^3$

$=64x^6+y^3$

95. **a.** $A=x^2-y^2$

b. $x^2-y^2=(x+y)(x-y)$

c. $A=x^2-y^2$

$=6^2-4^2=36-16=20\,\text{in}^2$

97. $x^2-y^2+x+y=(x+y)(x-y)+(x+y)$

$=(x+y)(x-y+1)$

99. x^3+y^3+x+y

$=(x+y)\left(x^2-xy+y^2\right)+x+y$

$=(x+y)\left(x^2-xy+y^2+1\right)$

101. $576a^5-9a^2-64a^3c^2+c^2$

$=9a^2(64a^3-1)-c^2(64a^3-1)$

$=(9a^2-c^2)(64a^3-1)$

$=(3a-c)(3a+c)(4a-1)(16a^2+4a+1)$

Problem Recognition Exercises:

1. A prime factor is an expression whose only factors are 1 and itself.

3. When factoring binomials, look for:
Difference of squares: a^2-b^2;
Difference of cubes: a^3-b^3; or
Sums of cubes: a^3+b^3.

5. Try factoring by grouping (2 terms and two terms) or grouping 3 terms and one term.

7. **a.** Trinomial

b. $6x^2-21x-45=3\left(2x^2-7x-15\right)$

$=3\left(2x^2-10x+3x-15\right)$

$$= 3\left[2x(x-5)+3(x-5)\right]$$
$$= 3(x-5)(2x+3)$$

9. a. Difference of squares

b. $8a^2-50=2\left(4a^2-25\right)$
$=2\left[(2a)^2-5^2\right]$
$=2(2a+5)(2a-5)$

11. a. Trinomial

b. $14u^2-11uv+2v^2$
$=14u^2-7uv-4uv+2v^2$
$=7u(2u-v)-2v(2u-v)$
$=(2u-v)(7u-2v)$

13. a. Difference of cubes

b. $16x^3-2=2\left(8x^3-1\right)=2\left[(2x)^3-1^3\right]$
$=2(2x-1)\left(4x^2+2x+1\right)$

15. a. Sum of cubes

b. $27y^3+125=(3y)^3+5^3$
$=(3y+5)\left(9y^2-15y+25\right)$

17. a. Sum of cubes

b. $128p^6+54q^3=2\left(64p^6+27q^3\right)$
$=2\left[\left(4p^2\right)^3+(3q)^3\right]$
$=2\left(4p^2+3q\right)\left(16p^4-12p^2q+9q^2\right)$

19. a. Difference of squares

b. $16a^4-1=\left(4a^2\right)-1^2$
$=\left(4a^2+1\right)\left(4a^2-1\right)$
$=\left(4a^2+1\right)(2a+1)(2a-1)$

21. a. Grouping

b. $p^2-12p+36-c^2=(p-6)^2-c^2$
$=(p-6+c)(p-6-c)$

23. a. Grouping

b. $12ax-6ay+4bx-2by$
$=2\left(6ax-3ay+2bx-by\right)$
$=2\left[3a(2x-y)+b(2x-y)\right]$
$=2(2x-y)(3a+b)$

25. a. Trinomial

b. $5y^2+14y-3=5y^2+15y-y-3$
$=5y(y+3)-(y+3)$
$=(y+3)(5y-1)$

27. a. Difference of squares

b. $t^2-100=t^2-10^2$
$=(t-10)(t+10)$

29. **a.** Sum of cubes

b. $y^3 + 27 = y^3 + 3^3$
$= (y+3)(y^2 - 3y + 9)$

31. **a.** Trinomial

b. $d^2 + 3d - 28 = (d+7)(d-4)$

33. **a.** Perfect square trinomial

b. $x^2 - 12x + 36 = x^2 - 2(x)(6) + (6)^2$
$= (x-6)^2$

35. **a.** Grouping

b. $2ax^2 - 5ax + 2bx - 5b$
$= ax(2x-5) + b(2x-5)$
$= (ax+b)(2x-5)$

37. **a.** Trinomial

b. $10y^2 + 3y - 4 = (2y-1)(5y+4)$

39. **a.** Difference of squares

b. $10p^2 - 640 = 10(p^2 - 64)$
$= 10(p-8)(p+8)$

41. **a.** Difference of cubes

b. $z^4 - 64z = z(z^3 - 64)$
$= z(z-4)(z^2 + 4z + 16)$

43. **a.** Trinomial

b. $b^3 - 4b^2 - 45b = b(b^2 - 4b - 45)$
$= b(b-9)(b+5)$

45. **a.** Perfect square trinomial

b. $9w^2 + 24wx + 16x^2$
$= (3w)^2 + 2(3w)(4x) + (4x)^2$
$= (3w+4x)^2$

47. **a.** Grouping

b. $60x^2 - 20x + 30ax - 10a$
$= 10(6x^2 - 2x + 3ax - a)$
$= 10[2x(3x-1) + a(3x-1)]$
$= 10(2x+a)(3x-1)$

49. **a.** Difference of squares

b. $w^4 - 16 = (w^2 - 4)(w^2 + 4)$
$= (w-2)(w+2)(w^2+4)$

51. **a.** Difference of cubes

b. $t^6 - 8 = (t^2)^3 - 2^3$
$= (t^2 - 2)(t^4 + 2t^2 + 4)$

53. **a.** Trinomial

b. $8p^2 - 22p + 5 = (4p-1)(2p-5)$

55. **a.** Perfect square trinomial

b. $36y^2 - 12y + 1$
$= (6y)^2 - 2(6y)(1) + (1)^2$
$= (6y-1)^2$

57. **a.** Sum of squares

b. $2x^2 + 50 = 2(x^2 + 25)$

59. **a.** Trinomial

b. $12r^2s^2 + 7rs^2 - 10s^2$

$= s^2(12r^2 + 7r - 10)$

$= s^2(4r + 5)(3r - 2)$

61. **a.** Trinomial

b. $x^2 + 8xy - 33y^2 = (x - 3y)(x + 11y)$

63. **a.** Sum of cubes

b. $m^6 + n^3 = (m^2)^3 + n^3$

$= (m^2 + n)(m^4 - m^2n + n^2)$

65. **a.** None of these

b. $x^2 - 4x = x(x - 4)$

67. $x^2(x + y) - y^2(x + y)$

$= (x + y)(x^2 - y^2)$

$= (x + y)(x + y)(x - y)$

$= (x + y)^2(x - y)$

69. $(a+3)^4 + 6(a+3)^5 = (a+3)^4(1 + 6(a+3))$

$= (a+3)^4(1 + 6a + 18)$

$= (a+3)^4(6a + 19)$

71. $24(3x+5)^3 - 30(3x+5)^2$

$= 6(3x+5)^2[4(3x+5) - 5]$

$= 6(3x+5)^2[12x + 15]$

$= 6(3x+5)^2 3(4x+5)$

$= 18(3x+5)^2(4x+5)$

73. $\frac{1}{100}x^2 + \frac{1}{35}x + \frac{1}{49}$

$= \left(\frac{1}{10}x\right)^2 + 2\left(\frac{1}{10}x\right)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2$

$= \left(\frac{1}{10}x + \frac{1}{7}\right)^2$

75. $\left(5x^2 - 1\right)^2 - 4\left(5x^2 - 1\right) - 5$

Let $u = 5x^2 - 1$

$u^2 - 4u - 5 = (u - 5)(u + 1)$

$= \left(5x^2 - 1 - 5\right)\left(5x^2 - 1 + 1\right)$

$= \left(5x^2 - 6\right)\left(5x^2\right)$

77. $16p^4 - q^4 = (4p^2)^2 - (q^2)^2$

$= (4p^2 + q^2)(4p^2 - q^2)$

$= (4p^2 + q^2)(2p + q)(2p - q)$

79. $y^3 + \frac{1}{64} = y^3 + \left(\frac{1}{4}\right)^3$

$= \left(y + \frac{1}{4}\right)\left(y^2 - \frac{1}{4}y + \frac{1}{16}\right)$

81. $6a^3 + a^2b - 6ab^2 - b^3$

$= a^2(6a+b) - b^2(6a+b)$

$= (6a+b)(a^2 - b^2)$

$= (6a+b)(a+b)(a-b)$

83. $\frac{1}{9}t^2 + \frac{1}{6}t + \frac{1}{16}$

$= \left(\frac{1}{3}t\right)^2 + 2\left(\frac{1}{3}t\right)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2$

$= \left(\frac{1}{3}t + \frac{1}{4}\right)^2$

85. $x^2 + 12x + 36 - a^2 = (x+6)^2 - a^2$

$= (x+6+a)(x+6-a)$

87. $p^2 + 2pq + q^2 - 81 = (p+q)^2 - 9^2$

$= (p+q+9)(p+q-9)$

89. $b^2 - (x^2 + 4x + 4) = b^2 - (x+2)^2$

$= (b + (x+2))(b - (x+2))$

$= (b + x + 2)(b - x - 2)$

91. $4 - u^2 + 2uv - v^2 = 4 - \left(u^2 - 2uv + v^2\right)$

$= 4 - (u-v)^2$

$= (2 + (u-v))(2 - (u-v))$

$= (2 + u - v)(2 - u + v)$

93. $6ax - by + 2bx - 3ay$

$= 6ax + 2bx - by - 3ay$

$= 2x(3a+b) - y(3a+b)$

$= (3a+b)(2x-y)$

95. $u^6 - 64$

$= (u^3)^2 - (8)^2$

$= (u^3 + 8)(u^3 - 8)$

$= (u+2)(u^2 - 2u + 4)(u-2)(u^2 + 2u + 4)$

$= (u+2)(u-2)(u^2 - 2u + 4)(u^2 + 2u + 4)$

97. $x^8 - 1 = (x^4)^2 - 1^2$

$= (x^4 + 1)(x^4 - 1)$

$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$

$= (x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$

99. $a^2 - b^2 + a + b = (a+b)(a-b) + (a+b)$

$= (a+b)(a-b+1)$

101. $5wx^3 + 5wy^3 - 2zx^3 - 2zy^3 = 5w(x^3 + y^3) - 2z(x^3 + y^3)$

$= (x^3 + y^3)(5w - 2z)$

$= (x+y)(x^2 - xy + y^2)(5w - 2z)$

Section 4.8 Practice Exercises

1. **a.** quadratic

b. 0; 0

c. Pythagorean; c^2

d. quadratic

e. $f(x)=0$; y

f. $x+1;\ x+2;\ x+2$

g. lw

h. $\frac{1}{2}bh$

3. $10x^2+3x=x\left(10x+3\right)$

5. $2p^2-9p-5=2p^2-10p+p-5$

$=2p\left(p-5\right)+\left(p-5\right)$

$=\left(p-5\right)\left(2p+1\right)$

7. $t^3-1=t^3-1^3=\left(t-1\right)\left(t^2+t+1\right)$

9. The equation must be set equal to 0, and the polynomial must be factored.

11. $2x\left(x-3\right)=0$ Correct form.

13. $3p^2-7p+4=0$ Incorrect form. The polynomial is not factored.

15. $a\left(a+3\right)^2=5$ Incorrect form. The equation is not set equal to 0.

17. **a.** $w^2-81=(w+9)(w-9)$

b. $w^2-81=0$

$(w+9)(w-9)=0$

$w+9=0$ or $w-9=0$

$w=-9$ or $w=9$ $\{-9,9\}$

19. **a.** $3x^2+14x-5=(3x-1)(x+5)$

b. $3x^2+14x-5=0$

$(3x-1)(x+5)=0$

$3x-1=0$ or $x+5=0$

$x=\frac{1}{3}$ or $x=-5$ $\left\{\frac{1}{3},-5\right\}$

21. $\left(x+3\right)\left(x+5\right)=0$

$x+3=0$ or $x+5=0$

$x=-3$ or $x=-5$ $\{-3,-5\}$

23. $\left(2w+9\right)\left(5w-1\right)=0$

$2w+9=0$ or $5w-1=0$

$2w=-9$ or $5w=1$

$w=-\frac{9}{2}$ or $w=\frac{1}{5}$ $\left\{-\frac{9}{2},\frac{1}{5}\right\}$

25. $x(x+4)(10x-3)=0$

$x=0$ or $x+4=0$ or $10x-3=0$

$x=0$ or $x=-4$ or $10x=3$

$x=0$ or $x=-4$ or $x=\frac{3}{10}$

$\left\{0,-4,\frac{3}{10}\right\}$

27. $0=5(y-0.4)(y+2.1)$

$5=0$ or $y-0.4=0$ or $y+2.1=0$

no solution $y=0.4$ or $y=-2.1$

$\{0.4,-2.1\}$

29. $x^2+6x-27=0$

$(x+9)(x-3)=0$

$x+9=0$ or $x-3=0$

$x=-9$ or $x=3$ $\{-9,3\}$

31. $2x^2+5x=3$

$2x^2+5x-3=0$

$2x^2+6x-x-3=0$

$2x(x+3)-(x+3)=0$

$(x+3)(2x-1)=0$

$x+3=0$ or $2x-1=0$

$x=-3$ or $2x=1$

$x=-3$ or $x=\frac{1}{2}$ $\left\{-3,\frac{1}{2}\right\}$

33. $10x^2=15x$

$10x^2-15x=0$

$5x(2x-3)=0$

$5x=0$ or $2x-3=0$

$x=0$ or $2x=3$

$x=0$ or $x=\frac{3}{2}$ $\left\{0,\frac{3}{2}\right\}$

35. $6(y-2)-3(y+1)=8$

$6y-12-3y-3=8$

$3y-15=8$

$3y=23$

$y=\frac{23}{3}$ $\left\{\frac{23}{3}\right\}$

37. $-9=y(y+6)$

$-9=y^2+6y$

$y^2+6y+9=0$

$(y+3)^2=0$

$y+3=0$

$y=-3$ $\{-3\}$

39. $9p^2-15p-6=0$

$3(3p^2-5p-2)=0$

$3(3p^2-6p+p-2)=0$

$3[3p(p-2)+(p-2)]=0$

$3(p-2)(3p+1)=0$

$3=0$ or $p-2=0$ or $3p+1=0$

$p=2$ or $3p=-1$

no solution $\quad p=2$ or $p=-\frac{1}{3}$

$\left\{2,-\frac{1}{3}\right\}$

41. $(x+1)(2x-1)(x-3)=0$

$x+1=0$ or $2x-1=0$ or $x-3=0$

$x=-1$ or $2x=1$ or $x=3$

$x=-1$ or $x=\frac{1}{2}$ or $x=3$

$\left\{-1,\frac{1}{2},3\right\}$

43. $(y-3)(y+4)=8$

$y^2+y-12=8$

$y^2+y-20=0$

$(y+5)(y-4)=0$

$y+5=0$ or $y-4=0$

$y=-5$ or $y=4$ $\{-5,4\}$

45. $(2a-1)(a-1)=6$

$2a^2-3a+1=6$

$2a^2-3a-5=0$

$(2a-5)(a+1)=0$

$2a-5=0$ or $a+1=0$

$2a=5$ or $a=-1$

$a=\frac{5}{2}$ or $a=-1$ $\left\{\frac{5}{2},-1\right\}$

47. $p^2+(p+7)^2=169$

$p^2+p^2+14p+49=169$

$2p^2+14p-120=0$

$2(p^2+7p-60)=0$

$2(p+12)(p-5)=0$

$2\neq 0$ or $p+12=0$ or $p-5=0$

$p=-12$ or $p=5$ $\{-12,5\}$

49. $3t(t+5)-t^2=2t^2+4t-1$

$3t^2+15t-t^2=2t^2+4t-1$

$11t=-1$

$t=-\frac{1}{11}$ $\left\{-\frac{1}{11}\right\}$

51. $2x^3-8x^2-24x=0$

$2x(x^2-4x-12)=0$

$2x(x-6)(x+2)=0$

$2x=0$ or $x-6=0$ or $x+2=0$

$x=0$ or $x=6$ or $x=-2$ $\{0,6,-2\}$

53. $w^3=16w$

$w^3-16w=0$

$w(w^2-16)=0$

$w(w+4)(w-4)=0$

55. $0=2x^3+5x^2-18x-45$

$0=x^2(2x+5)-9(2x+5)$

$0=(2x+5)(x^2-9)$

$0=(2x+5)(x+3)(x-3)$

$w=0$ or $w+4=0$ or $w-4=0$

$w=0$ or $x=-4$ or $x=4$

$\{0,-4,4\}$

$2x+5=0$ or $x+3=0$ or $x-3=0$

$2x=-5$ or $x=-3$ or $x=3$

$x=-\frac{5}{2}$ or $x=-3$ or $x=3$

$\left\{-\frac{5}{2},-3,3\right\}$

57. Let x = the number

$x^2+5=30$

$x^2-25=0$

$(x+5)(x-5)=0$

$x+5=0$ or $x-5=0$

$x=-5$ or $x=5$

59. Let x = the number

$x^2=x+12$

$x^2-x-12=0$

$(x+3)(x-4)=0$

$x+3=0$ or $x-4=0$

$x=-3$ or $x=4$

61. Let x = the first consecutive integer

$x+1$ = the second consecutive integer

$x(x+1)=42$

$x^2+x=42$

$x^2+x-42=0$

$(x+7)(x-6)=0$

$x+7=0$ or $x-6=0$

$x=-7$ or $x=6$

$x+1=-7+1=-6$ or $x+1=6+1=7$

The consecutive integers are –7 and –6 or 6 and 7.

63. Let x = the first consecutive odd integer

$x+2$ = second consecutive odd integer

$x(x+2)=63$

$x^2+2x=63$

$x^2+2x-63=0$

$(x+9)(x-7)=0$

$x+9=0$ or $x-7=0$

$x=-9$ or $x=7$

$x+2=-9+2=-7$ or $x+2=7+2=9$

The consecutive odd integers are –9 and –7 or 7 and 9.

65. Let x = the length

$x-2$ = the width

67. Let x = the width

$x+5$ = the length

$$x(x-2)=35$$
$$x^2-2x=35$$
$$x^2-2x-35=0$$
$$(x+5)(x-7)=0$$
$$x+5=0 \quad \text{or} \quad x-7=0$$
$$x \neq -5 \quad \text{or} \quad x=7$$
$$\text{or} \quad x-2=7-2=5$$

The length is 7 ft and the width is 5 ft.

$$x(x+5)=300$$
$$x^2+5x=300$$
$$x^2+5x-300=0$$
$$(x+20)(x-15)=0$$
$$x+20=0 \quad \text{or} \quad x-15=0$$
$$x \neq -20 \quad \text{or} \quad x=15$$
$$\text{or} \quad x+5=15+5=20$$

The width is 15 yd and the length is 20 yd.

69. a. Let b = the base of the triangle

$b+1$ = the height of the triangle

$$\frac{1}{2}b(b+1+2)=20$$
$$b(b+3)=40$$
$$b^2+3b=40$$
$$b^2+3b-40=0$$

b. $(b+8)(b-5)=0$

$$b+8=0 \quad \text{or} \quad b-5=0$$
$$b \neq -8 \quad \text{or} \quad b=5$$
$$b+1=5+1=6$$

The base is 5 in and the height is 6 in.

$$A=\frac{1}{2}(5)(6)=15 \text{ in}^2$$

The area is 15 in^2.

71. Let h = the height of the triangle

$2h$ = the base of the triangle

$$\frac{1}{2}(2h)(h)=25$$
$$h^2=25$$
$$h^2-25=0$$
$$(h+5)(h-5)=0$$
$$h+5=0 \quad \text{or} \quad h-5=0$$
$$h \neq -5 \quad \text{or} \quad h=5$$
$$2h=2(5)=10$$

The height is 5 ft and the base is 10 ft.

73. Let x = the first positive consecutive integer

$x+1$ = second pos consecutive integer

$$x^2+(x+1)^2=41$$
$$x^2+x^2+2x+1=41$$
$$2x^2+2x-40=0$$

75. a. Let x = the northern leg

$x-2$ = the eastern leg

$$x^2+(x-2)^2=10^2$$
$$x^2+x^2-4x+4=100$$
$$2x^2-4x-96=0$$
$$2(x^2-2x-48)=0$$
$$2(x+6)(x-8)=0$$

$$2\left(x^2+x-20\right)=0$$
$$2\left(x+5\right)\left(x-4\right)=0$$
$$x+5=0 \text{ or } x-4=0$$
$$x\neq -5 \text{ or } x=4$$
$$x+1=4+1=5$$

The consecutive positive integers are 4 and 5.

$$x+6=0 \text{ or } x-8=0$$
$$x\neq -6 \text{ or } x=8$$
$$x-2=8-2=6$$

b. The alternative route is 8 mi + 6 mi =14 mi.

$$t=\frac{d}{r}=\frac{10}{40}=\frac{1}{4}=0.25 \text{ hr}$$
$$t=\frac{d}{r}=\frac{14}{60}=\frac{7}{30}\approx 0.23 \text{ hr}$$

The alternative route using superhighways takes less time.

77. Let x = the first consecutive even integer

$x+2$ = second consecutive even integer

$x+4$ = third consecutive even integer

$$x^2+\left(x+2\right)^2=\left(x+4\right)^2$$
$$x^2+x^2+4x+4=x^2+8x+16$$
$$x^2-4x-12=0$$
$$\left(x+2\right)\left(x-6\right)=0$$
$$x+2=0 \text{ or } x-6=0$$
$$x\neq -2 \text{ or } x=6$$
$$x+2=6+2=8$$
$$x+4=6+4=10$$

The lengths of the sides are 6 m, 8 m, and 10 m.

79. Let r = the radius of the circle

$$\pi r^2=2\pi r$$
$$r^2=2r$$
$$r^2-2r=0$$
$$r\left(r-2\right)=0$$
$$r=0 \text{ or } r-2=0$$
$$r=0 \text{ or } r=2$$

The radius is 2 units.

81. **a.** $f\left(x\right)=x^2-3x=0$

$$x\left(x-3\right)=0$$
$$x=0 \text{ or } x-3=0$$
$$x=0 \text{ or } x=3$$

b. $f\left(0\right)=0^2-3\left(0\right)=0-0=0$

83. **a.** $f\left(x\right)=x^2-6x-7=0$

$$\left(x-7\right)\left(x+1\right)=0$$
$$x-7=0 \text{ or } x+1=0$$
$$x=7 \text{ or } x=-1$$

b. $f\left(0\right)=0^2-6\left(0\right)-7=0-0-7=-7$

85. $f(x)=\frac{1}{2}(x-2)(x+1)(2x)=0$

$\frac{1}{2}\neq 0$ or $x-2=0$ or $x+1=0$ or $2x=0$

$x=2$ or $x=-1$ or $x=0$

$f(0)=\frac{1}{2}(0-2)(0+1)(2\cdot 0)$

$=\frac{1}{2}(-2)(1)(0)=0$

x-intercepts: (2, 0), (–1, 0), (0, 0)

y-intercept: (0, 0)

87. $f(x)=x^2-2x+1=0$

$(x-1)^2=0$

$x-1=0$

$x=1$

$f(0)=0^2-2(0)+1=0-0+1=1$

x-intercepts: (1, 0)

y-intercept: (0, 1)

89. $g(x)=(x+3)(x-3)=0$

$(x+3)=0$ or $x-3=0$

$x=-3$ or $x=3$

x-intercepts: (–3, 0), (3,0)

Graph d.

91. $f(x)=4(x+1)=0$

$4\neq 0$ or $x+1=0$

$x=-1$

x-intercepts: (–1, 0)

Graph a.

93. **a.** The function is in the form

$s(t)=at^2+bt+c$

b. $s(t)=-4.9t^2+490t=0$

$-4.9t(t-100)=0$

$-4.9t=0$ or $t-100=0$

$t=0$ or $t=100$

c. t-intercepts (0, 0), (100, 0)

d. At 0 sec and 100 sec, the rocket is at ground level (height = 0).

$s(t)=-4.9t^2+490t=485.1$

$-4.9t^2+490t-485.1=0$

$-4.9(t^2-100t+99)=0$

$-4.9(t-1)(t-99)=0$

$-4.9\neq 0$ or $t-1=0$ or $t-99=0$

$t=1$ or $t=99$

The height is 485.1 m at 1 sec and 99 sec.

95. $f(x)=x^2-7x+10=0$

$f(x)=(x-5)(x-2)=0$

$x-5=0$ or $x-2=0$

$x=5$ or $x=2$

$x = 5$ and $x = 2$ represent the x-intercepts.

97. $f(x)=x^2+2x+1=0$

$f(x)=(x+1)^2=0$

$x+1=0$

$x=-1$

$x = -1$represents the x-intercept.

99. $f(x) = -x^2 - 6x - 5 = 0$

$f(x) = -(x^2 + 6x + 5) = 0$

$f(x) = -(x+1)(x+5) = 0$

$x + 1 = 0 \quad \text{or} \quad x + 5 = 0$

$x = -1 \quad \text{or} \quad x = -5$

$x = -1$ and $x = -5$ represent the x-intercepts.

101. $SA = 2\pi r^2 + 2\pi rh$

$2\pi r^2 + 2\pi r(7) = 156\pi$

$r^2 + 7r = 78$

$r^2 + 7r - 78 = 0$

$(r+13)(r-6) = 0$

$r + 13 = 0 \quad \text{or} \quad r - 6 = 0$

$r = -13 \quad \text{or} \quad r = 6$

The radius is 6 ft.

103. Let l = the length w = the width

$2l + 2w = 28$

$2w = 28 - 2l$

$w = 14 - l$

$A = l(14 - l) = 48$

$14l - l^2 = 48$

$0 = l^2 - 14l + 48$

$0 = (l-8)(l-6)$

$l - 8 = 0 \quad \text{or} \quad l - 6 = 0$

$l = 8 \quad \text{or} \quad l = 6$

$w = 14 - 8 = 6$

The length is 8 ft and the width is 6 ft.

105. $x = 2$ and $x = -2$

$(x-2)(x+2) = 0$

$x^2 - 4 = 0$

107. $x = 0$ and $x = -3$

$(x-0)(x+3) = 0$

$x^2 + 3x = 0$