

Chapter 5 Rational Expressions and Functions

Section 5.1 Practice Exercises

1. a. rational
b. denominator

- c. $\frac{p}{q}$
d. 1; -1

3.
$$k(x) = \frac{-3}{x+4}$$

$$k(0) = \frac{-3}{0+4} = -\frac{3}{4}$$

$$k(-1) = \frac{-3}{-1+4} = -\frac{3}{3} = -1$$

$$k(2) = \frac{-3}{2+4} = -\frac{3}{6} = -\frac{1}{2}$$

$$k(-4) = \frac{-3}{-4+4}$$

$$= -\frac{3}{0} \text{ is undefined}$$

5.
$$n(a) = \frac{3a+1}{a^2+1}$$

$$n(1) = \frac{3(1)+1}{(1)^2+1} = \frac{3+1}{1+1} = \frac{4}{2} = 2$$

$$n(0) = \frac{3(0)+1}{(0)^2+1} = \frac{0+1}{0+1} = \frac{1}{1} = 1$$

$$n\left(-\frac{1}{3}\right) = \frac{3\left(-\frac{1}{3}\right)+1}{\left(-\frac{1}{3}\right)^2+1} = \frac{-1+1}{\frac{1}{9}+1} = \frac{0}{\frac{10}{9}} = 0$$

$$n(-1) = \frac{3(-1)+1}{(-1)^2+1} = \frac{-3+1}{1+1} = \frac{-2}{2} = -1$$

7.
$$f(x) = \frac{9}{x}$$

$$x \neq 0$$

a. $\{x \mid x \text{ is a real number and } x \neq 0\}$
b. $(-\infty, 0) \cup (0, \infty)$

9.
$$h(v) = \frac{v+1}{v-7}$$

$$v-7 \neq 0$$

$$v \neq 7$$

a. $\{v \mid v \text{ is a real number and } v \neq 7\}$
b. $(-\infty, 7) \cup (7, \infty)$

11.
$$k(x) = \frac{3x-1}{2x-5}$$

$$2x-5 \neq 0$$

$$2x \neq 5$$

$$x \neq \frac{5}{2}$$

13.
$$f(q) = \frac{q+1}{q^2+6q-27}$$

$$q^2+6q-27 \neq 0$$

$$(q+9)(q-3) \neq 0$$

$$q+9 \neq 0 \quad \text{or} \quad q-3 \neq 0$$

$$q \neq -9 \quad \text{or} \quad q \neq 3$$

a. $\left\{x \mid x \text{ is a real number and } x \neq \frac{5}{2}\right\}$

b. $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

15.

$$h(c) = \frac{c}{c^2 + 25}$$

Because c^2 is nonnegative for any real number c , the denominator $c^2 + 25$ cannot equal zero; therefore, no real numbers are excluded from the domain.

a. $\{c \mid c \text{ is a real number}\}$

b. $(-\infty, \infty)$

19.

$$p(x) = \frac{x-5}{3}$$

a. $\{x \mid x \text{ is a real number}\}$

b. $(-\infty, \infty)$

23.

$$q(x) = \frac{1}{x-4}$$

$$x-4=0 \Rightarrow x=4$$

$$D: (-\infty, 4) \cup (4, \infty)$$

Graph: d

27.

$$\frac{x^2 + 6x + 8}{x^2 + 3x - 4}$$

a. $\frac{(x+4)(x+2)}{(x+4)(x-1)}$

a. $\left\{q \mid q \text{ is a real number and } q \neq -9, q \neq 3\right\}$

b. $(-\infty, -9) \cup (-9, 3) \cup (3, \infty)$

17.

$$f(x) = \frac{x+5}{x^2 - 25}$$

$$x^2 - 25 \neq 0$$

$$(x+5)(x-5) \neq 0$$

$$x+5 \neq 0$$

$$\text{or } x-5 \neq 0$$

$$x \neq -5$$

$$\text{or } x \neq 5$$

a. $\left\{x \mid x \text{ is a real number and } x \neq -5, x \neq 5\right\}$

b. $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

21.

$$m(x) = \frac{1}{x+4}$$

$$x+4=0$$

$$x=-4$$

$$D: (-\infty, -4) \cup (-4, \infty)$$

Graph: b

25.

a. $\frac{8x}{4y} = \frac{2x}{y}$

b. $\frac{8+x}{4+y}$ cannot be simplified

29.

$$\frac{x^2 - 18x + 81}{x^2 - 81}$$

a. $\frac{(x-9)(x-9)}{(x+9)(x-9)}$

Section 5.1 Rational Expressions and Rational Functions

$$\begin{aligned} \text{b. } (x+4)(x-1) &\neq 0 \\ x+4 &\neq 0 \text{ or } x-1 \neq 0 \\ x &\neq -4 \text{ or } x \neq 1 \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{\cancel{(x+4)}(x+2)}{\cancel{(x+4)}(x-1)} &= \frac{x+2}{x-1} \\ &\text{provided } x \neq -4, x \neq 1 \end{aligned}$$

$$\begin{aligned} \text{b. } (x+9)(x-9) &\neq 0 \\ x+9 &\neq 0 \text{ or } x-9 \neq 0 \\ x &\neq -9 \text{ or } x \neq 9 \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{(x-9)\cancel{(x-9)}}{(x+9)\cancel{(x-9)}} &= \frac{x-9}{x+9} \\ &\text{provided } x \neq -9, x \neq 9 \end{aligned}$$

$$\begin{aligned} \text{31. } \frac{100x^3y^5}{36xy^8} &= \frac{25}{9}x^{3-1}y^{5-8} = \frac{25}{9}x^2y^{-3} \\ &= \frac{25x^2}{9y^3} \text{ provided } x \neq 0, y \neq 0 \end{aligned}$$

$$\begin{aligned} \text{33. } \frac{7w^{11}z^6}{14w^3z^3} &= \frac{1}{2}w^{11-3}z^{6-3} = \frac{1}{2}w^8z^3 \\ &= \frac{w^8z^3}{2} \text{ provided } w \neq 0, z \neq 0 \end{aligned}$$

$$\begin{aligned} \text{35. } \frac{-3m^4n}{12m^6n^4} &= -\frac{1}{4}m^{4-6}n^{1-4} = -\frac{1}{4}m^{-2}n^{-3} \\ &= -\frac{1}{4m^2n^3} \text{ provided } m \neq 0, n \neq 0 \end{aligned}$$

$$\text{37. } \frac{6a+18}{9a+27} = \frac{6\cancel{(a+3)}}{9\cancel{(a+3)}} = \frac{2}{3} \text{ provided } a \neq -3$$

$$\begin{aligned} \text{39. } \frac{x-5}{x^2-25} &= \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}} = \frac{1}{x+5} \\ &\text{provided } x \neq -5, x \neq 5 \end{aligned}$$

$$\begin{aligned} \text{41. } \frac{-7c}{21c^2-35c} &= \frac{-1 \cdot \cancel{7c}}{\cancel{7c}(3c-5)} = -\frac{1}{3c-5} \\ &\text{provided } c \neq 0, c \neq \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{43. } \frac{2t^2+7t-4}{-2t^2-5t+3} &= \frac{(2t-1)(t+4)}{-(2t^2+5t-3)} \\ &= \frac{\cancel{(2t-1)}(t+4)}{-\cancel{(2t-1)}(t+3)} \\ &= -\frac{t+4}{t+3} \text{ provided } t \neq \frac{1}{2}, t \neq -3 \end{aligned}$$

$$\begin{aligned} \text{45. } \frac{(p+1)(2p-1)^4}{(p+1)^2(2p-1)^2} &= (p+1)^{1-2}(2p-1)^{4-2} \\ &= (p+1)^{-1}(2p-1)^2 \\ &= \frac{(2p-1)^2}{p+1} \text{ provided } p \neq \frac{1}{2}, p \neq -1 \end{aligned}$$

$$\begin{aligned} \text{47. } \frac{9-z^2}{2z^2+z-15} &= \frac{\cancel{(3+z)}(3-z)}{(2z-5)\cancel{(z+3)}} \\ &= \frac{3-z}{2z-5} \\ &\text{provided } z \neq \frac{5}{2}, z \neq -3 \end{aligned}$$

$$\begin{aligned} \text{49. } \frac{2z^3+128}{16+8z+z^2} &= \frac{2(z^3+64)}{z^2+8z+16} \\ &= \frac{2\cancel{(z+4)}(z^2-4z+16)}{(z+4)\cancel{(z+4)}} = \frac{2(z^2-4z+16)}{z+4} \\ &\text{provided } z \neq -4 \end{aligned}$$

$$\begin{aligned}
 51. \quad \frac{10x^3 - 25x^2 + 4x - 10}{-4 - 10x^2} &= \frac{5x^2(2x-5) + 2(2x-5)}{-2(5x^2+2)} \\
 &= \frac{(2x-5)\cancel{(5x^2+2)}}{-2\cancel{(5x^2+2)}} = -\frac{2x-5}{2}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{r+6}{6+r} &= \frac{r+6}{r+6} = 1 \\
 &\text{provided } r \neq -6
 \end{aligned}$$

$$\begin{aligned}
 55. \quad \frac{b+8}{-b-8} &= \frac{\cancel{b+8}}{-\cancel{(b+8)}} \\
 &= -1 \quad \text{provided } b \neq -8
 \end{aligned}$$

$$\begin{aligned}
 57. \quad \frac{10-x}{x-10} &= \frac{-\cancel{(x-10)}}{\cancel{x-10}} \\
 &= -1 \\
 &\text{provided } x \neq 10
 \end{aligned}$$

$$\begin{aligned}
 59. \quad \frac{2t-2}{1-t} &= \frac{2\cancel{(t-1)}}{-\cancel{(t-1)}} \\
 &= -2 \\
 &\text{provided } t \neq 1
 \end{aligned}$$

$$61. \quad \frac{c+4}{c-4} \text{ cannot be simplified}$$

$$\begin{aligned}
 63. \quad \frac{y-x}{12x^2-12y^2} &= \frac{-(x-y)}{12(x^2-y^2)} = \frac{-\cancel{(x-y)}}{12(x+y)\cancel{(x-y)}} \\
 &= -\frac{1}{12(x+y)} \quad \text{provided } x \neq y, x \neq -y
 \end{aligned}$$

$$\begin{aligned}
 65. \quad \frac{t^2-1}{t^2+7t+6} &= \frac{(t-1)\cancel{(t+1)}}{(t+6)\cancel{(t+1)}} \\
 &= \frac{t-1}{t+6} \\
 &\text{provided } t \neq -6, t \neq -1
 \end{aligned}$$

$$\begin{aligned}
 67. \quad \frac{8p+8}{2p^2-4p-6} &= \frac{8(p+1)}{2(p^2-2p-3)} \\
 &= \frac{\cancel{2} \cdot 4\cancel{(p+1)}}{\cancel{2}(p-3)\cancel{(p+1)}} = \frac{4}{p-3} \\
 &\text{provided } p \neq 3, p \neq -1
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \frac{-16a^2bc^4}{8ab^2c^4} &= -\frac{16}{8}a^{2-1}b^{1-2}c^{4-4} \\
 &= -2a^1b^{-1}c^0 \\
 &= -\frac{2a}{b} \quad \text{provided } a \neq 0, b \neq 0, c \neq 0
 \end{aligned}$$

$$\begin{aligned}
 71. \quad \frac{x^2-y^2}{8y-8x} &= \frac{\cancel{(x-y)}(x+y)}{-8\cancel{(x-y)}} \\
 &= -\frac{x+y}{8} \quad \text{provided } x \neq y
 \end{aligned}$$

$$73. \quad \frac{b+4}{2b^2+5b-12} = \frac{\cancel{b+4}}{(2b-3)(\cancel{b+4})}$$

$$= \frac{1}{2b-3}$$

provided $b \neq \frac{3}{2}, b \neq -4$

$$75. \quad \frac{-2x+34}{-4x+6} = \frac{\cancel{-2}(x-17)}{\cancel{-2}(2x-3)}$$

$$= \frac{x-17}{2x-3}$$

provided $x \neq \frac{3}{2}$

$$77. \quad \frac{(a-2)^2(a-5)^3}{(a-2)^3(a-5)} = (a-2)^{2-3}(a-5)^{3-1}$$

$$= (a-2)^{-1}(a-5)^2$$

$$= \frac{(a-5)^2}{a-2}$$

provided $a \neq 2, a \neq 5$

$$79. \quad \frac{4x-2x^2}{5x-10} = \frac{-2x(\cancel{x-2})}{5(\cancel{x-2})}$$

$$= -\frac{2x}{5}$$

provided $x \neq 2$

$$81. \quad \frac{x^3-2x^2-25x+50}{x^3+5x^2-4x-20}$$

$$= \frac{x^2(x-2)-25(x-2)}{x^2(x+5)-4(x+5)} = \frac{(x-2)(x^2-25)}{(x+5)(x^2-4)}$$

$$= \frac{(\cancel{x-2})(x+5)(x-5)}{(\cancel{x+5})(x+2)(\cancel{x-2})}$$

$$= \frac{x-5}{x+2} \quad \text{provided } x \neq -5, x \neq -2, x \neq 2$$

$$83. \quad \frac{t^3+8}{3t^2+t-10} = \frac{(\cancel{t+2})(t^2-2t+4)}{(3t-5)(\cancel{t+2})}$$

$$= \frac{t^2-2t+4}{3t-5}$$

provided $t \neq \frac{5}{3}, t \neq -2$

$$85. \quad \text{For example: } \frac{1}{x-2}$$

$$87. \quad \text{For example: } f(x) = \frac{1}{x+5}$$

Section 5.2 Practice Exercises

$$1. \quad \text{a. } \frac{pr}{qs}$$

$$\text{b. } \frac{ps}{qr}$$

$$3. \quad \frac{t^2-5t-6}{t^2-7t+6} = \frac{(\cancel{t-6})(t+1)}{(\cancel{t-6})(t-1)} = \frac{t+1}{t-1}$$

$$5. \quad \frac{2-p}{p^2-p-2} = \frac{-1(\cancel{p-2})}{(\cancel{p-2})(p+1)} = -\frac{1}{p+1}$$

$$7. \frac{7x+14}{7x^2-7x-42} = \frac{7(x+2)}{7(x^2-x-6)}$$

$$= \frac{\cancel{7}(x+2)}{\cancel{7}(x-3)\cancel{(x+2)}} = \frac{1}{x-3}$$

$$9. \frac{a^3b^2c^5}{2a^3bc^2} = \frac{1}{2}a^{3-3}b^{2-1}c^{5-2}$$

$$= \frac{1}{2}a^0bc^3$$

$$= \frac{bc^3}{2}$$

$$11. \frac{16}{z^7} \cdot \frac{z^4}{8} = \frac{16z^4}{8z^7} = 2z^{4-7}$$

$$= 2z^{-3} = \frac{2}{z^3}$$

$$13. \frac{27r^5}{7s} \cdot \frac{28rs^3}{9r^3s^2} = \frac{\cancel{9} \cdot 3 \cancel{r^5} \cdot r^2}{\cancel{7} \cancel{s}} \cdot \frac{\cancel{7} \cdot 4r \cancel{s^3} \cdot \cancel{s}}{\cancel{9} \cancel{r^3} \cancel{s^2}}$$

$$= 3 \cdot 4 \cdot r^2 \cdot r = 12r^{2+1} = 12r^3$$

$$15. \frac{x^2y}{x^2-4x-5} \cdot \frac{2x^2-13x+15}{xy^3}$$

$$= \frac{\cancel{x} \cdot x \cdot \cancel{y}}{(x-5)(x+1)} \cdot \frac{(2x-3)\cancel{(x-5)}}{\cancel{x} \cdot \cancel{y} \cdot y^2}$$

$$= \frac{x(2x-3)}{y^2(x+1)}$$

$$17. \frac{10w-8}{w+2} \cdot \frac{3w^2-w-14}{25w^2-16}$$

$$= \frac{2(5w-4)}{\cancel{w+2}} \cdot \frac{(3w-7)\cancel{(w+2)}}{(5w+4)\cancel{(5w-4)}}$$

$$= \frac{2(3w-7)}{5w+4}$$

$$19. \frac{3x-15}{4x^2-2x} \cdot \frac{10x-20x^2}{5-x}$$

$$= \frac{3\cancel{(x-5)}}{\cancel{2x}(2x-1)} \cdot \frac{\cancel{5} \cdot 2\cancel{x}(2x-1)}{\cancel{5}\cancel{(x-5)}} = 15$$

$$21. y(y^2-4) \cdot \frac{y}{y+2} = \frac{y^2\cancel{(y+2)}(y-2)}{\cancel{y+2}}$$

$$= y^2(y-2)$$

$$23. \frac{2a}{7b^3} \div \frac{10a^5}{77} = \frac{2a}{7b^3} \cdot \frac{77}{10a^5}$$

$$= \frac{\cancel{2a}}{\cancel{7}b^3} \cdot \frac{\cancel{7} \cdot 11}{\cancel{2a} \cdot 5a^4} = \frac{11}{5a^4b^3}$$

$$25. \frac{(r+3)^2}{4r^3s} \div \frac{r+3}{rs} = \frac{(r+3)^2}{4r^3s} \cdot \frac{rs}{r+3}$$

$$= \frac{(r+3)\cancel{(r+3)}}{4r^2 \cdot \cancel{r} \cdot \cancel{s}} \cdot \frac{\cancel{r} \cdot \cancel{s}}{\cancel{r+3}} = \frac{r+3}{4r^2}$$

$$27. \frac{6p+7}{p+2} \div (36p^2-49)$$

$$= \frac{6p+7}{p+2} \cdot \frac{1}{36p^2-49}$$

$$29. \frac{b^2-6b+9}{b^2-b-6} \div \frac{b^2-9}{4}$$

$$= \frac{b^2-6b+9}{b^2-b-6} \cdot \frac{4}{b^2-9}$$

$$\begin{aligned}
 &= \frac{\cancel{6p+7}}{p+2} \cdot \frac{1}{(\cancel{6p+7})(6p-7)} &= \frac{(\cancel{b-3})(\cancel{b-3})}{(\cancel{b-3})(b+2)} \cdot \frac{4}{(b+3)(\cancel{b-3})} \\
 &= \frac{1}{(p+2)(6p-7)} &= \frac{4}{(b+2)(b+3)}
 \end{aligned}$$

$$\begin{aligned}
 31. \quad \frac{6s^2 + st - 2t^2}{6s^2 - 5st + t^2} \div \frac{3s^2 + 17st + 10t^2}{6s^2 + 13st - 5t^2} &= \frac{6s^2 + st - 2t^2}{6s^2 - 5st + t^2} \cdot \frac{6s^2 + 13st - 5t^2}{3s^2 + 17st + 10t^2} \\
 &= \frac{(\cancel{3s+2t})(\cancel{2s-t})}{(\cancel{3s-t})(\cancel{2s-t})} \cdot \frac{(\cancel{3s-t})(2s+5t)}{(\cancel{3s+2t})(s+5t)} = \frac{2s+5t}{s+5t}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad \frac{a^3 + a + a^2 + 1}{a^3 + a^2 + ab^2 + b^2} \div \frac{a^3 + a + a^2b + b}{2a^2 + 2ab + ab^2 + b^3} &= \frac{a^3 + a + a^2 + 1}{a^3 + a^2 + ab^2 + b^2} \cdot \frac{2a^2 + 2ab + ab^2 + b^3}{a^3 + a + a^2b + b} \\
 &= \frac{a(a^2+1) + (a^2+1)}{a^2(a+1) + b^2(a+1)} \cdot \frac{2a(a+b) + b^2(a+b)}{a(a^2+1) + b(a^2+1)} = \frac{(\cancel{a^2+1})(\cancel{a+1})}{(\cancel{a^2+1})(\cancel{a+b})} \cdot \frac{(\cancel{a+b})(2a+b^2)}{(\cancel{a^2+1})(\cancel{a+b})} = \frac{2a+b^2}{a^2+b^2}
 \end{aligned}$$

$$\begin{aligned}
 35. \quad \frac{8x - 4x^2}{xy - 2y + 3x - 6} \div \frac{3x + 6}{y + 3} &= \frac{8x - 4x^2}{y(x-2) + 3(x-2)} \cdot \frac{y+3}{3x+6} \\
 &= \frac{-4x(\cancel{x-2})}{(\cancel{x-2})(y+3)} \cdot \frac{\cancel{y+3}}{3(x+2)} = \frac{-4x}{3(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 37. \quad \frac{3x^5}{2x^2y^7} \div \frac{4x^3y}{6y^6} &= \frac{3x^5}{2x^2y^7} \cdot \frac{6y^6}{4x^3y} \\
 &= \frac{\cancel{3}x^{\cancel{5}}x^{\cancel{3}}}{\cancel{2}x^{\cancel{2}}y^{\cancel{7}}y} \cdot \frac{\cancel{2} \cdot \cancel{3}y^{\cancel{6}}}{4x^{\cancel{3}}y} \\
 &= \frac{9}{4y^2}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad \frac{4y}{7} \div \frac{y^2}{14} \cdot \frac{3}{y} &= \frac{4y}{7} \cdot \frac{14}{y^2} \cdot \frac{3}{y} \\
 &= \frac{\cancel{4} \cancel{7} \cdot 2 \cdot 3}{\cancel{7} y^2 \cancel{y}} = \frac{24}{y^2}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad \frac{6a^2 + ab - b^2}{10a^2 + 5ab} \cdot \frac{2a^3 + 4a^2b}{3a^2 + 5ab - 2b^2} &= \frac{(\cancel{3a-b})(\cancel{2a+b})}{5a(\cancel{2a+b})} \cdot \frac{2a^2(\cancel{a+2b})}{(\cancel{3a-b})(\cancel{a+2b})} = \frac{2a}{5}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad (2x^2 + 8) \div \frac{x^4 - 16}{x^2 + x - 6} &= \frac{2x^2 + 8}{1} \cdot \frac{x^2 + x - 6}{x^4 - 16} = \frac{2(x^2 + 4)}{1} \cdot \frac{(x+3)(x-2)}{(x^2+4)(x^2-4)} \\
 &= \frac{2(\cancel{x^2+4})}{1} \cdot \frac{(x+3)(\cancel{x-2})}{(\cancel{x^2+4})(x+2)(\cancel{x-2})} = \frac{2(x+3)}{x+2}
 \end{aligned}$$

$$\begin{aligned}
45. \quad & \frac{m^2 - n^2}{(m-n)^2} \div \frac{m^2 - 2mn + n^2}{m^2 - mn + n^2} \cdot \frac{(m-n)^4}{m^3 + n^3} \\
&= \frac{m^2 - n^2}{(m-n)^2} \cdot \frac{m^2 - mn + n^2}{m^2 - 2mn + n^2} \cdot \frac{(m-n)^4}{m^3 + n^3} \\
&= \frac{\cancel{(m+n)}(m-n)}{\cancel{(m-n)}^2} \cdot \frac{\cancel{m^2 - mn + n^2}}{\cancel{(m-n)}^2} \cdot \frac{\cancel{(m-n)}^2 \cancel{(m-n)}^2}{\cancel{(m+n)} \cancel{(m^2 - mn + n^2)}} \\
&= m - n
\end{aligned}$$

$$\begin{aligned}
47. \quad & \frac{x^2 - 6xy + 9y^2}{x^2 - 4y^2} \cdot \frac{x^2 - 5xy + 6y^2}{3y - x} \div \frac{x^2 - 9y^2}{x + 2y} \\
&= \frac{x^2 - 6xy + 9y^2}{x^2 - 4y^2} \cdot \frac{x^2 - 5xy + 6y^2}{3y - x} \cdot \frac{x + 2y}{x^2 - 9y^2} \\
&= \frac{\cancel{(x-3y)} \cancel{(x-3y)}}{\cancel{(x+2y)} \cancel{(x-2y)}} \cdot \frac{\cancel{(x-3y)} \cancel{(x-2y)}}{-(\cancel{x-3y})} \cdot \frac{\cancel{x+2y}}{\cancel{(x+3y)} \cancel{(x-3y)}} \\
&= -\frac{x-3y}{x+3y} \text{ or } \frac{3y-x}{x+3y}
\end{aligned}$$

$$\begin{aligned}
49. \quad & \frac{25m^2 - 1}{125m^3 - 1} \div \frac{5m+1}{25m^2 + 5m + 1} = \frac{25m^2 - 1}{125m^3 - 1} \cdot \frac{25m^2 + 5m + 1}{5m+1} \\
&= \frac{\cancel{(5m+1)} \cancel{(5m-1)}}{\cancel{(5m-1)} \cancel{(25m^2 + 5m + 1)}} \cdot \frac{\cancel{25m^2 + 5m + 1}}{\cancel{5m+1}} = 1
\end{aligned}$$

$$\begin{aligned}
51. \quad & \frac{2a^2 + ab - 8a - 4b}{2a^2 - 6a + ab - 3b} \cdot \frac{a^2 - 6a + 9}{a^2 - 16} = \frac{a(2a+b) - 4(2a+b)}{2a(a-3) + b(a-3)} \cdot \frac{(a-3)(a-3)}{(a+4)(a-4)} \\
&= \frac{\cancel{(2a+b)} \cancel{(a-4)}}{\cancel{(a-3)} \cancel{(2a+b)}} \cdot \frac{(a-3) \cancel{(a-3)}}{(a+4) \cancel{(a-4)}} = \frac{a-3}{a+4}
\end{aligned}$$

$$\begin{aligned}
53. \quad & \frac{45}{2x+1} \cdot (8x+4) \div \frac{27}{4x+2} = \frac{5 \cdot \cancel{9}}{\cancel{2x+1}} \cdot \frac{4 \cancel{(2x+1)}}{1} \cdot \frac{2(2x+1)}{\cancel{9} \cdot 3} \\
&= \frac{40(2x+1)}{3}
\end{aligned}$$

$$55. \frac{2x^2 - 11x - 6}{3x - 2} \div \frac{2x^2 - 5x - 3}{3x^2 - 7x - 6} = \frac{2x^2 - 11x - 6}{3x - 2} \cdot \frac{3x^2 - 7x - 6}{2x^2 - 5x - 3}$$

$$= \frac{\cancel{(2x+1)}(x-6)}{3x-2} \cdot \frac{(3x+2)\cancel{(x-3)}}{\cancel{(2x+1)}\cancel{(x-3)}} = \frac{(x-6)(3x+2)}{3x-2}$$

$$57. A = \frac{1}{2} \left(\frac{k^2}{2h^2} \right) \left(\frac{8}{hk} \right)$$

$$= \frac{1}{\cancel{2}} \left(\frac{k \cdot \cancel{k}}{\cancel{2}h^2} \right) \left(\frac{\cancel{2} \cdot \cancel{2} \cdot 2}{h\cancel{k}} \right) = \frac{2k}{h^3} \text{ cm}^2$$

$$59. A = \frac{x^2}{x-3} \cdot \frac{5x-15}{4x}$$

$$= \frac{x \cdot \cancel{x}}{\cancel{x-3}} \cdot \frac{5\cancel{(x-3)}}{4\cancel{x}} = \frac{5x}{4} \text{ ft}^2$$

Section 5.3 Practice Exercises

1. a. $\frac{p+r}{q}; \frac{p-r}{q}$.

b. least common denominator

$$3. \frac{9b+9}{4b+8} \cdot \frac{2b+4}{3b-3} = \frac{\cancel{3} \cdot 3(b+1)}{\cancel{2} \cdot 2(b+2)} \cdot \frac{\cancel{2}(b+2)}{\cancel{3}(b-1)} = \frac{3(b+1)}{2(b-1)}$$

$$5. \frac{(5-a)^2}{10a-2} \cdot \frac{25a^2-1}{a^2-10a+25} = \frac{(-1)^2 \cancel{(a-5)}^2}{2\cancel{(a-1)}} \cdot \frac{(5a+1)\cancel{(5a-1)}}{\cancel{(a-5)}^2} = \frac{5a+1}{2}$$

$$7. \frac{3}{5x} + \frac{7}{5x} = \frac{10}{5x} = \frac{\cancel{5} \cdot 2}{\cancel{5}x}$$

$$= \frac{2}{x}$$

$$9. \frac{x}{x^2-2x-3} - \frac{3}{x^2-2x-3} = \frac{x-3}{x^2-2x-3}$$

$$= \frac{\cancel{x-3}}{\cancel{(x-3)}(x+1)} = \frac{1}{x+1}$$

$$11. \frac{5x-1}{(2x+9)(x-6)} - \frac{3x-6}{(2x+9)(x-6)} = \frac{5x-1-(3x-6)}{(2x+9)(x-6)} = \frac{5x-1-3x+6}{(2x+9)(x-6)} = \frac{2x+5}{(2x+9)(x-6)}$$

$$13. \frac{x+2}{x-5} + \frac{x-12}{x-5} = \frac{2x-10}{x-5}$$

$$= \frac{2\cancel{(x-5)}}{\cancel{x-5}} = 2$$

$$15. \frac{5}{8} = \frac{5}{2^3},$$

$$\frac{3}{20x} = \frac{3}{5 \cdot 2^2 x} \quad \text{LCD} = 2^3 \cdot 5 \cdot x = 40x$$

$$17. \frac{-5}{6m^4} = \frac{-5}{2 \cdot 3 \cdot m^4}, \quad \frac{1}{15mn^7} = \frac{1}{3 \cdot 5 \cdot mn^7}$$

$$\text{LCD} = 2 \cdot 3 \cdot 5 \cdot m^4 n^7 = 30m^4 n^7$$

$$19. \frac{6}{(x-4)(x+2)}, \quad \frac{-8}{(x-4)(x-6)}$$

$$\text{LCD} = (x-4)(x+2)(x-6)$$

$$21. \frac{3}{x(x-1)(x+7)^2}, \quad \frac{-1}{x^2(x+7)}$$

$$\text{LCD} = x^2(x-1)(x+7)^2$$

$$23. \frac{5}{x-6}, \quad \frac{x-5}{x^2-8x+12} = \frac{x-5}{(x-6)(x-2)}$$

$$\text{LCD} = (x-6)(x-2)$$

$$25. \frac{3a}{a-4}, \quad \frac{5}{4-a} = \frac{5(-1)}{(4-a)(-1)} = \frac{-5}{a-4}$$

$$\text{LCD} = a-4 \text{ or } 4-a$$

$$27. \frac{5}{3x} = \frac{5}{9x^2y}$$

$$\frac{5}{3x} \cdot \frac{3xy}{3xy} = \frac{15xy}{9x^2y}$$

$$29. \frac{2x}{x-1} = \frac{2x}{x(x-1)(x+2)}$$

$$\frac{2x}{x-1} \cdot \frac{x(x+2)}{x(x+2)} = \frac{2x^2(x+2)}{x(x-1)(x+2)}$$

$$= \frac{2x^3+4x^2}{x(x-1)(x+2)}$$

$$31. \frac{y}{y+6} = \frac{y}{y^2+5y-6}$$

$$\frac{y}{y+6} = \frac{y}{(y+6)(y-1)}$$

$$\frac{y}{y+6} \cdot \frac{y-1}{y-1} = \frac{y^2-y}{(y+6)(y-1)}$$

$$33. \frac{4}{3p} - \frac{5}{2p^2} \quad \text{LCD} = 2 \cdot 3 \cdot p^2 = 6p^2$$

$$= \frac{4}{3p} \cdot \frac{2p}{2p} - \frac{5}{2p^2} \cdot \frac{3}{3} = \frac{8p}{6p^2} - \frac{15}{6p^2}$$

$$= \frac{8p-15}{6p^2}$$

$$35. \frac{s-1}{s} - \frac{t+1}{t} \quad \text{LCD} = st$$

$$= \frac{s-1}{s} \cdot \frac{t}{t} - \frac{t+1}{t} \cdot \frac{s}{s} = \frac{st-t}{st} - \frac{st+s}{st}$$

$$= \frac{st-t-st-s}{st} = \frac{-t-s}{st}$$

$$37. \frac{4a-2}{3a+12} - \frac{a-2}{a+4} = \frac{4a-2}{3(a+4)} - \frac{a-2}{a+4}$$

$$\text{LCD} = 3(a+4)$$

$$= \frac{4a-2}{3(a+4)} - \frac{a-2}{a+4} \cdot \frac{3}{3}$$

$$= \frac{4a-2-3(a-2)}{3(a+4)}$$

$$39. \frac{10}{b(b+5)} + \frac{2}{b} \quad \text{LCD} = b(b+5)$$

$$= \frac{10}{b(b+5)} + \frac{2}{b} \cdot \frac{b+5}{b+5} = \frac{10+2(b+5)}{b(b+5)}$$

$$= \frac{10+2b+10}{b(b+5)}$$

$$= \frac{2b+20}{b(b+5)}$$

$$= \frac{4a-2-3a+6}{3(a+4)} = \frac{\cancel{a+4}}{3(\cancel{a+4})} = \frac{1}{3}$$

$$\begin{aligned} 41. \quad \frac{x-2}{x-6} - \frac{x+2}{6-x} &= \frac{x-2}{x-6} - \frac{x+2}{6-x} \cdot \frac{(-1)}{(-1)} \\ &= \frac{x-2}{x-6} - \frac{-(x+2)}{x-6} \\ &= \frac{x-2+x+2}{x-6} \\ &= \frac{2x}{x-6} \end{aligned}$$

$$\begin{aligned} 43. \quad \frac{6b}{b-4} - \frac{1}{b+1} \quad \text{LCD} &= (b-4)(b+1) \\ &= \frac{6b}{b-4} \cdot \frac{b+1}{b+1} - \frac{1}{b+1} \cdot \frac{b-4}{b-4} \\ &= \frac{6b(b+1) - 1(b-4)}{(b-4)(b+1)} \\ &= \frac{6b^2 + 6b - b + 4}{(b-4)(b+1)} = \frac{6b^2 + 5b + 4}{(b-4)(b+1)} \end{aligned}$$

$$\begin{aligned} 45. \quad \frac{2}{2x+1} + \frac{4}{x-2} \quad \text{LCD} &= (2x+1)(x-2) \\ &= \frac{2}{2x+1} \cdot \frac{x-2}{x-2} + \frac{4}{x-2} \cdot \frac{2x+1}{2x+1} = \frac{2x-4+8x+4}{(2x+1)(x-2)} = \frac{10x}{(2x+1)(x-2)} \end{aligned}$$

$$\begin{aligned} 47. \quad \frac{y-2}{y-4} + \frac{2y^2-15y+12}{y^2-16} &= \frac{y-2}{y-4} + \frac{2y^2-15y+12}{(y+4)(y-4)} \quad \text{LCD} = (y+4)(y-4) \\ &= \frac{y-2}{y-4} \cdot \frac{y+4}{y+4} + \frac{2y^2-15y+12}{(y+4)(y-4)} = \frac{y^2+2y-8+2y^2-15y+12}{(y+4)(y-4)} = \frac{3y^2-13y+4}{(y+4)(y-4)} \\ &= \frac{(3y-1)\cancel{(y-4)}}{(y+4)\cancel{(y-4)}} = \frac{3y-1}{y+4} \end{aligned}$$

$$\begin{aligned} 49. \quad \frac{x+2}{x^2-36} - \frac{x}{x^2+9x+18} &= \frac{x+2}{(x+6)(x-6)} - \frac{x}{(x+6)(x+3)} \quad \text{LCD} = (x+6)(x-6)(x+3) \\ &= \frac{x+2}{(x+6)(x-6)} \cdot \frac{x+3}{x+3} - \frac{x}{(x+6)(x+3)} \cdot \frac{x-6}{x-6} \\ &= \frac{(x+2)(x+3) - x(x-6)}{(x+6)(x-6)(x+3)} \\ &= \frac{x^2+5x+6-x^2+6x}{(x+6)(x-6)(x+3)} \\ &= \frac{11x+6}{(x+6)(x-6)(x+3)} \end{aligned}$$

$$\begin{aligned}
 51. \quad \frac{5}{w} + \frac{8}{-w} &= \frac{5}{w} + \frac{8}{-w} \cdot \frac{(-1)}{(-1)} \\
 &= \frac{5}{w} + \frac{-8}{w} = -\frac{3}{w}
 \end{aligned}$$

$$\begin{aligned}
 52. \quad \frac{4}{y} + \frac{5}{-y} &= \frac{4}{y} + \frac{5}{-y} \cdot \frac{(-1)}{(-1)} \\
 &= \frac{4}{y} + \frac{-5}{y} = -\frac{1}{y}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{n}{5-n} + \frac{2n-5}{n-5} &= \frac{n}{5-n} \cdot \frac{(-1)}{(-1)} + \frac{2n-5}{n-5} \\
 &= \frac{-n}{n-5} + \frac{2n-5}{n-5} \\
 &= \frac{n-5}{n-5} = 1
 \end{aligned}$$

$$\begin{aligned}
 55. \quad \frac{2}{3x-15} + \frac{x}{25-x^2} &= \frac{2}{3x-15} + \frac{x}{25-x^2} \cdot \frac{(-1)}{(-1)} \\
 &= \frac{2}{3(x-5)} + \frac{-x}{(x+5)(x-5)} \quad \text{LCD} = 3(x+5)(x-5) \\
 &= \frac{2}{3(x-5)} \cdot \frac{x+5}{x+5} + \frac{-x}{(x+5)(x-5)} \cdot \frac{3}{3} \\
 &= \frac{2x+10-3x}{3(x+5)(x-5)} = \frac{10-x}{3(x+5)(x-5)}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad \frac{m}{20+9m+m^2} - \frac{4}{12+7m+m^2} &= \frac{m}{(m+5)(m+4)} - \frac{4}{(m+4)(m+3)} \\
 \text{LCD} &= (m+5)(m+4)(m+3) \\
 &= \frac{m}{(m+5)(m+4)} \cdot \frac{m+3}{m+3} - \frac{4}{(m+4)(m+3)} \cdot \frac{m+5}{m+5} \\
 &= \frac{m^2+3m}{(m+5)(m+4)(m+3)} - \frac{4m+20}{(m+5)(m+4)(m+3)} \\
 &= \frac{m^2+3m-4m-20}{(m+5)(m+4)(m+3)} = \frac{m^2-m-20}{(m+5)(m+4)(m+3)} \\
 &= \frac{(m-5)\cancel{(m+4)}}{(m+5)\cancel{(m+4)}(m+3)} = \frac{m-5}{(m+5)(m+3)}
 \end{aligned}$$

$$\begin{aligned}
 59. \quad \frac{x+3}{x^2} + \frac{x+5}{2x} \quad \text{LCD} &= 2x^2 \\
 &= \frac{x+3}{x^2} \cdot \frac{2}{2} + \frac{x+5}{2x} \cdot \frac{x}{x} \\
 &= \frac{2x+6}{2x^2} + \frac{x^2+5x}{2x^2} = \frac{x^2+7x+6}{2x^2}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad w+2 + \frac{1}{w-2} \quad \text{LCD} &= w-2 \\
 &= (w+2) \cdot \frac{w-2}{w-2} + \frac{1}{w-2} \\
 &= \frac{w^2-4+1}{w-2} = \frac{w^2-3}{w-2}
 \end{aligned}$$

$$63. \quad \frac{9}{x^2-2x+1} - \frac{x-3}{x^2-x} = \frac{9}{(x-1)^2} - \frac{x-3}{x(x-1)} \quad \text{LCD} = x(x-1)^2$$

$$\begin{aligned}
&= \frac{9}{(x-1)^2} \cdot \frac{x}{x} - \frac{x-3}{x(x-1)} \cdot \frac{x-1}{x-1} = \frac{9x - (x^2 - 4x + 3)}{x(x-1)^2} \\
&= \frac{9x - x^2 + 4x - 3}{x(x-1)^2} = \frac{-x^2 + 13x - 3}{x(x-1)^2}
\end{aligned}$$

$$\begin{aligned}
65. \quad \frac{t+1}{t+3} - \frac{t-2}{t-3} + \frac{6}{t^2-9} &= \frac{t+1}{t+3} - \frac{t-2}{t-3} + \frac{6}{(t+3)(t-3)} \quad \text{LCD} = (t+3)(t-3) \\
&= \frac{t+1}{t+3} \cdot \frac{t-3}{t-3} - \frac{t-2}{t-3} \cdot \frac{t+3}{t+3} + \frac{6}{(t+3)(t-3)} = \frac{t^2 - 2t - 3 - (t^2 + t - 6) + 6}{(t+3)(t-3)} \\
&= \frac{t^2 - 2t - 3 - t^2 - t + 6 + 6}{(t+3)(t-3)} = \frac{-3t + 9}{(t+3)(t-3)} = \frac{-3\cancel{(t-3)}}{(t+3)\cancel{(t-3)}} = -\frac{3}{t+3}
\end{aligned}$$

$$\begin{aligned}
67. \quad (x-1) \cdot \left[\frac{3}{x^2-1} + \frac{x}{2x-2} \right] &= \frac{3\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} + \frac{x\cancel{(x-1)}}{2\cancel{(x-1)}} = \frac{3}{x+1} + \frac{x}{2} \quad \text{LCD} = 2(x+1) \\
&= \frac{3}{x+1} \cdot \frac{2}{2} + \frac{x}{2} \cdot \frac{x+1}{x+1} = \frac{6+x^2+x}{2(x+1)} = \frac{x^2+x+6}{2(x+1)}
\end{aligned}$$

$$\begin{aligned}
69. \quad \frac{3z}{z-3} - \frac{z}{z+4} \quad \text{LCD} &= (z-3)(z+4) \\
&= \frac{3z}{z-3} \cdot \frac{z+4}{z+4} - \frac{z}{z+4} \cdot \frac{z-3}{z-3} = \frac{3z^2 + 12z - (z^2 - 3z)}{(z-3)(z+4)} = \frac{3z^2 + 12z - z^2 + 3z}{(z-3)(z+4)} = \frac{2z^2 + 15z}{(z-3)(z+4)}
\end{aligned}$$

$$\begin{aligned}
71. \quad \frac{2x}{x^2-y^2} - \frac{1}{x-y} + \frac{1}{y-x} &= \frac{2x}{(x+y)(x-y)} - \frac{1}{x-y} + \frac{1}{y-x} \quad \text{LCD} = (x+y)(x-y) \\
&= \frac{2x}{(x+y)(x-y)} - \frac{1}{x-y} \cdot \frac{x+y}{x+y} + \frac{1}{y-x} \cdot \frac{(-1)}{(-1)} \cdot \frac{x+y}{x+y} \\
&= \frac{2x - 1(x+y) - 1(x+y)}{(x+y)(x-y)} = \frac{2x - x - y - x - y}{(x+y)(x-y)} \\
&= \frac{-2y}{(x+y)(x-y)}
\end{aligned}$$

$$\begin{aligned}
73. \quad (2p+1) \cdot \left[\frac{2p}{6p+3} - \frac{1}{p+4} \right] &= \frac{2p\cancel{(2p+1)}}{3\cancel{(2p+1)}} - \frac{2p+1}{p+4} = \frac{2p}{3} - \frac{2p+1}{p+4} \quad \text{LCD} = 3(p+4)
\end{aligned}$$

$$\begin{aligned}
&= \frac{2p}{3} \cdot \frac{p+4}{p+4} - \frac{2p+1}{p+4} \cdot \frac{3}{3} = \frac{2p^2+8p-(6p+3)}{3(p+4)} = \frac{2p^2+8p-6p-3}{3(p+4)} \\
&= \frac{2p^2+2p-3}{3(p+4)}
\end{aligned}$$

75. $\frac{1}{x+5} + \frac{3}{(x+5)^2} - \frac{2}{(x+5)^3}$ LCD = $(x+5)^3$

$$\begin{aligned}
&= \frac{1}{x+5} \cdot \frac{(x+5)^2}{(x+5)^2} + \frac{3}{(x+5)^2} \cdot \frac{x+5}{x+5} - \frac{2}{(x+5)^3} = \frac{(x+5)^2}{(x+5)^3} + \frac{3x+15}{(x+5)^3} - \frac{2}{(x+5)^3} \\
&= \frac{x^2+10x+25+3x+15-2}{(x+5)^3} = \frac{x^2+13x+38}{(x+5)^3}
\end{aligned}$$

77. $\frac{-10}{z^2-6z+5} + \frac{15}{z^2-4z-5}$ LCD = $(z-5)(z-1)(z+1)$

$$\begin{aligned}
&= \frac{-10}{(z-5)(z-1)} + \frac{15}{(z-5)(z+1)} \\
&= \frac{-10}{(z-5)(z-1)} \cdot \frac{z+1}{z+1} + \frac{15}{(z-5)(z+1)} \cdot \frac{z-1}{z-1} = \frac{-10z-10}{(z-5)(z-1)(z+1)} + \frac{15z-15}{(z-5)(z-1)(z+1)} \\
&= \frac{5z-25}{(z-5)(z-1)(z+1)} \\
&= \frac{5\cancel{(z-5)}}{\cancel{(z-5)}(z-1)(z+1)} = \frac{5}{(z-1)(z+1)}
\end{aligned}$$

79. $\frac{5}{x^2-4} + \frac{2}{x^3-8}$

$$\begin{aligned}
&= \frac{5}{(x-2)(x+2)} + \frac{2}{(x-2)(x^2+2x+4)} \\
\text{LCD} &= (x-2)(x+2)(x^2+2x+4) \\
&= \frac{5}{(x-2)(x+2)} \cdot \frac{x^2+2x+4}{x^2+2x+4} + \frac{2}{(x-2)(x^2+2x+4)} \cdot \frac{x+2}{x+2} \\
&= \frac{5x^2+10x+20+2x+4}{(x-2)(x+2)(x^2+2x+4)} \\
&= \frac{5x^2+12x+24}{(x-2)(x+2)(x^2+2x+4)}
\end{aligned}$$

$$\begin{aligned}
 81. \quad \frac{2}{3x} + \frac{x+1}{x} + \frac{6}{x^2} \quad \text{LCD} &= 3x^2 \\
 &= \frac{2}{3x} \cdot \frac{x}{x} + \frac{x+1}{x} \cdot \frac{3x}{3x} + \frac{6}{x^2} \cdot \frac{3}{3} \\
 &= \frac{2x+3x^2+3x+18}{3x^2} \\
 &= \frac{3x^2+5x+18}{3x^2} \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 83. \quad 2\left(\frac{5}{x-3}\right) + 2\left(\frac{2x}{x+5}\right) \quad \text{LCD} &= (x-3)(x+5) \\
 &= \frac{10}{x-3} \cdot \frac{x+5}{x+5} + \frac{4x}{x+5} \cdot \frac{x-3}{x-3} \\
 &= \frac{10x+50+4x^2-12x}{(x-3)(x+5)} \\
 &= \frac{4x^2-2x+50}{(x-3)(x+5)} \text{ m}
 \end{aligned}$$

Section 5.4 Practice Exercises

1. complex

$$\begin{aligned}
 3. \quad \frac{25a^3b^3c}{15a^4bc} &= \frac{25}{15} a^{3-4} b^{3-1} c^{1-1} = \frac{5}{3} a^{-1} b^2 c^0 \\
 &= \frac{5b^2}{3a}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{5}{x^2} + \frac{3}{2x} \quad \text{LCD} &= 2x^2 \\
 &= \frac{5}{x^2} \cdot \frac{2}{2} + \frac{3}{2x} \cdot \frac{x}{x} = \frac{10+3x}{2x^2}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{3}{a-5} - \frac{1}{a+1} \quad \text{LCD} &= (a-5)(a+1) \\
 &= \frac{3}{a-5} \cdot \frac{a+1}{a+1} - \frac{1}{a+1} \cdot \frac{a-5}{a-5} \\
 &= \frac{3a+3-a+5}{(a-5)(a+1)} = \frac{2a+8}{(a-5)(a+1)}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{5x^2}{9y^2} &= \frac{5x^2}{9y^2} \cdot \frac{\cancel{y^2}}{\cancel{y^2}} \\
 &= \frac{5x^2}{27}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{\frac{x-6}{3x}}{\frac{3x-18}{9}} &= \frac{x-6}{3x} \cdot \frac{9}{3x-18} \\
 &= \frac{\cancel{x-6}}{\cancel{3}x} \cdot \frac{\cancel{3} \cdot \cancel{3}}{\cancel{3}(x-6)} = \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{\frac{2}{3} + \frac{1}{6}}{\frac{1}{2} - \frac{1}{4}} &= \frac{\frac{4}{6} + \frac{1}{6}}{\frac{2}{4} - \frac{1}{4}} = \frac{\frac{5}{6}}{\frac{1}{4}} = \frac{5}{6} \cdot \frac{4}{1} \\
 &= \frac{5 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 3 \cdot 1} = \frac{10}{3}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad 8 - \frac{5}{2x} &= 8 \cdot \frac{2x}{2x} - \frac{5}{2x} = \frac{16x-5}{2x} \\
 \frac{5}{8x} - 2 &= \frac{5}{8x} - 2 \cdot \frac{8x}{8x} = \frac{5-16x}{8x}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{\frac{7y}{y+3}}{\frac{1}{4y+12}} &= \frac{7y}{y+3} \cdot \frac{4y+12}{4(y+3)} \quad \text{LCD} = 4(y+3) \\
 &= \frac{7y}{y+3} \cdot \frac{4(y+3)}{4(y+3)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\cancel{16x-5} \cdot \cancel{2} \cdot 4x}{\cancel{2}x \cdot \cancel{-(16x-5)}} \\
 &= \frac{4}{-1} = -4
 \end{aligned}$$

$$\frac{4(\cancel{y+3})\left(\frac{7y}{\cancel{y+3}}\right)}{\cancel{4}(\cancel{y+3})\left(\frac{1}{\cancel{4}(\cancel{y+3})}\right)} = \frac{4(7y)}{1} = 28y$$

19. $\frac{1+\frac{1}{3}}{\frac{5}{6}-1}$ LCD = 6

$$\begin{aligned}
 &\frac{6\left(1+\frac{1}{3}\right)}{6\left(\frac{5}{6}-1\right)} = \frac{6 \cdot 1 + 6\left(\frac{1}{3}\right)}{6\left(\frac{5}{6}\right) - 6 \cdot 1} \\
 &= \frac{6+2}{5-6} \\
 &= \frac{8}{-1} = -8
 \end{aligned}$$

21. $\frac{\frac{3q}{p}-q}{q-\frac{q}{p}}$ LCD = p

$$\begin{aligned}
 &\frac{p\left(\frac{3q}{p}-q\right)}{p\left(q-\frac{q}{p}\right)} = \frac{\cancel{p}\left(\frac{3q}{\cancel{p}}-pq\right)}{pq-\cancel{p}\left(\frac{q}{\cancel{p}}\right)} = \frac{3q-pq}{pq-q} \\
 &= \frac{\cancel{q}(3-p)}{\cancel{q}(p-1)} = \frac{3-p}{p-1}
 \end{aligned}$$

23. $\frac{\frac{2}{a}+\frac{3}{a^2}}{\frac{4}{a^2}-\frac{9}{a}}$ LCD = a²

$$\begin{aligned}
 &\frac{a^2\left(\frac{2}{a}+\frac{3}{a^2}\right)}{a^2\left(\frac{4}{a^2}-\frac{9}{a}\right)} = \frac{a^2\left(\frac{2}{a}\right)+a^2\left(\frac{3}{a^2}\right)}{a^2\left(\frac{4}{a^2}\right)-a^2\left(\frac{9}{a}\right)} \\
 &= \frac{2a+3}{4-9a}
 \end{aligned}$$

25. $\frac{t^{-1}-1}{1-t^{-2}} = \frac{\frac{1}{t}-1}{1-\frac{1}{t^2}}$ LCD = t²

$$\begin{aligned}
 &\frac{t^2\left(\frac{1}{t}-1\right)}{t^2\left(1-\frac{1}{t^2}\right)} = \frac{t^2\left(\frac{1}{t}\right)-t^2(1)}{t^2(1)-t^2\left(\frac{1}{t^2}\right)} = \frac{t-t^2}{t^2-1} \\
 &= \frac{-t(\cancel{t-1})}{(t+1)(\cancel{t-1})} = -\frac{t}{t+1}
 \end{aligned}$$

27. $\frac{-8}{\frac{6w}{w-1}-4}$ LCD = w-1

$$\begin{aligned}
 &\frac{(w-1)(-8)}{(w-1)\left(\frac{6w}{w-1}-4\right)} = \frac{(w-1)(-8)}{(\cancel{w-1})\left(\frac{6w}{\cancel{w-1}}\right)-(w-1)4} = \frac{-8w+8}{6w-4w+4} = \frac{-8w+8}{2w+4} \\
 &= \frac{-4 \cdot \cancel{2}(w-1)}{\cancel{2}(w+2)} = -\frac{4(w-1)}{w+2}
 \end{aligned}$$

29. $\frac{\frac{y}{y+3}}{\frac{y}{y+3} + y}$ LCD = $y+3$

$$\frac{(y+3)\left(\frac{y}{y+3}\right)}{(y+3)\left(\frac{y}{y+3} + y\right)} = \frac{(\cancel{y+3})\left(\frac{y}{\cancel{y+3}}\right)}{(\cancel{y+3})\left(\frac{y}{\cancel{y+3}}\right) + (y+3)y} = \frac{y}{y+y^2+3y} = \frac{y}{y^2+4y}$$

$$= \frac{y}{y(y+4)} = \frac{1}{y+4}$$

31. $1 - \frac{1}{x} - \frac{6}{x^2}$ LCD = x^2

$$\frac{1 - \frac{1}{x} - \frac{6}{x^2}}{1 - \frac{4}{x} + \frac{3}{x^2}} = \frac{x^2\left(1 - \frac{1}{x} - \frac{6}{x^2}\right)}{x^2\left(1 - \frac{4}{x} + \frac{3}{x^2}\right)} = \frac{x^2(1) - x^2\left(\frac{1}{x}\right) - x^2\left(\frac{6}{x^2}\right)}{x^2(1) - x^2\left(\frac{4}{x}\right) + x^2\left(\frac{3}{x^2}\right)} = \frac{x^2 - x - 6}{x^2 - 4x + 3}$$

$$= \frac{(\cancel{x-3})(x+2)}{(\cancel{x-3})(x-1)} = \frac{x+2}{x-1}$$

33. $2 - \frac{2}{t+1}$ LCD = $t(t+1)$

$$\frac{2 - \frac{2}{t+1}}{2 + \frac{2}{t}} = \frac{t(t+1)\left(2 - \frac{2}{t+1}\right)}{t(t+1)\left(2 + \frac{2}{t}\right)} = \frac{t(t+1)(2) - t(\cancel{t+1})\left(\frac{2}{\cancel{t+1}}\right)}{t(t+1)(2) + \cancel{t}(t+1)\left(\frac{2}{\cancel{t}}\right)}$$

$$= \frac{2t^2 + 2t - 2t}{2t^2 + 2t + 2t + 2}$$

$$= \frac{\cancel{2}t^2}{\cancel{2}(t^2 + 2t + 1)} = \frac{t^2}{(t+1)^2}$$

$$\begin{aligned}
35. \quad & \frac{\frac{2}{a} - \frac{3}{a+1}}{\frac{2}{a+1} - \frac{3}{a}} \quad \text{LCD} = a(a+1) \\
& \frac{a(a+1)\left(\frac{2}{a} - \frac{3}{a+1}\right)}{a(a+1)\left(\frac{2}{a+1} - \frac{3}{a}\right)} = \frac{\cancel{a}(a+1)\left(\frac{2}{\cancel{a}}\right) - a(\cancel{a+1})\left(\frac{3}{\cancel{a+1}}\right)}{a(\cancel{a+1})\left(\frac{2}{\cancel{a+1}}\right) - \cancel{a}(a+1)\left(\frac{3}{\cancel{a}}\right)} = \frac{2a+2-3a}{2a-3a-3} = \frac{-a+2}{-a-3}
\end{aligned}$$

$$\begin{aligned}
37. \quad & \frac{\frac{1}{y+2} + \frac{4}{y-3}}{\frac{2}{y-3} - \frac{7}{y+2}} \quad \text{LCD} = (y+2)(y-3) \\
& \frac{(y+2)(y-3)\left(\frac{1}{y+2} + \frac{4}{y-3}\right)}{(y+2)(y-3)\left(\frac{2}{y-3} - \frac{7}{y+2}\right)} = \frac{(\cancel{y+2})(y-3)\left(\frac{1}{\cancel{y+2}}\right) + (y+2)(\cancel{y-3})\left(\frac{4}{\cancel{y-3}}\right)}{(y+2)(\cancel{y-3})\left(\frac{2}{\cancel{y-3}}\right) - (\cancel{y+2})(y-3)\left(\frac{7}{\cancel{y+2}}\right)} \\
& = \frac{y-3+4y+8}{2y+4-7y+21} = \frac{5y+5}{-5y+25} = \frac{\cancel{5}(y+1)}{-\cancel{5}(y-5)} = -\frac{y+1}{y-5}
\end{aligned}$$

$$\begin{aligned}
39. \quad & \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \quad \text{LCD} = x(x+h) \\
& \frac{x(x+h)\left(\frac{2}{x+h} - \frac{2}{x}\right)}{x(x+h)(h)} = \frac{x(\cancel{x+h})\left(\frac{2}{\cancel{x+h}}\right) - \cancel{x}(x+h)\left(\frac{2}{\cancel{x}}\right)}{x(x+h)(h)} = \frac{2x-2x-2h}{x(x+h)(h)} \\
& = \frac{-2\cancel{h}}{x(x+h)(\cancel{h})} = -\frac{2}{x(x+h)}
\end{aligned}$$

$$\begin{aligned}
41. \quad & \frac{x^{-2}}{x+3x^{-1}} = \frac{\frac{1}{x^2}}{x+\frac{3}{x}} \quad \text{LCD} = x^2 \\
& \frac{x^2\left(\frac{1}{x^2}\right)}{x^2\left(x+\frac{3}{x}\right)} = \frac{x^2\left(\frac{1}{x^2}\right)}{x^2(x)+x^2\left(\frac{3}{x}\right)} = \frac{1}{x^3+3x} = \frac{1}{x(x^2+3)}
\end{aligned}$$

$$43. \quad \frac{2a^{-1} + 3b^{-2}}{a^{-1} - b^{-1}} = \frac{\frac{2}{a} + \frac{3}{b^2}}{\frac{1}{a} - \frac{1}{b}} \quad \text{LCD} = ab^2$$

$$\frac{ab^2 \left(\frac{2}{a} + \frac{3}{b^2} \right)}{ab^2 \left(\frac{1}{a} - \frac{1}{b} \right)} = \frac{ab^2 \left(\frac{2}{a} \right) + ab^2 \left(\frac{3}{b^2} \right)}{ab^2 \left(\frac{1}{a} \right) - ab^2 \left(\frac{1}{b} \right)} = \frac{2b^2 + 3}{b^2 - ab}$$

$$45. \quad \frac{\frac{1}{4+h} - \frac{1}{4}}{h} \quad \text{LCD} = 4(4+h)$$

$$\frac{4(4+h) \left(\frac{1}{4+h} - \frac{1}{4} \right)}{4(4+h)h}$$

$$= \frac{4 - (4+h)}{4h(4+h)}$$

$$= \frac{-h}{4h(4+h)}$$

$$= \frac{-1}{4(4+h)}$$

$$47. \quad \frac{\frac{6}{x+h} - \frac{6}{x}}{h} \quad \text{LCD} = x(x+h)$$

$$\frac{x(x+h) \left(\frac{6}{x+h} - \frac{6}{x} \right)}{x(x+h)h} = \frac{6x - 6(x+h)}{xh(x+h)}$$

$$= \frac{6x - 6x - 6h}{xh(x+h)}$$

$$= \frac{-6h}{xh(x+h)}$$

$$= \frac{-6}{x(x+h)}$$

$$49. \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$51. \quad m = \frac{-3 - \frac{3}{5}}{-1 - \left(-\frac{3}{7} \right)} = \frac{-3 - \frac{3}{5}}{-1 + \frac{3}{7}} \quad \text{LCD} = 35$$

$$= \frac{35 \left(-3 - \frac{3}{5} \right)}{35 \left(-1 + \frac{3}{7} \right)} = \frac{-105 - 21}{-35 + 15}$$

$$= \frac{-126}{-20} = \frac{63}{10}$$

$$53. \quad m = \frac{\frac{1}{6} - \frac{1}{3}}{\frac{1}{8} - \frac{1}{4}} \quad \text{LCD} = 24$$

$$= \frac{24 \left(\frac{1}{6} - \frac{1}{3} \right)}{24 \left(\frac{1}{8} - \frac{1}{4} \right)} = \frac{4 - 8}{3 - 6}$$

$$= \frac{-4}{-3} = \frac{4}{3}$$

$$\begin{aligned}
 55. \quad (x^{-1} + y^{-1})^{-1} &= \frac{1}{x^{-1} + y^{-1}} \\
 &= \frac{1}{\frac{1}{x} + \frac{1}{y}} \quad \text{LCD} = xy \\
 &= \frac{xy(1)}{xy\left(\frac{1}{x} + \frac{1}{y}\right)} = \frac{xy}{y+x} \\
 &= \frac{xy}{x+y}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad \frac{x}{1 - \left(1 - \frac{1}{x}\right)^{-1}} &= \frac{x}{1 - \frac{1}{1 - \frac{1}{x}}} = \frac{x}{1 - \frac{1}{\frac{x}{x} - \frac{1}{x}}} \\
 &= \frac{x}{1 - \frac{1}{\frac{x-1}{x}}} = \frac{x}{1 - 1 \cdot \frac{x}{x-1}} \\
 &= \frac{x}{\frac{x-1}{x-1} - \frac{x}{x-1}} = \frac{x}{\frac{x-1-x}{x-1}} = x \cdot \frac{x-1}{-1} = -x(x-1)
 \end{aligned}$$

Problem Recognition Exercises

$$\begin{aligned}
 1. \quad \frac{2}{2y-3} - \frac{3}{2y} + 1 \quad \text{LCD} &= 2y(2y-3) \\
 &= \frac{2}{2y-3} \cdot \frac{2y}{2y} - \frac{3}{2y} \cdot \frac{2y-3}{2y-3} + 1 \cdot \frac{2y(2y-3)}{2y(2y-3)} \\
 &= \frac{4y-3(2y-3)+2y(2y-3)}{2y(2y-3)} \\
 &= \frac{4y-6y+9+4y^2-6y}{2y(2y-3)} \\
 &= \frac{4y^2-8y+9}{2y(2y-3)}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{5x^2-6x+1}{x^2-1} \div \frac{16x^2-9}{4x^2+7x+3} - \frac{x}{4x-3} \\
 &= \frac{5x^2-6x+1}{x^2-1} \cdot \frac{4x^2+7x+3}{16x^2-9} - \frac{x}{4x-3} \\
 &= \frac{(5x-1)\cancel{(x-1)}}{\cancel{(x+1)}\cancel{(x-1)}} \cdot \frac{(4x+3)\cancel{(x+1)}}{\cancel{(4x+3)}(4x-3)} - \frac{x}{4x-3} \\
 &= \frac{5x-1}{4x-3} - \frac{x}{4x-3} \\
 &= \frac{4x-1}{4x-3}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{4}{y+1} + \frac{y+2}{y^2-1} - \frac{3}{y-1} &= \frac{4}{y+1} + \frac{y+2}{(y+1)(y-1)} - \frac{3}{y-1} \quad \text{LCD} = (y+1)(y-1) \\
 &= \frac{4}{y+1} \cdot \frac{y-1}{y-1} + \frac{y+2}{(y+1)(y-1)} - \frac{3}{y-1} \cdot \frac{y+1}{y+1} \\
 &= \frac{4(y-1)+y+2-3(y+1)}{(y+1)(y-1)} \\
 &= \frac{4y-4+y+2-3y-3}{(y+1)(y-1)} \\
 &= \frac{2y-5}{(y+1)(y-1)}
 \end{aligned}$$

$$7. \frac{a^2-16}{2x+6} \cdot \frac{x+3}{a-4} = \frac{(a+4)\cancel{(a-4)}}{2\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{\cancel{a-4}}$$

$$= \frac{a+4}{2}$$

$$9. \frac{2+\frac{1}{a}}{4-\frac{1}{a^2}} \quad \text{LCD} = a^2$$

$$\frac{a^2\left(2+\frac{1}{a}\right)}{a^2\left(4-\frac{1}{a^2}\right)} = \frac{a^2(2)+a^2\left(\frac{1}{a}\right)}{a^2(4)-a^2\left(\frac{1}{a^2}\right)}$$

$$= \frac{2a^2+a}{4a^2-1} = \frac{a\cancel{(2a+1)}}{\cancel{(2a+1)}(2a-1)} = \frac{a}{2a-1}$$

$$11. \frac{6xy}{x^2-y^2} + \frac{x+y}{y-x} = \frac{6xy}{(x+y)(x-y)} + \frac{x+y}{y-x}$$

$$\text{LCD} = (x+y)(x-y)$$

$$= \frac{6xy}{(x+y)(x-y)} + \frac{x+y}{y-x} \cdot \frac{(-1)}{(-1)} \cdot \frac{x+y}{x+y}$$

$$= \frac{6xy - (x^2 + 2xy + y^2)}{(x+y)(x-y)}$$

$$= \frac{6xy - x^2 - 2xy - y^2}{(x+y)(x-y)}$$

$$= \frac{-x^2 + 4xy - y^2}{(x+y)(x-y)}$$

$$13. \frac{3}{x-2} - \frac{x-2}{6} \quad \text{LCD} = 6(x-2)$$

$$= \frac{3}{x-2} \cdot \frac{6}{6} - \frac{x-2}{6} \cdot \frac{x-2}{x-2}$$

$$= \frac{18 - (x^2 - 4x + 4)}{6(x-2)}$$

$$= \frac{18 - x^2 + 4x - 4}{6(x-2)}$$

$$= \frac{-x^2 + 4x + 14}{6(x-2)}$$

$$15. \frac{1}{w-1} - \frac{w+2}{3w-3} = \frac{1}{w-1} - \frac{w+2}{3(w-1)}$$

$$\text{LCD} = 3(w-1)$$

$$= \frac{3}{3} \cdot \frac{1}{w-1} - \frac{w+2}{3(w-1)} = \frac{3-w-2}{3(w-1)}$$

$$= \frac{1-w}{3(w-1)}$$

$$= \frac{-1\cancel{(w-1)}}{3\cancel{(w-1)}} = -\frac{1}{3}$$

$$17. \frac{y+\frac{2}{y}-3}{1-\frac{2}{y}} \quad \text{LCD} = y$$

$$\frac{y\left(y+\frac{2}{y}-3\right)}{y\left(1-\frac{2}{y}\right)} = \frac{y(y)+y\left(\frac{2}{y}\right)-y(3)}{y(1)-y\left(\frac{2}{y}\right)}$$

$$= \frac{y^2+2-3y}{y-2} = \frac{\cancel{(y-2)}(y-1)}{\cancel{y-2}} = y-1$$

$$\begin{aligned}
 19. \quad & \frac{4x^2 + 22x + 24}{4x + 4} \cdot \frac{6x + 6}{4x^2 - 9} \\
 &= \frac{2(2x^2 + 11x + 12)}{4(x + 1)} \cdot \frac{6(x + 1)}{(2x - 3)(2x + 3)} \\
 &= \frac{\cancel{2}(\cancel{2x + 3})(x + 4)}{\cancel{2} \cdot \cancel{2}(x + 1)} \cdot \frac{\cancel{2} \cdot 3(\cancel{x + 1})}{(2x - 3)(\cancel{2x + 3})} \\
 &= \frac{3(x + 4)}{2x - 3}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{3x - 1}{4} + \frac{7}{6x - 2} = \frac{3x - 1}{4} + \frac{7}{2(3x - 1)} \\
 & \text{LCD} = 4(3x - 1) \\
 &= \frac{3x - 1}{4} \cdot \frac{3x - 1}{3x - 1} + \frac{7}{2(3x - 1)} \cdot \frac{2}{2} \\
 &= \frac{9x^2 - 6x + 1 + 14}{4(3x - 1)} = \frac{9x^2 - 6x + 15}{4(3x - 1)}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & (y + 2) \cdot \frac{2y + 1}{y^2 - 4} - \frac{y - 2}{y + 3} = \frac{\cancel{(y + 2)}(2y + 1)}{\cancel{(y + 2)}(y - 2)} - \frac{y - 2}{y + 3} = \frac{2y + 1}{y - 2} - \frac{y - 2}{y + 3} \\
 & \text{LCD} = (y - 2)(y + 3) \\
 & \left(\frac{y + 3}{y + 3} \right) \left(\frac{2y + 1}{y - 2} \right) - \left(\frac{y - 2}{y - 2} \right) \left(\frac{y - 2}{y + 3} \right) = \frac{2y^2 + 7y + 3 - (y^2 - 4y + 4)}{(y - 2)(y + 3)} \\
 &= \frac{2y^2 + 7y + 3 - y^2 + 4y - 4}{(y - 2)(y + 3)} = \frac{y^2 + 11y - 1}{(y - 2)(y + 3)}
 \end{aligned}$$

Section 5.5 Practice Exercises

1. a. rational.
- b. denominator
- c. No.

$$\begin{aligned}
 3. \quad & \frac{3}{y^2 - 1} - \frac{2}{y^2 - 2y + 1} = \frac{3}{(y + 1)(y - 1)} - \frac{2}{(y - 1)^2} \quad \text{LCD} = (y - 1)^2(y + 1) \\
 &= \frac{3}{(y + 1)(y - 1)} \cdot \frac{y - 1}{y - 1} - \frac{2}{(y - 1)^2} \cdot \frac{y + 1}{y + 1} \\
 &= \frac{3y - 3 - 2y - 2}{(y - 1)^2(y + 1)} = \frac{y - 5}{(y - 1)^2(y + 1)}
 \end{aligned}$$

$$5. \quad \frac{2t^2 + 7t + 3}{4t^2 - 1} \div (t + 3) = \frac{2t^2 + 7t + 3}{4t^2 - 1} \cdot \frac{1}{t + 3} = \frac{\cancel{(2t + 1)}(\cancel{t + 3})}{\cancel{(2t + 1)}(2t - 1)} \cdot \frac{1}{\cancel{t + 3}} = \frac{1}{2t - 1}$$

$$\begin{aligned}
 7. \quad \frac{x+y}{x^{-1}+y^{-1}} &= \frac{x+y}{\frac{1}{x}+\frac{1}{y}} \quad \text{LCD} = xy \\
 \frac{xy(x+y)}{xy\left(\frac{1}{x}+\frac{1}{y}\right)} &= \frac{xy(x)+xy(y)}{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)} \\
 &= \frac{x^2y+xy^2}{y+x} \\
 &= \frac{xy(\cancel{x+y})}{\cancel{x+y}} = xy
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{x+2}{3} - \frac{x-4}{4} &= \frac{1}{2} \quad \text{LCD} = 12 \\
 12\left(\frac{x+2}{3} - \frac{x-4}{4}\right) &= 12\left(\frac{1}{2}\right) \\
 12\left(\frac{x+2}{3}\right) - 12\left(\frac{x-4}{4}\right) &= 12\left(\frac{1}{2}\right) \\
 4(x+2) - 3(x-4) &= 6 \\
 4x+8 - 3x+12 &= 6 \\
 x+20 &= 6 \\
 x &= -14 \quad \{-14\}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{3y}{4} - 2 &= \frac{5y}{6} \quad \text{LCD} = 12 \\
 12\left(\frac{3y}{4} - 2\right) &= 12\left(\frac{5y}{6}\right) \\
 12\left(\frac{3y}{4}\right) - 12(2) &= 12\left(\frac{5y}{6}\right) \\
 9y - 24 &= 10y \\
 -24 &= y \quad \{-24\}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{5}{4p} - \frac{7}{6} + 3 &= 0 \quad \text{LCD} = 12p \quad \text{so } p \neq 0 \\
 12p\left(\frac{5}{4p} - \frac{7}{6} + 3\right) &= 12p(0) \\
 12p\left(\frac{5}{4p}\right) - 12p\left(\frac{7}{6}\right) + 12p(3) &= 0 \\
 15 - 14p + 36p &= 0 \\
 15 + 22p = 0 &\Rightarrow 22p = -15 \\
 p &= -\frac{15}{22} \quad \left\{-\frac{15}{22}\right\}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{1}{2} - \frac{3}{2x} &= \frac{4}{x} - \frac{5}{12} \quad \text{LCD} = 12x \quad \text{so } x \neq 0 \\
 12x\left(\frac{1}{2} - \frac{3}{2x}\right) &= 12x\left(\frac{4}{x} - \frac{5}{12}\right) \\
 12x\left(\frac{1}{2}\right) - 12x\left(\frac{3}{2x}\right) &= 12x\left(\frac{4}{x}\right) - 12x\left(\frac{5}{12}\right) \\
 6x - 18 &= 48 - 5x \\
 11x &= 66 \\
 x &= 6 \quad \{6\}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{3}{x-4} + 2 &= \frac{5}{x-4} \\
 \text{LCD} = x-4 &\quad \text{so } x \neq 4 \\
 (x-4)\left(\frac{3}{x-4} + 2\right) &= (\cancel{x-4})\left(\frac{5}{\cancel{x-4}}\right) \\
 (\cancel{x-4})\left(\frac{3}{\cancel{x-4}}\right) + (x-4)(2) &= 5 \\
 3 + 2x - 8 &= 5 \\
 2x - 5 &= 5 \\
 2x &= 10 \\
 x &= 5 \quad \{5\}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{1}{3} + \frac{2}{w-3} &= 1 \quad \text{LCD} = 3(w-3) \quad \text{so } w \neq 3 \\
 21. \quad \frac{12}{x} - \frac{12}{x-5} &= \frac{2}{x} \quad \text{LCD} = x(x-5) \quad \text{so } x \neq 0 \text{ or } x \neq 5
 \end{aligned}$$

$$\begin{aligned}
3(w-3)\left(\frac{1}{3} + \frac{2}{w-3}\right) &= 3(w-3)(1) \\
\cancel{3}(w-3)\left(\frac{1}{\cancel{3}}\right) + 3(\cancel{w-3})\left(\frac{2}{\cancel{w-3}}\right) & \\
&= 3w-9 \\
w-3+6 &= 3w-9 \\
w+3 &= 3w-9 \\
-2w &= -12 \Rightarrow w=6 \quad \{6\}
\end{aligned}$$

$$\begin{aligned}
x(x-5)\left(\frac{12}{x} - \frac{12}{x-5}\right) &= \cancel{x}(x-5)\left(\frac{2}{\cancel{x}}\right) \\
\cancel{x}(x-5)\left(\frac{12}{\cancel{x}}\right) - x(\cancel{x-5})\left(\frac{12}{\cancel{x-5}}\right) &= 2x-10 \\
12x-60-12x &= 2x-10 \\
-60 &= 2x-10 \\
-50 &= 2x \\
x &= -25 \quad \{-25\}
\end{aligned}$$

23. $\frac{3}{a^2} - \frac{4}{a} = -1$ LCD = a^2 so $a \neq 0$

$$\begin{aligned}
a^2\left(\frac{3}{a^2} - \frac{4}{a}\right) &= a^2(-1) \\
\cancel{a^2}\left(\frac{3}{\cancel{a^2}}\right) - a \cdot \cancel{a}\left(\frac{4}{\cancel{a}}\right) &= -a^2 \\
3-4a &= -a^2 \\
a^2-4a+3 &= 0 \\
(a-3)(a-1) &= 0 \\
a-3=0 \text{ or } a-1=0 & \\
a=3 \text{ or } a=1 & \\
\{3,1\} &
\end{aligned}$$

25. $\frac{1}{4}a - 4a^{-1} = 0$

$$\begin{aligned}
\frac{a}{4} - \frac{4}{a} &= 0 \quad \text{LCD} = 4a \quad \text{so } a \neq 0 \\
4a\left(\frac{a}{4} - \frac{4}{a}\right) &= 4a(0) \\
\cancel{4}a\left(\frac{a}{\cancel{4}}\right) - 4\cancel{a}\left(\frac{4}{\cancel{a}}\right) &= 0 \\
a^2-16 &= 0 \\
(a+4)(a-4) &= 0 \\
a+4=0 \text{ or } a-4=0 & \\
a=-4 \text{ or } a=4 & \quad \{-4,4\}
\end{aligned}$$

27. $\frac{y}{y+3} + \frac{2}{y^2+3y} = \frac{6}{y}$

$$\begin{aligned}
\frac{y}{y+3} + \frac{2}{y(y+3)} &= \frac{6}{y} \\
\text{LCD} = y(y+3) \text{ so } y \neq 0, y \neq -3 & \\
y(y+3)\left(\frac{y}{y+3} + \frac{2}{y(y+3)}\right) &= y(y+3)\frac{6}{y} \\
\cancel{y}(\cancel{y+3})\left(\frac{y}{\cancel{y+3}}\right) + \cancel{y}(\cancel{y+3})\left(\frac{2}{\cancel{y}(y+3)}\right) &= \cancel{y}(y+3)\frac{6}{\cancel{y}} \\
y^2+2 &= (y+3)6 \\
y^2+2 &= 6y+18 \\
y^2-6y-16 &= 0
\end{aligned}$$

$$\begin{aligned}(y-8)(y+2) &= 0 \\ y-8=0 \text{ or } y+2=0 \\ y=8 \text{ or } y=-2 \quad \{8, -2\}\end{aligned}$$

$$\begin{aligned}29. \quad \frac{4}{t-2} - \frac{8}{t^2-2t} &= -2 \\ \frac{4}{t-2} - \frac{8}{t(t-2)} &= -2 \quad \text{LCD} = t(t-2) \quad \text{so } t \neq 0 \text{ or } t \neq 2\end{aligned}$$

$$t(t-2) \left(\frac{4}{t-2} - \frac{8}{t(t-2)} \right) = t(t-2)(-2)$$

$$t(\cancel{t-2}) \left(\frac{4}{\cancel{t-2}} \right) - \cancel{t}(\cancel{t-2}) \left(\frac{8}{\cancel{t}(\cancel{t-2})} \right) = -2t(t-2)$$

$$4t - 8 = -2t^2 + 4t$$

$$2t^2 - 8 = 0$$

$$2(t^2 - 4) = 0$$

$$2(t+2)(t-2) = 0$$

$$2 \neq 0 \text{ or } t+2=0 \text{ or } t-2=0$$

$$t = -2 \text{ or } t = 2$$

$\{-2\}$ is the solution. ($t = 2$ does not check because the denominator is zero.)

$$\begin{aligned}31. \quad \frac{6}{5y+10} - \frac{1}{y-5} &= \frac{4}{y^2-3y-10} \\ \frac{6}{5(y+2)} - \frac{1}{y-5} &= \frac{4}{(y-5)(y+2)} \quad \text{LCD} = 5(y-5)(y+2) \quad \text{so } y \neq 5 \text{ or } y \neq -2\end{aligned}$$

$$5(y-5)(y+2) \left(\frac{6}{5(y+2)} - \frac{1}{y-5} \right) = 5(y-5)(y+2) \left(\frac{4}{(y-5)(y+2)} \right)$$

$$\begin{aligned}\cancel{5}(y-5)(\cancel{y+2}) \left(\frac{6}{\cancel{5}(\cancel{y+2})} \right) - 5(\cancel{y-5})(y+2) \left(\frac{1}{\cancel{y-5}} \right) \\ = 5(\cancel{y-5})(\cancel{y+2}) \left(\frac{4}{(\cancel{y-5})(\cancel{y+2})} \right)\end{aligned}$$

$$6y - 30 - 5y - 10 = 20$$

$$y - 40 = 20 \Rightarrow y = 60$$

$\{60\}$ is the solution.

$$33. \quad \frac{x}{x-5} + \frac{1}{5} = \frac{5}{x-5} \quad \text{LCD} = 5(x-5) \quad \text{so } x \neq 5$$

$$5(x-5)\left(\frac{x}{x-5} + \frac{1}{5}\right) = 5(x-5)\left(\frac{5}{x-5}\right)$$

$$5(\cancel{x-5})\left(\frac{x}{\cancel{x-5}}\right) + \cancel{5}(x-5)\left(\frac{1}{\cancel{5}}\right) = 5(\cancel{x-5})\left(\frac{5}{\cancel{x-5}}\right)$$

$$5x + x - 5 = 25$$

$$6x - 5 = 25$$

$$6x = 30$$

$$x = 5$$

{ } No solution. ($x = 5$ does not check because it makes the denominator zero.)

$$35. \quad \frac{6}{x^2 - 4x + 3} - \frac{1}{x-3} = \frac{1}{4x-4}$$

$$\frac{6}{(x-3)(x-1)} - \frac{1}{x-3} = \frac{1}{4(x-1)} \quad \text{LCD} = 4(x-3)(x-1) \quad \text{so } x \neq 3 \text{ or } x \neq 1$$

$$4(x-3)(x-1)\left(\frac{6}{(x-3)(x-1)} - \frac{1}{x-3}\right) = \cancel{4}(x-3)(\cancel{x-1})\left(\frac{1}{\cancel{4}(\cancel{x-1})}\right)$$

$$4(\cancel{x-3})(\cancel{x-1})\left(\frac{6}{(\cancel{x-3})(\cancel{x-1})}\right) - 4(\cancel{x-3})(x-1)\left(\frac{1}{\cancel{x-3}}\right) = x-3$$

$$24 - 4x + 4 = x - 3$$

$$-4x + 28 = x - 3$$

$$-5x = -31$$

$$x = \frac{31}{5} \quad \left\{\frac{31}{5}\right\} \text{ is the solution.}$$

$$37. \quad \frac{1}{k+2} - \frac{4}{k-2} - \frac{k^2}{4-k^2} = 0$$

$$\frac{1}{k+2} - \frac{4}{k-2} + \frac{k^2}{(k+2)(k-2)} = 0 \quad \text{LCD} = (k+2)(k-2) \quad \text{so } k \neq -2 \text{ or } k \neq 2$$

$$(k+2)(k-2)\left(\frac{1}{k+2} - \frac{4}{k-2} + \frac{k^2}{(k+2)(k-2)}\right) = (k+2)(k-2)(0)$$

$$\begin{aligned}
 (\cancel{k+2})(k-2)\left(\frac{1}{\cancel{k+2}}\right) - (k+2)(\cancel{k-2})\left(\frac{4}{\cancel{k-2}}\right) + (\cancel{k+2})(\cancel{k-2})\left(\frac{k^2}{(\cancel{k+2})(\cancel{k-2})}\right) &= 0 \\
 k-2-4k-8+k^2 &= 0 \\
 k^2-3k-10 &= 0 \\
 (k-5)(k+2) &= 0 \\
 k-5=0 \text{ or } k+2=0 & \\
 k=5 \text{ or } k=-2 &
 \end{aligned}$$

{5} is the solution. ($k = -2$ does not check because the denominator is zero.)

$$\begin{aligned}
 39. \quad \frac{5}{x^2-7x+12} &= \frac{2}{x-3} + \frac{5}{x-4} \\
 \frac{5}{(x-4)(x-3)} &= \frac{2}{x-3} + \frac{5}{x-4} \quad \text{LCD} = (x-4)(x-3) \text{ so } x \neq 4 \text{ or } x \neq 3 \\
 (x-4)(x-3)\left(\frac{5}{(x-4)(x-3)}\right) &= (x-4)(x-3)\left(\frac{2}{x-3} + \frac{5}{x-4}\right) \\
 (\cancel{x-4})(\cancel{x-3})\left(\frac{5}{(\cancel{x-4})(\cancel{x-3})}\right) &= (x-4)(\cancel{x-3})\left(\frac{2}{\cancel{x-3}}\right) + (\cancel{x-4})(x-3)\left(\frac{5}{\cancel{x-4}}\right) \\
 5 &= 2x-8+5x-15 \\
 5 &= 7x-23 \\
 28 &= 7x \\
 x &= 4
 \end{aligned}$$

{ } no solution. ($x = 4$ does not check because the denominator is zero.)

$$\begin{aligned}
 41. \quad \frac{4}{x^2+7x+12} - \frac{7}{x^2+8x+15} &= \frac{1}{x^2+9x+20} \\
 \frac{4}{(x+4)(x+3)} - \frac{7}{(x+5)(x+3)} &= \frac{1}{(x+5)(x+4)} \\
 \text{LCD} &= (x+4)(x+3)(x+5) \text{ so } x \neq -4, x \neq -3, x \neq -5 \\
 (x+4)(x+3)(x+5)\left(\frac{4}{(x+4)(x+3)} - \frac{7}{(x+5)(x+3)}\right) & \\
 &= (x+4)(x+3)(x+5)\left(\frac{1}{(x+5)(x+4)}\right)
 \end{aligned}$$

$$\begin{aligned} & (\cancel{x+4})(\cancel{x+3})(x+5) \frac{4}{(\cancel{x+4})(\cancel{x+3})} - (x+4)(\cancel{x+3})(\cancel{x+5}) \frac{7}{(\cancel{x+5})(\cancel{x+3})} \\ & = (\cancel{x+4})(x+3)(\cancel{x+5}) \frac{1}{(\cancel{x+5})(\cancel{x+4})} \end{aligned}$$

$$4x + 20 - 7x - 28 = x + 3$$

$$-3x - 8 = x + 3$$

$$-4x = 11$$

$$x = -\frac{11}{4} \quad \left\{ -\frac{11}{4} \right\} \text{ is the solution.}$$

43. $K = \frac{ma}{F}$ for m
 $KF = ma$
 $\frac{KF}{a} = m$

45. $K = \frac{IR}{E}$ for E
 $KE = IR$
 $E = \frac{IR}{K}$

47. $I = \frac{E}{R+r}$ for R
 $I(R+r) = E$
 $IR + Ir = E$
 $IR = E - Ir$
 $R = \frac{E - Ir}{I}$ or $R = \frac{E}{I} - r$

49. $h = \frac{2A}{B+b}$ for B
 $h(B+b) = 2A$
 $hB + hb = 2A$
 $hB = 2A - hb$
 $B = \frac{2A - hb}{h}$ or $B = \frac{2A}{h} - b$

51. $x = \frac{at+b}{t}$ for t
 $xt = at + b$
 $xt - at = b$
 $t(x-a) = b$
 $t = \frac{b}{x-a}$

53. $\frac{x-y}{xy} = z$ for x
 $x-y = xyz$
 $x - xyz = y$
 $x(1-yz) = y$
 $x = \frac{y}{1-yz}$

55. $a+b = \frac{2A}{h}$ for h
 $h(a+b) = 2A$
 $h = \frac{2A}{a+b}$

$$\begin{aligned}
 57. \quad \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \quad \text{for } R \\
 RR_1R_2\left(\frac{1}{R}\right) &= RR_1R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\
 \cancel{R}R_1R_2\left(\frac{1}{\cancel{R}}\right) &= R\cancel{R}_1R_2\left(\frac{1}{\cancel{R}_1}\right) \\
 &\quad + RR_1\cancel{R}_2\left(\frac{1}{\cancel{R}_2}\right) \\
 R_1R_2 &= RR_2 + RR_1 = R(R_2 + R_1) \\
 \frac{R_1R_2}{R_2 + R_1} &= R
 \end{aligned}$$

$$\begin{aligned}
 61. \quad \frac{3}{x+2} + \frac{2}{x} &= \frac{-4}{x^2+2x} \\
 \frac{3}{x+2} + \frac{2}{x} &= \frac{-4}{x(x+2)} \\
 \text{LCD} &= x(x+2) \text{ so } x \neq 0, x \neq -2 \\
 x(x+2)\left(\frac{3}{x+2} + \frac{2}{x}\right) & \\
 &= x(x+2)\frac{-4}{x(x+2)} \\
 \cancel{x}(\cancel{x+2})\frac{3}{\cancel{x+2}} + \cancel{x}(\cancel{x+2})\frac{2}{\cancel{x}} & \\
 &= \cancel{x}(\cancel{x+2})\frac{-4}{\cancel{x}(\cancel{x+2})} \\
 3x+2x+4 &= -4 \\
 5x+4 &= -4 \Rightarrow 5x = -8 \\
 x &= -\frac{8}{5} \quad \left\{-\frac{8}{5}\right\}
 \end{aligned}$$

$$\begin{aligned}
 65. \quad \frac{2}{v-1} - \frac{4}{v+5} &= \frac{3}{v^2+4v-5} \\
 \frac{2}{v-1} - \frac{4}{v+5} &= \frac{3}{(v+5)(v-1)} \quad \text{LCD} = (v+5)(v-1) \quad \text{so } v \neq -5 \text{ or } v \neq 1
 \end{aligned}$$

$$\begin{aligned}
 59. \quad v &= \frac{s_2 - s_1}{t_2 - t_1} \quad \text{for } t_2 \\
 v(t_2 - t_1) &= s_2 - s_1 \\
 vt_2 - vt_1 &= s_2 - s_1 \\
 vt_2 &= s_2 - s_1 + vt_1 \\
 t_2 &= \frac{s_2 - s_1 + vt_1}{v}
 \end{aligned}$$

$$\begin{aligned}
 63. \quad 4c(c+1) &= 3(c^2+4) \\
 4c^2+4c &= 3c^2+12 \\
 c^2+4c-12 &= 0 \\
 (c+6)(c-2) &= 0 \\
 c+6=0 \text{ or } c-2=0 & \\
 c=-6 \text{ or } c=2 & \quad \{-6, 2\}
 \end{aligned}$$

$$\begin{aligned}
(v+5)(v-1)\left(\frac{2}{v-1}-\frac{4}{v+5}\right) &= (v+5)(v-1)\left(\frac{3}{(v+5)(v-1)}\right) \\
(v+5)\cancel{(v-1)}\left(\frac{2}{\cancel{v-1}}\right) - \cancel{(v+5)}(v-1)\left(\frac{4}{\cancel{v+5}}\right) &= \cancel{(v+5)}\cancel{(v-1)}\left(\frac{3}{\cancel{(v+5)}\cancel{(v-1)}}\right) \\
2v+10-4v+4 &= 3 \\
-2v+14 &= 3 \Rightarrow -2v = -11 \\
v &= \frac{11}{2} \quad \left\{\frac{11}{2}\right\}
\end{aligned}$$

67. $5(x-9) = 3(x+4) - 2(4x+1)$
 $5x - 45 = 3x + 12 - 8x - 2$
 $5x - 45 = -5x + 10$
 $10x - 45 = 10$
 $10x = 55$
 $x = \frac{55}{10}$
 $x = \frac{11}{2} \quad \left\{\frac{11}{2}\right\}$

69. $\frac{3y}{10} - \frac{5}{2y} = \frac{y}{5}$ LCD = 10y so $y \neq 0$
 $10y\left(\frac{3y}{10} - \frac{5}{2y}\right) = 10y\left(\frac{y}{5}\right)$
 $\cancel{10}y\left(\frac{3y}{\cancel{10}}\right) - 5 \cdot \cancel{2}y\left(\frac{5}{\cancel{2}y}\right) = \cancel{2} \cdot 2y\left(\frac{y}{\cancel{2}}\right)$
 $3y^2 - 25 = 2y^2 \Rightarrow y^2 - 25 = 0$
 $(y-5)(y+5) = 0$
 $y-5 = 0$ or $y+5 = 0$
 $y = 5$ or $y = -5 \quad \{-5, 5\}$

71. $\frac{1}{2}(4d-1) + \frac{2}{3}(2d+2) = \frac{5}{6}(4d+1)$
LCD = 6
 $6\left[\frac{1}{2}(4d-1) + \frac{2}{3}(2d+2)\right]$
 $= 6\left[\frac{5}{6}(4d+1)\right]$
 $3(4d-1) + 4(2d+2) = 5(4d+1)$
 $12d - 3 + 8d + 8 = 20d + 5$
 $20d + 5 = 20d + 5$
 $5 = 5$
 $\{d \mid d \text{ is a real number}\}$

73. $8t^{-1} + 2 = 3t^{-1}$
 $\frac{8}{t} + 2 = \frac{3}{t}$ LCD = t so $t \neq 0$
 $t\left(\frac{8}{t} + 2\right) = t\left(\frac{3}{t}\right)$
 $\cancel{t}\left(\frac{8}{\cancel{t}}\right) + t(2) = \cancel{t}\left(\frac{3}{\cancel{t}}\right)$
 $8 + 2t = 3$
 $2t = -5$
 $t = -\frac{5}{2} \quad \left\{-\frac{5}{2}\right\}$

75. $\frac{y-1}{11-3} = \frac{1}{2}$

77. $\frac{2-(-2)}{x-4} = 4$

Problem Recognition Exercises: Rational Equations vs. Expressions

$$\begin{array}{l} \frac{y-1}{8} = \frac{1}{2} \quad \text{LCD} = 8 \\ \cancel{8} \left(\frac{y-1}{\cancel{8}} \right) = 4 \cdot \cancel{2} \left(\frac{1}{\cancel{2}} \right) \\ y-1 = 4 \\ y = 5 \end{array} \qquad \begin{array}{l} \frac{4}{x-4} = 4 \quad \text{LCD} = x-4 \\ (\cancel{x-4}) \left(\frac{4}{\cancel{x-4}} \right) = (x-4)(4) \\ 4 = 4x - 16 \\ 20 = 4x \Rightarrow x = 5 \end{array}$$

Problem Recognition Exercises

1. a. $\frac{3}{w-5} + \frac{10}{w^2-25} - \frac{1}{w+5} = \frac{3}{w-5} + \frac{10}{(w+5)(w-5)} - \frac{1}{w+5} \quad \text{LCD} = (w+5)(w-5)$

$$= \frac{3}{w-5} \cdot \frac{w+5}{w+5} + \frac{10}{(w+5)(w-5)} - \frac{1}{w+5} \cdot \frac{w-5}{w-5} = \frac{3w+15+10-w+5}{(w+5)(w-5)} = \frac{2w+30}{(w+5)(w-5)}$$

b. $\frac{3}{w-5} + \frac{10}{w^2-25} - \frac{1}{w+5} = 0$

$$\frac{2w+30}{(w+5)(w-5)} = 0$$

$$(\cancel{w+5})(\cancel{w-5}) \left(\frac{2w+30}{(\cancel{w+5})(\cancel{w-5})} \right) = (w+5)(w-5)(0)$$

$$2w+30 = 0$$

$$2w = -30$$

$$w = -15 \quad \{-15\}$$

c. The problem in part (a) is an expression, and the problem in part (b) is an equation.

3. $\frac{2}{a^2+4a+3} + \frac{1}{a+3} = \frac{2}{(a+3)(a+1)} + \frac{1}{a+3} \quad \text{LCD} = (a+3)(a+1)$

$$= \frac{2}{(a+3)(a+1)} + \frac{1}{a+3} \cdot \frac{a+1}{a+1} = \frac{2+a+1}{(a+3)(a+1)} = \frac{\cancel{a+3}}{(\cancel{a+3})(a+1)} = \frac{1}{a+1}$$

5. $\frac{7}{y^2-y-2} + \frac{1}{y+1} - \frac{3}{y-2} = 0$

$$\frac{7}{(y-2)(y+1)} + \frac{1}{y+1} - \frac{3}{y-2} = 0 \quad \text{LCD} = (y-2)(y+1) \quad \text{so } y \neq 2 \text{ or } y \neq -1$$

$$(y-2)(y+1)\left(\frac{7}{(y-2)(y+1)} + \frac{1}{y+1} - \frac{3}{y-2}\right) = (y-2)(y+1)(0)$$

$$\begin{aligned} (\cancel{y-2})(\cancel{y+1})\left(\frac{7}{(\cancel{y-2})(\cancel{y+1})}\right) + (y-2)(\cancel{y+1})\left(\frac{1}{\cancel{y+1}}\right) - (\cancel{y-2})(y+1)\left(\frac{3}{\cancel{y-2}}\right) &= 0 \\ 7 + y - 2 - 3y - 3 &= 0 \\ -2y + 2 &= 0 \\ -2y &= -2 \\ y &= 1 \end{aligned}$$

{1} is the solution.

7.
$$\frac{x}{x-1} - \frac{12}{x^2-x} = \frac{x}{x-1} - \frac{12}{x(x-1)}$$

LCD = $x(x-1)$

$$\begin{aligned} &= \frac{x}{x-1} \cdot \frac{x}{x} - \frac{12}{x(x-1)} \\ &= \frac{x^2-12}{x(x-1)} \end{aligned}$$

9.
$$\frac{3}{w} - 5 = \frac{7}{w} - 1$$

LCD = w so $w \neq 0$

$$\begin{aligned} w\left(\frac{3}{w} - 5\right) &= w\left(\frac{7}{w} - 1\right) \\ \cancel{w}\left(\frac{3}{\cancel{w}}\right) - w(5) &= \cancel{w}\left(\frac{7}{\cancel{w}}\right) - w(1) \\ 3 - 5w &= 7 - w \\ -4w &= 4 \Rightarrow w = -1 \quad \{-1\} \end{aligned}$$

11.
$$\frac{4p+1}{8p-12} + \frac{p-3}{2p-3} = \frac{4p+1}{4(2p-3)} + \frac{p-3}{2p-3}$$

LCD = $4(2p-3)$

$$\begin{aligned} &= \frac{4p+1}{4(2p-3)} + \frac{p-3}{2p-3} \cdot \frac{4}{4} \\ &= \frac{4p+1+4p-12}{4(2p-3)} = \frac{8p-11}{4(2p-3)} \end{aligned}$$

13.
$$\frac{1}{2x^2} + \frac{1}{6x} \quad \text{LCD} = 6x^2$$

$$\begin{aligned} &= \frac{1}{2x^2} \cdot \frac{3}{3} + \frac{1}{6x} \cdot \frac{x}{x} \\ &= \frac{3+x}{6x^2} \end{aligned}$$

15.
$$\frac{3}{2t} + \frac{2}{3t^2} = \frac{-1}{t}$$

LCD = $6t^2$ so $t \neq 0$

$$6t^2\left(\frac{3}{2t} + \frac{2}{3t^2}\right) = 6t^2\left(\frac{-1}{t}\right)$$

17.
$$\frac{3}{c^2+4c+3} - \frac{2}{c^2+6c+9}$$

$$\begin{aligned} &= \frac{3}{(c+3)(c+1)} - \frac{2}{(c+3)^2} \\ \text{LCD} &= (c+3)^2(c+1) \end{aligned}$$

$$\begin{aligned}
\cancel{2t} \cdot 3t \left(\frac{3}{\cancel{2t}} \right) + 2 \cdot \cancel{3t} \left(\frac{2}{\cancel{3t}} \right) &= \frac{3}{(c+3)(c+1)} \cdot \frac{c+3}{c+3} - \frac{2}{(c+3)^2} \cdot \frac{c+1}{c+1} \\
&= 6t \cdot \left(\frac{-1}{t} \right) &= \frac{3c+9-2c-2}{(c+3)^2(c+1)} \\
9t+4 = -6t \Rightarrow 4 = -15t & &= \frac{c+7}{(c+3)^2(c+1)} \\
-\frac{4}{15} = t & \left\{ -\frac{4}{15} \right\}
\end{aligned}$$

19. $\frac{4}{w-4} - \frac{36}{2w^2-7w-4} = \frac{3}{2w+1}$

$\frac{4}{w-4} - \frac{36}{(2w+1)(w-4)} = \frac{3}{2w+1}$ LCD = $(2w+1)(w-4)$ so $w \neq -\frac{1}{2}$ or $w \neq 4$

$$(2w+1)(w-4) \left(\frac{4}{w-4} - \frac{36}{(2w+1)(w-4)} \right) = (2w+1)(w-4) \left(\frac{3}{2w+1} \right)$$

$$(2w+1) \cancel{(w-4)} \left(\frac{4}{\cancel{w-4}} \right) - \cancel{(2w+1)} \cancel{(w-4)} \left(\frac{36}{\cancel{(2w+1)} \cancel{(w-4)}} \right) = \cancel{(2w+1)} \cancel{(w-4)} \left(\frac{3}{\cancel{2w+1}} \right)$$

$$8w+4-36=3w-12$$

$$8w-32=3w-12$$

$$5w-32=-12$$

$$5w=20$$

$$w=4$$

{ } no solution. ($w=4$ does not check because the denominator is zero.)

Section 5.6 Practice Exercises

1. a. proportion

b. proportional

3. $2 + \frac{6}{x} = x + 7$ LCD = x so $x \neq 0$

$$x \left(2 + \frac{6}{x} \right) = x(x+7)$$

$$x(2) + \cancel{x} \left(\frac{6}{\cancel{x}} \right) = x^2 + 7x$$

$$2x+6 = x^2+7x$$

5. $\frac{4}{5t-1} + \frac{1}{10t-2} = \frac{4}{5t-1} + \frac{1}{2(5t-1)}$

$$\text{LCD} = 2(5t-1)$$

$$= \frac{4}{5t-1} \cdot \frac{2}{2} + \frac{1}{2(5t-1)}$$

$$= \frac{8+1}{2(5t-1)}$$

$$\begin{aligned}
0 &= x^2 + 5x - 6 & &= \frac{9}{2(5t-1)} \\
(x+6)(x-1) &= 0 \\
x+6=0 \text{ or } x-1 &= 0 \\
x &= -6 \text{ or } x=1 \quad \{-6, 1\}
\end{aligned}$$

$$\begin{aligned}
7. \quad \frac{5}{w-2} &= 7 - \frac{10}{w+2} \quad \text{LCD} = (w+2)(w-2) \quad \text{so } w \neq -2 \text{ or } w \neq 2 \\
(w+2)(w-2) \left(\frac{5}{w-2} \right) &= (w+2)(w-2) \left(7 - \frac{10}{w+2} \right) \\
(w+2) \left(\cancel{w-2} \right) \left(\frac{5}{\cancel{w-2}} \right) &= (w+2)(w-2)(7) - (\cancel{w+2})(w-2) \left(\frac{10}{\cancel{w+2}} \right) \\
5w+10 &= 7(w^2-4) - 10w+20 \\
5w+10 &= 7w^2 - 28 - 10w + 20 \\
0 &= 7w^2 - 15w - 18 \\
(7w+6)(w-3) &= 0 \\
7w+6=0 \text{ or } w-3 &= 0 \\
7w &= -6 \text{ or } w=3 \\
w &= -\frac{6}{7} \text{ or } w=3 \quad \left\{ -\frac{6}{7}, 3 \right\}
\end{aligned}$$

$$\begin{aligned}
9. \quad \frac{8p^2-32}{p^2-4p+4} \cdot \frac{3p^2-3p-6}{2p^2+20p+32} &= \frac{8(p^2-4)}{p^2-4p+4} \cdot \frac{3(p^2-p-2)}{2(p^2+10p+16)} \\
&= \frac{4 \cdot \cancel{(p+2)} \cdot \cancel{(p-2)}}{(\cancel{p-2})(\cancel{p-2})} \cdot \frac{3 \cdot \cancel{(p-2)} \cdot (p+1)}{\cancel{(p+8)} \cdot \cancel{(p+2)}} = \frac{12(p+1)}{p+8}
\end{aligned}$$

$$\begin{aligned}
11. \quad \frac{y}{6} &= \frac{20}{15} \quad \text{LCD} = 30 \\
30 \left(\frac{y}{6} \right) &= 30 \left(\frac{20}{15} \right) \\
5y &= 40 \\
y &= 8 \quad \{8\}
\end{aligned}$$

$$\begin{aligned}
13. \quad \frac{9}{75} &= \frac{m}{50} \quad \text{LCD} = 150 \\
150 \left(\frac{9}{75} \right) &= 150 \left(\frac{m}{50} \right) \\
18 &= 3m \\
m &= 6 \quad \{6\}
\end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{p-1}{4} &= \frac{p+3}{3} & \text{LCD} &= 12 \\
 12\left(\frac{p-1}{4}\right) &= 12\left(\frac{p+3}{3}\right) \\
 3(p-1) &= 4(p+3) \\
 3p-3 &= 4p+12 \\
 -15 &= p & \{-15\}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{x+1}{5} &= \frac{4}{15} & \text{LCD} &= 15 \\
 15\left(\frac{x+1}{5}\right) &= 15\left(\frac{4}{15}\right) \\
 3(x+1) &= 4 \\
 3x+3 &= 4 \\
 3x &= 1 \\
 x &= \frac{1}{3} & \left\{\frac{1}{3}\right\}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{5-2x}{x} &= \frac{1}{4} & \text{LCD} &= 4x \\
 4x\left(\frac{5-2x}{x}\right) &= 4x\left(\frac{1}{4}\right) \\
 4(5-2x) &= x \\
 20-8x &= x \\
 20 &= 9x \\
 x &= \frac{20}{9} & \left\{\frac{20}{9}\right\}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{2}{y-1} &= \frac{y-3}{4} & \text{LCD} &= 4(y-1) \\
 4\left(\frac{2}{y-1}\right) &= 4(y-1)\left(\frac{y-3}{4}\right) \\
 8 &= y^2 - 4y + 3 \\
 0 &= y^2 - 4y - 5 \\
 (y-5)(y+1) &= 0 \\
 y-5=0 & \text{ or } y+1=0 \\
 y=5 & \text{ or } y=-1 & \{5, -1\}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \frac{1}{49w} &= \frac{w}{9} \\
 \text{LCD} &= 9 \cdot 49w \\
 9 \cdot 49w\left(\frac{1}{49w}\right) &= 9 \cdot 49w\left(\frac{w}{9}\right) \\
 9 &= 49w^2 \\
 0 &= 49w^2 - 9 \\
 (7w+3)(7w-3) &= 0 \\
 7w+3=0 & \text{ or } 7w-3=0 \\
 7w=-3 & \text{ or } 7w=3 \\
 w=-\frac{3}{7} & \text{ or } w=\frac{3}{7} & \left\{-\frac{3}{7}, \frac{3}{7}\right\}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \frac{x+3}{5x+26} &= \frac{2}{x+4} \\
 \text{LCD} &= (5x+26)(x+4) \\
 (5x+26)(x+4)\left(\frac{x+3}{5x+26}\right) &= (5x+26)(x+4)\left(\frac{2}{x+4}\right) \\
 x^2+7x+12 &= 10x+52 \\
 x^2-3x-40 &= 0 \\
 (x-8)(x+5) &= 0 \\
 x-8=0 & \text{ or } x+5=0 \\
 x=8 & \text{ or } x=-5 & \{8, -5\}
 \end{aligned}$$

27. Let a = the number of adults

29. Let x = the number of grams of fat

$$\frac{3}{1} = \frac{18}{a}$$

LCD = a

$$a\left(\frac{3}{1}\right) = a\left(\frac{18}{a}\right)$$

$$3a = 18$$

$$a = 6$$

6 adults must be on the staff.

$$\frac{3.5}{21.0} = \frac{14}{x}$$

LCD = $21x$

$$21x\left(\frac{3.5}{21.0}\right) = 21x\left(\frac{14}{x}\right)$$

$$3.5x = 294$$

$$x = 84$$

The 14-oz box of candy contains 84 g of fat.

31. Let x = the number of fish

$$\frac{8}{1840} = \frac{x}{230,000}$$

LCD = 230,000

$$230,000\left(\frac{8}{1840}\right) = 230,000\left(\frac{x}{230,000}\right)$$

$$1000 = x$$

1000 swordfish were caught.

33. Let x = the number of gallons of gas

$$\frac{243}{4.5} = \frac{621}{x} \quad \text{LCD} = 4.5x$$

$$4.5x\left(\frac{243}{4.5}\right) = 4.5x\left(\frac{621}{x}\right)$$

$$243x = 2794.5$$

$$x = 11.5$$

Pam needs 11.5 gallons of gas.

35. Let x = the total number of bison

$$\frac{x}{200} = \frac{120}{6}$$

LCD = 600

$$600\left(\frac{x}{200}\right) = 600\left(\frac{120}{6}\right)$$

$$3x = 12000$$

$$x = 4000$$

There are approximately 4000 bison in the park.

37. Let x = the number of men

$186 - x$ = the number of women

$$\frac{1}{5} = \frac{x}{186 - x}$$

LCD = $186 - x$

$$5(186 - x)\left(\frac{1}{5}\right) = 5(186 - x)\left(\frac{x}{186 - x}\right)$$

$$186 - x = 5x$$

$$186 = 6x$$

$$x = 31$$

There are 31 men enrolled.

39. Let x = the number of women

$1095 - x$ = the number of men

$$\frac{119}{100} = \frac{1095 - x}{x}$$

LCD = $100x$

41. $\frac{11.2}{a} = \frac{14}{10}$

$$10a\left(\frac{11.2}{a}\right) = 10a\left(\frac{14}{10}\right)$$

Section 5.6 Applications of Rational Equations and Proportions

$$100x\left(\frac{119}{100}\right) = 100x\left(\frac{1095-x}{x}\right)$$

$$119x = 109,500 - 100x$$

$$219x = 109,500$$

$$x = 500$$

$$1095 - x = 1095 - 500 = 595$$

There are 595 men and 500 women in the group.

$$112 = 14a$$

$$a = 8 \text{ ft}$$

$$\frac{b}{6} = \frac{14}{10}$$

$$30\left(\frac{b}{6}\right) = 30\left(\frac{14}{10}\right)$$

$$5b = 42$$

$$b = 8.4 \text{ ft}$$

43.
$$\frac{1.75}{5} = \frac{4.55}{y}$$

$$5y\left(\frac{1.75}{5}\right) = 5y\left(\frac{4.55}{y}\right)$$

$$1.75y = 22.75$$

$$y = 13 \text{ in}$$

$$(1.75)^2 + z^2 = (4.55)^2$$

$$3.0625 + z^2 = 20.7025$$

$$z^2 = 17.64$$

$$z = 4.2 \text{ in}$$

$$\frac{1.75}{5} = \frac{4.2}{x}$$

$$5x\left(\frac{1.75}{5}\right) = 5x\left(\frac{4.2}{x}\right)$$

$$1.75x = 21$$

$$x = 12 \text{ in}$$

45. Let x = the number

$$\frac{1}{x} + 5 = \frac{16}{3} \quad \text{LCD} = 3x \text{ so } x \neq 0$$

$$3x\left(\frac{1}{x} + 5\right) = 3x\left(\frac{16}{3}\right)$$

$$3\cancel{x}\left(\frac{1}{\cancel{x}}\right) + 3x(5) = \cancel{3}x\left(\frac{16}{\cancel{3}}\right)$$

$$3 + 15x = 16x$$

$$3 = x$$

47. Let x = the number

$$7 - \frac{1}{x} = \frac{9}{2}$$

$$\text{LCD} = 2x \text{ so } x \neq 0$$

$$2x\left(7 - \frac{1}{x}\right) = 2x\left(\frac{9}{2}\right)$$

$$2x(7) - 2\cancel{x}\left(\frac{1}{\cancel{x}}\right) = \cancel{2}x\left(\frac{9}{\cancel{2}}\right)$$

$$14x - 2 = 9x$$

$$5x = 2 \Rightarrow x = \frac{2}{5}$$

49. a. $x + 7$

b. $\frac{48}{x}$

c. $\frac{83}{x+7}$

51. Let x = the speed in rainstorm

$x + 20$ = the speed in sunny weather

	Distance	Rate	Time
Rain	80	x	$80/x$

$$\text{Sunny} \quad \frac{120}{x+20} \quad \frac{120}{(x+20)}$$

$$(\text{Time rain}) = (\text{Time sunny})$$

$$\frac{80}{x} = \frac{120}{x+20} \quad \text{LCD} = x(x+20)$$

$$x(x+20)\left(\frac{80}{x}\right) = x(x+20)\left(\frac{120}{x+20}\right)$$

$$80x + 1600 = 120x$$

$$1600 = 40x$$

$$x = 40$$

$$x + 20 = 40 + 20 = 60$$

The motorist drives 40 mph in the rainstorm and 60 mph in sunny weather.

53. Let x = the speed of the Broadmoor truck

$x + 6.4$ = the speed of the Wescott truck

	Distance	Rate	Time
Broadmoor	88	x	$88/x$
Wescott	96	$x + 6.4$	$96/(x + 6.4)$

$$(\text{Time Broadmoor}) = (\text{Time Wescott})$$

$$\frac{88}{x} = \frac{96}{x+6.4}$$

$$\text{LCD} = x(x+6.4)$$

$$x(x+6.4)\left(\frac{88}{x}\right) = x(x+6.4)\left(\frac{96}{x+6.4}\right)$$

$$88x + 563.2 = 96x$$

$$563.2 = 8x$$

$$x = 70.4$$

$$x + 6.4 = 70.4 + 6.4$$

$$= 76.8$$

The Broadmoor truck travels 70.4 mph and the Wescott truck travels 76.8 mph.

55. Let x = the speed against the wind

$x + 5$ = the speed with the wind

$$\frac{30}{x} + \frac{30}{x+5} = 5 \quad \text{LCD} = x(x+5)$$

$$x(x+5)\left(\frac{30}{x} + \frac{30}{x+5}\right) = x(x+5)(5)$$

$$30(x+5)+30x=5x(x+5)$$

$$30x+150+30x=5x^2+25x$$

$$5x^2-35x-150=0$$

$$5(x^2-7x-30)=0$$

$$5(x-10)(x+3)=0$$

$$x-10=0 \text{ or } x+3=0$$

$$x=10 \text{ or } x=-3$$

The cyclist rides at a speed of 10 mph against the wind.

57. Let x = Celeste's walking speed

$x + 2$ = speed on moving walkway

	Distance	Rate	Time
Off walkway	100	x	$100/x$
On walkway	140	$x + 2$	$140/(x + 2)$

(Time off walkway)+(Time on walkway) = 40

$$\frac{100}{x} + \frac{140}{x+2} = 40 \quad \text{LCD} = x(x+2)$$

$$x(x+2)\left(\frac{100}{x} + \frac{140}{x+2}\right) = x(x+2)(40)$$

$$100(x+2)+140x=40x(x+2)$$

$$100x+200+140x=40x^2+80x$$

$$240x+200=40x^2+80x$$

$$0=40x^2-160x-200$$

$$0=40(x^2-4x-5)$$

$$0=40(x-5)(x+1)$$

$$x-5=0 \text{ or } x+1=0$$

$$x=5 \text{ or } x \neq -1$$

$$x+2=5+2=7$$

Celeste walks 5 ft/sec on the ground and travels 7 ft/sec while on the moving walkway.

59. Let x = Joe's speed

$x + 2$ = Beatrice's speed

	Distance	Rate	Time
Joe	12	x	$12/x$
Beatrice	12	$x + 2$	$12/(x + 2)$

$$(\text{Joe's time}) - (\text{Beatrice's time}) = 0.5$$

$$\frac{12}{x} - \frac{12}{x+2} = \frac{1}{2} \quad \text{LCD} = 2x(x+2)$$

$$2x(x+2)\left(\frac{12}{x} - \frac{12}{x+2}\right) = 2x(x+2)\left(\frac{1}{2}\right)$$

$$24(x+2) - 24x = x(x+2)$$

$$24x + 48 - 24x = x^2 + 2x$$

$$48 = x^2 + 2x$$

$$0 = x^2 + 2x - 48$$

$$0 = (x-6)(x+8)$$

$$x-6=0 \quad \text{or} \quad x+8=0$$

$$x=6 \quad \text{or} \quad x \neq -8$$

$$x+2=6+2=8$$

Joe runs at 6 mph and Beatrice runs at 8 mph.

61. Work Rate Time Portion of Job Comp

Paint#1	1/6	x	(1/6)x
---------	-----	---	--------

Paint#2	1/8	x	(1/8)x
---------	-----	---	--------

$$(\text{Paint\#1 Part}) + (\text{Paint\#2 Part}) = (1 \text{ Job})$$

$$\frac{1}{6}x + \frac{1}{8}x = 1 \quad \text{LCD} = 24$$

$$24\left(\frac{1}{6}x + \frac{1}{8}x\right) = 24(1)$$

$$4x + 3x = 24$$

$$7x = 24$$

$$x = \frac{24}{7} \text{ hr or } 3\frac{3}{7} \text{ hr}$$

Together, the painters can paint the room in $3\frac{3}{7}$ hr.

63. Work Rate Time Portion of Job Comp

Joel	1/12	x	(1/12)x
------	------	---	---------

Michael	1/15	x	(1/15)x
---------	------	---	---------

$$(\text{Joel's Part}) + (\text{Michael's Part}) = (1 \text{ Job})$$

$$\frac{1}{12}x + \frac{1}{15}x = 1 \quad \text{LCD} = 60$$

$$60\left(\frac{1}{12}x + \frac{1}{15}x\right) = 60(1)$$

$$5x + 4x = 60$$

$$9x = 60$$

$$x = \frac{60}{9} = \frac{20}{3} \text{ hr or } 6\frac{2}{3} \text{ hr}$$

Together, they can fence the yard in $6\frac{2}{3}$ hr.

65. a. Work Rate Time Part of Job Comp

$$\text{Old} \quad \frac{1}{30} \quad 12 \quad \frac{(1/30) 12}{\underline{\hspace{2cm}}}$$

$$\text{New} \quad \frac{1}{x} \quad 12 \quad \frac{(1/x) 12}{\underline{\hspace{2cm}}}$$

(Old Part) + (New Part) = (1 Job)

$$\frac{1}{30} \cdot 12 + \frac{1}{x} \cdot 12 = 1 \quad \text{LCD} = 30x$$

$$30x\left(\frac{1}{30} \cdot 12 + \frac{1}{x} \cdot 12\right) = 30x(1)$$

$$12x + 360 = 30x$$

$$360 = 18x$$

$$x = 20 \text{ hr}$$

The new pump will take 20 hr.

b. The technician should return at noon on Friday.

67. Work Rate Time Part of Job Comp

$$\text{Gus} \quad \frac{1}{x} \quad 4 \quad \frac{(1/x) 4}{\underline{\hspace{2cm}}}$$

$$\text{Sid} \quad \frac{1}{(2x)} \quad 4 \quad \frac{(1/(2x)) 4}{\underline{\hspace{2cm}}}$$

(Gus's Part) + (Sid's Part) = (1 Job)

$$\frac{1}{x} \cdot 4 + \frac{1}{2x} \cdot 4 = 1 \quad \text{LCD} = 2x$$

$$2x\left(\frac{1}{x} \cdot 4 + \frac{1}{2x} \cdot 4\right) = 2x(1)$$

$$8 + 4 = 2x$$

$$12 = 2x$$

$$x = 6 \text{ hr}$$

$$2x = 2(6) = 12 \text{ hr}$$

Gus would take 6 hr and Sid would take 12 hr to dig the garden.