Chapter 6 Roots, Radicals, and Root Functions

Section 6.1 Practice Exercises

1. **a.** b; a

b. principal

c. b^n; a

d. index; radicand

e. cube

f. is not; is

g. even; odd

h. Pythagorean; c^2

i. $[0, \infty)$; $(-\infty, \infty)$

j. -5 and -4

3. **a.** 8 is a square root of 64 because $8^2 = 64$.
-8 is a square root of 64 because $(-8)^2 = 64$.

b. $\sqrt{64} = 8$

c. There are two square roots for every positive number. $\sqrt{64}$ identifies the positive square root.

5. **a.** $\sqrt{81} = 9$

b. $-\sqrt{81} = -9$

7. There is no real number b such that $b^2 = -36$.

9. $\sqrt{49} = 7$

11. $-\sqrt{49} = -7$

13. $\sqrt{-49}$ is not a real number.

15. $\sqrt{\frac{64}{9}} = \frac{8}{3}$

17. $\sqrt{0.81} = 0.9$

19. $-\sqrt{0.16} = -0.4$

21. **a.** $\sqrt{64} = 8$

b. $\sqrt[3]{64} = 4$

c. $-\sqrt{64} = -8$

d. $-\sqrt[3]{64} = -4$

e. $\sqrt{-64}$ is not a real number.

f. $\sqrt[3]{-64} = -4$

23. $\sqrt[3]{-27} = -3$

25. $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

27. $\sqrt[5]{32}=2$

29. $\sqrt[3]{-\frac{125}{64}}=-\frac{5}{4}$

31. $\sqrt[4]{-1}$ is not a real number.

33. $\sqrt[6]{1{,}000{,}000}=10$

35. $-\sqrt[3]{0.008}=-0.2$

37. $\sqrt[4]{0.0625}=0.5$

39. $\sqrt{a^2}=|a|$

41. $\sqrt[3]{a^3}=a$

43. $\sqrt[6]{a^6}=|a|$

45. $\sqrt{(x+1)^2}=|x+1|$

47. $\sqrt{x^2y^4}=\sqrt{x^2\left(y^2\right)^2}=|x|y^2$

49. $-\sqrt[3]{\frac{x^3}{y^3}}=-\frac{x}{y}, y\neq 0$

51. $\frac{2}{\sqrt[4]{x^4}}=\frac{2}{|x|}, x\neq 0$

53. $\sqrt[3]{(-92)^3}=-92$

55. $\sqrt[10]{(-2)^{10}}=|-2|=2$

57. $\sqrt[7]{(-923)^7}=-923$

59. $\sqrt{y^8}=\sqrt{\left(y^4\right)^2}=y^4$

61. $\sqrt{\frac{a^6}{b^2}}=\sqrt{\frac{\left(a^3\right)^2}{b^2}}=\frac{a^3}{b}$

63. $-\sqrt{\frac{25}{q^2}}=-\frac{5}{q}$

65. $\sqrt{9x^2y^4z^2}=\sqrt{9x^2\left(y^2\right)^2z^2}=3xy^2z$

67. $\sqrt{\frac{h^2k^4}{16}}=\sqrt{\frac{h^2\left(k^2\right)^2}{16}}=\frac{hk^2}{4}$

69. $-\sqrt[3]{\frac{t^3}{27}}=-\frac{t}{3}$

71. $\sqrt[5]{32y^{10}} = \sqrt[5]{32\left(y^2\right)^5} = 2y^2$

73. $\sqrt[6]{64p^{12}q^{18}} = \sqrt[6]{64\left(p^2\right)^6\left(q^3\right)^6} = 2p^2q^3$

75.
$$a^2 + b^2 = c^2$$
$$12^2 + b^2 = 15^2$$
$$144 + b^2 = 225$$
$$b^2 = 81 \Rightarrow b = 9 \text{ cm}$$

77.
$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 = c^2$$
$$144 + 25 = c^2$$
$$169 = c^2 \Rightarrow c = 13 \text{ ft}$$

79.
$$a^2 + b^2 = c^2$$
$$4^2 + 3^2 = c^2$$
$$16 + 9 = c^2$$
$$25 = c^2 \Rightarrow c = 5 \text{ mi}$$
They were 5 mi apart.

81.
$$a^2 + b^2 = c^2$$
$$20^2 + 15^2 = c^2$$
$$400 + 225 = c^2$$
$$625 = c^2 \Rightarrow c = 25 \text{ mi}$$
They are 25 mi apart.

83. $h(x) = \sqrt{x-2}$

a. $h(0) = \sqrt{0-2} = \sqrt{-2}$
not a real number

b. $h(1) = \sqrt{1-2} = \sqrt{-1}$
not a real number

c. $h(2) = \sqrt{2-2} = \sqrt{0} = 0$

d. $h(3) = \sqrt{3-2} = \sqrt{1} = 1$

e. $h(6) = \sqrt{6-2} = \sqrt{4} = 2$
$x - 2 \geq 0$
$x \geq 2$ Domain: $[2, \infty)$

85. $g(x) = \sqrt[3]{x-2}$

a. $g(-6) = \sqrt[3]{-6-2} = \sqrt[3]{-8} = -2$

b. $g(1) = \sqrt[3]{1-2} = \sqrt[3]{-1} = -1$

c. $g(2) = \sqrt[3]{2-2} = \sqrt[3]{0} = 0$

d. $g(3) = \sqrt[3]{3-2} = \sqrt[3]{1} = 1$

Domain: $(-\infty, \infty)$

87.
$$f(x) = \sqrt{5-2x}$$
$$5 - 2x \geq 0$$
$$-2x \geq -5$$
$$x \leq \frac{5}{2}$$
Domain: $\left(-\infty, \frac{5}{2}\right]$

89. $k(x) = \sqrt[3]{4x-7}$
Domain: $(-\infty, \infty)$

91. $M(x) = \sqrt{x-5} + 3$

$x - 5 \geq 0$

$x \geq 5$

Domain: $[5, \infty)$

93. $F(x) = \sqrt[3]{x+7} - 2$

Domain: $(-\infty, \infty)$

95. a. $f(x) = \sqrt{1-x}$

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

$(-\infty, 1]$

b. Create a table of ordered pairs where x values are taken to be less than or equal to 1.

x	$f(x)$
1	0
0	1
–3	2
–8	3
–15	4

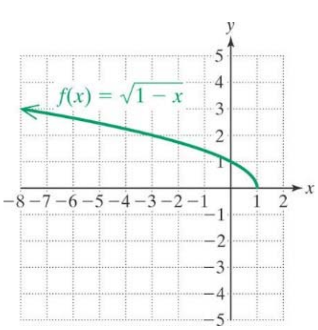
$f(1) = \sqrt{1-1} = \sqrt{0} = 0$

$f(0) = \sqrt{1-0} = \sqrt{1} = 1$

$f(-3) = \sqrt{1-(-3)} = \sqrt{4} = 2$

$f(-8) = \sqrt{1-(-8)} = \sqrt{9} = 3$

$f(-15) = \sqrt{1-(-15)} = \sqrt{16} = 4$



97. a. $f(x) = \sqrt{x+3}$

$x + 3 \geq 0$

$x \geq -3$

$[-3, \infty)$

b. Create a table of ordered pairs where x values are taken to be greater than or equal to -3.

x	$f(x)$
–3	0
–2	1
1	2
6	3
13	4

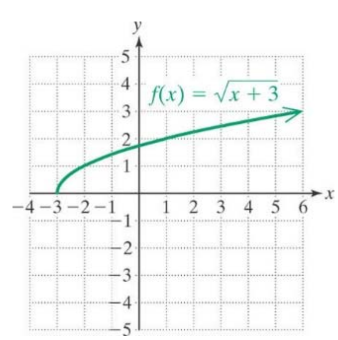
$f(-3) = \sqrt{-3+3} = \sqrt{0} = 0$

$f(-2) = \sqrt{-2+3} = \sqrt{1} = 1$

$f(1) = \sqrt{1+3} = \sqrt{4} = 2$

$f(6) = \sqrt{6+3} = \sqrt{9} = 3$

$f(13) = \sqrt{13+3} = \sqrt{16} = 4$



99. **a.** $f(x)=\sqrt{x}+2$

$x \geq 0$

$[0, \infty)$

b. Create a table of ordered pairs where x values are taken to be greater than or equal to 0.

x	$f(x)$
0	2
1	3
4	4
9	5
16	6

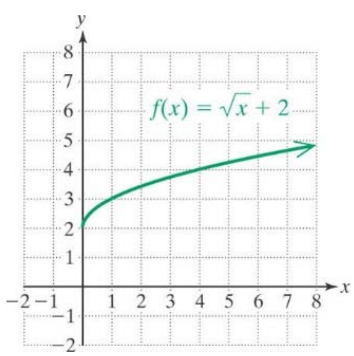
$f(0)=\sqrt{0}+2=0+2=2$

$f(1)=\sqrt{1}+2=1+2=3$

$f(4)=\sqrt{4}+2=2+2=4$

$f(9)=\sqrt{9}+2=3+2=5$

$f(16)=\sqrt{16}+2=4+2=6$



101. **a.** $f(x)=\sqrt[3]{x-1}$

The index is odd; therefore the domain is all real numbers.

$(-\infty, \infty)$

b. Create a table of ordered pairs where x values are taken to be all real numbers.

x	$f(x)$
–7	–2
0	–1
1	0
2	1
9	2

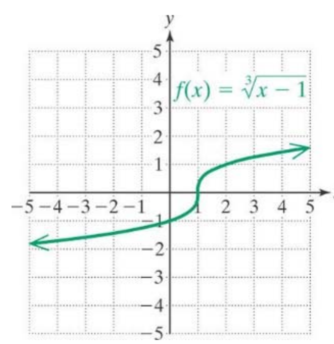
$f(-7)=\sqrt[3]{-7-1}=\sqrt[3]{-8}=-2$

$f(0)=\sqrt[3]{0-1}=\sqrt[3]{-1}=-1$

$f(1)=\sqrt[3]{1-1}=\sqrt[3]{0}=0$

$f(2)=\sqrt[3]{2-1}=\sqrt[3]{1}=1$

$f(9)=\sqrt[3]{9-1}=\sqrt[3]{8}=2$



103. $q+p^2$

105. $\frac{6}{\sqrt[3]{x}}$

107. $s^2 = 64$
$s = \sqrt{64}$
$= 8$ in.

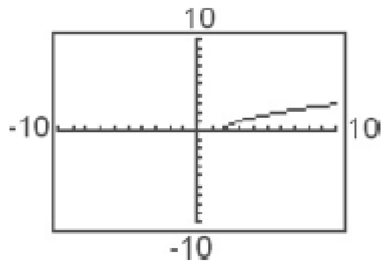
109. $\sqrt{69} \approx 8.3066$

111. $2 + \sqrt[3]{5} \approx 2 + 1.7100$
$= 3.7100$

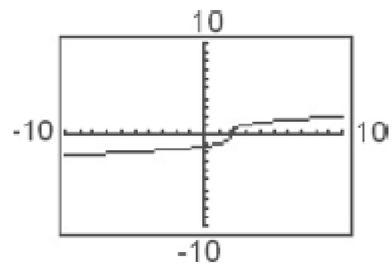
113. $7\sqrt[4]{25} \approx 15.6525$

115. $\dfrac{3 - \sqrt{19}}{11} \approx -0.1235$

117.



119.



Section 6.2 Practice Exercises

1. **a.** $\sqrt[n]{a}$

b. $\left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$

c. $\dfrac{1}{\sqrt{x}}$

d. 2; $\dfrac{1}{2}$

3. $\sqrt{25} = 5$

5. $\sqrt[4]{81} = 3$

7. $144^{1/2} = \sqrt{144} = 12$

9. $-144^{1/2} = -\sqrt{144} = -12$

11. $(-144)^{1/2} = \sqrt{-144}$ is not a real number.

13. $(-64)^{1/3} = \sqrt[3]{-64} = -4$

15. $25^{-1/2} = \dfrac{1}{25^{1/2}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

17. $-49^{-1/2} = -\dfrac{1}{49^{1/2}} = -\dfrac{1}{\sqrt{49}} = -\dfrac{1}{7}$

19. $a^{m/n} = \sqrt[n]{a^m}$; The numerator of the exponent represents the power of the base. The denominator of the exponent represents the index of the radical.

21. **a.** $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

b. $-16^{3/4} = -\left(\sqrt[4]{16}\right)^3 = -\left(2^3\right) = -8$

c. $\left(-16\right)^{3/4} = \left(\sqrt[4]{-16}\right)^3$ is not a real num

d. $16^{-3/4} = \dfrac{1}{16^{3/4}} = \dfrac{1}{\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

e. $-16^{-3/4} = \dfrac{1}{-16^{3/4}} = \dfrac{1}{-\left(\sqrt[4]{16}\right)^3}$

$= \dfrac{1}{-\left(2^3\right)} = \dfrac{1}{8}$

f. $\left(-16\right)^{-3/4} = \dfrac{1}{\left(-16\right)^{3/4}}$

$= \dfrac{1}{\left(\sqrt[4]{-16}\right)^3}$

is not a real number.

23. **a.** $25^{3/2} = \left(\sqrt{25}\right)^3 = 5^3 = 125$

b. $-25^{3/2} = -\left(\sqrt{25}\right)^3 = -\left(5^3\right) = -125$

c. $\left(-25\right)^{3/2} = \left(\sqrt{-25}\right)^3$ is not a real numbe

d. $25^{-3/2} = \dfrac{1}{25^{3/2}} = \dfrac{1}{\left(\sqrt{25}\right)^3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

e. $-25^{-3/2} = \dfrac{1}{-25^{3/2}} = \dfrac{1}{-\left(\sqrt{25}\right)^3} = \dfrac{1}{-\left(5^3\right)}$

$= -\dfrac{1}{125}$

f. $\left(-25\right)^{-3/2} = \dfrac{1}{\left(-25\right)^{3/2}}$

$= \dfrac{1}{\left(\sqrt{-25}\right)^3}$

is not a real number.

25. $64^{-3/2} = \dfrac{1}{64^{3/2}} = \dfrac{1}{\left(\sqrt{64}\right)^3} = \dfrac{1}{8^3} = \dfrac{1}{512}$

27. $243^{3/5} = \left(\sqrt[5]{243}\right)^3 = 3^3 = 27$

29. $-27^{-4/3} = \dfrac{1}{-27^{4/3}} = \dfrac{1}{-\left(\sqrt[3]{27}\right)^4}$

$= \dfrac{1}{-\left(3\right)^4}$

$= -\dfrac{1}{81}$

31. $\left(\dfrac{100}{9}\right)^{-3/2} = \dfrac{1}{\left(\dfrac{100}{9}\right)^{3/2}} = \dfrac{1}{\left(\sqrt{\dfrac{100}{9}}\right)^3}$

$= \dfrac{1}{\left(\dfrac{10}{3}\right)^3} = \dfrac{1}{\dfrac{1000}{27}}$

$= \dfrac{27}{1000}$

33. $(-4)^{-3/2} = \dfrac{1}{(-4)^{3/2}} = \dfrac{1}{\left(\sqrt{-4}\right)^3}$ is not a real number

35. $(-8)^{1/3} = \sqrt[3]{-8} = -2$

37. $-8^{1/3} = -\sqrt[3]{8} = -2$

39. $\dfrac{1}{36^{-1/2}} = 36^{1/2} = \sqrt{36} = 6$

41. $\dfrac{1}{1000^{-1/3}} = 1000^{1/3} = \sqrt[3]{1000} = 10$

43. $\left(\dfrac{1}{8}\right)^{2/3} + \left(\dfrac{1}{4}\right)^{1/2} = \left(\sqrt[3]{\dfrac{1}{8}}\right)^2 + \sqrt{\dfrac{1}{4}}$
$= \left(\dfrac{1}{2}\right)^2 + \dfrac{1}{2} = \dfrac{1}{4} + \dfrac{1}{2} = \dfrac{3}{4}$

45. $\left(\dfrac{1}{16}\right)^{-3/4} - \left(\dfrac{1}{49}\right)^{-1/2} = 16^{3/4} - 49^{1/2}$
$= \left(\sqrt[4]{16}\right)^3 - \sqrt{49} = (2)^3 - 7$
$= 8 - 7 = 1$

47. $\left(\dfrac{1}{4}\right)^{1/2} + \left(\dfrac{1}{64}\right)^{-1/3} = \left(\dfrac{1}{4}\right)^{1/2} + (64)^{1/3}$
$= \sqrt{\dfrac{1}{4}} + \sqrt[3]{64} = \dfrac{1}{2} + 4$
$= \dfrac{9}{2}$

49. $q^{2/3} = \sqrt[3]{q^2}$

51. $6y^{3/4} = 6\sqrt[4]{y^3}$

53. $x^{2/3}y^{1/3} = \left(x^2y\right)^{1/3} = \sqrt[3]{x^2y}$

55. $6r^{-2/5} = 6 \cdot \dfrac{1}{r^{2/5}} = \dfrac{6}{\sqrt[5]{r^2}}$

57. $\sqrt[3]{x} = x^{1/3}$

59. $10\sqrt{b} = 10b^{1/2}$

61. $\sqrt[3]{y^2} = y^{2/3}$

63. $\sqrt[4]{a^2b^3} = \left(a^2b^3\right)^{1/4}$

65. $x^{1/4}x^{-5/4} = x^{1/4+(-5/4)} = x^{-1} = \dfrac{1}{x}$

67. $\dfrac{p^{5/3}}{p^{2/3}} = p^{(5/3)-(2/3)} = p^1 = p$

69. $\left(y^{1/5}\right)^{10}=y^{(1/5)(10)}=y^2$

71. $6^{-1/5}6^{3/5}=6^{(-1/5)+(3/5)}=6^{2/5}$

73. $\dfrac{4t^{-1/3}}{t^{4/3}}=4t^{(-1/3)-(4/3)}=4t^{-5/3}=\dfrac{4}{t^{5/3}}$

75. $\left(a^{1/3}a^{1/4}\right)^{12}=\left(a^{1/3}\right)^{12}\left(a^{1/4}\right)^{12}=a^{12/3}a^{12/4}$

$=a^4a^3=a^{3+4}=a^7$

77. $\left(5a^2c^{-1/2}d^{1/2}\right)^2$

$=5^2\left(a^2\right)^2\left(c^{-1/2}\right)^2\left(d^{1/2}\right)^2$

$=5^2a^4c^{-2/2}d^{2/2}=25a^4c^{-1}d^1$

$=\dfrac{25a^4d}{c}$

79. $\left(\dfrac{x^{-2/3}}{y^{-3/4}}\right)^{12}=\dfrac{\left(x^{-2/3}\right)^{12}}{\left(y^{-3/4}\right)^{12}}=\dfrac{x^{-24/3}}{y^{-36/4}}$

$=\dfrac{x^{-8}}{y^{-9}}=\dfrac{y^9}{x^8}$

81. $\left(\dfrac{16w^{-2}z}{2wz^{-8}}\right)^{1/3}=\left(8w^{-2-1}z^{1-(-8)}\right)^{1/3}$

$=\left(8w^{-3}z^9\right)^{1/3}$

$=8^{1/3}\left(w^{-3}\right)^{1/3}\left(z^9\right)^{1/3}$

$=\sqrt[3]{8}w^{-3/3}z^{9/3}$

$=2w^{-1}z^3$

$=\dfrac{2z^3}{w}$

83. $\left(25x^2y^4z^6\right)^{1/2}=25^{1/2}\left(x^2\right)^{1/2}\left(y^4\right)^{1/2}\left(z^6\right)^{1/2}$

$=\sqrt{25}x^{2/2}y^{4/2}z^{6/2}$

$=5xy^2z^3$

85. $\left(x^2y^{-1/3}\right)^6\left(x^{1/2}yz^{2/3}\right)^2$

$=\left(x^2\right)^6\left(y^{-1/3}\right)^6\left(x^{1/2}\right)^2y^2\left(z^{2/3}\right)^2$

$=x^{12}y^{-2}x^1y^2z^{4/3}=x^{12+1}y^{-2+2}z^{4/3}$

$=x^{13}y^0z^{4/3}$

$=x^{13}z^{4/3}$

87. $\left(\dfrac{x^{3m}y^{2m}}{z^{5m}}\right)^{1/m}=\dfrac{\left(x^{3m}\right)^{1/m}\left(y^{2m}\right)^{1/m}}{\left(z^{5m}\right)^{1/m}}$

$=\dfrac{x^{3m/m}y^{2m/m}}{z^{5m/m}}=\dfrac{x^3y^2}{z^5}$

89. **a.** $r=\left(\frac{A}{P}\right)^{1/t}-1$

$r=\left(\frac{16{,}802}{10{,}000}\right)^{1/5}-1\approx 0.109=10.9\%$

b. $r=\left(\frac{18{,}000}{10{,}000}\right)^{1/7}-1\approx 0.088=8.8\%$

c. The account in part (a).

91. $r=\left(\frac{3V}{4\pi}\right)^{1/3}$

$r=\left(\frac{3(85)}{4\pi}\right)^{1/3}=\sqrt[3]{\frac{3(85)}{4\pi}}\approx 2.7\text{ in}$

93. $\sqrt[6]{y^3}=y^{3/6}$
$=y^{1/2}$
$=\sqrt{y}$

95. $\sqrt[12]{z^3}=z^{3/12}=z^{1/4}=\sqrt[4]{z}$

97. $\sqrt[9]{x^6}=x^{6/9}=x^{2/3}=\sqrt[3]{x^2}$

99. $\sqrt[6]{x^3y^6}=\left(x^3y^6\right)^{1/6}=x^{3/6}y^{6/6}$
$=x^{1/2}y=y\sqrt{x}$

101. $\sqrt{16x^8y^6}=16^{1/2}x^{8/2}y^{6/2}$
$=4x^4y^3$

103. $\sqrt[3]{8x^3y^2z}=2x\sqrt[3]{y^2z}$

105. $\sqrt{\sqrt[3]{x}}=\sqrt{x^{1/3}}=\left(x^{1/3}\right)^{1/2}=x^{(1/3)(1/2)}=x^{1/6}=\sqrt[6]{x}$

107. $\sqrt[5]{\sqrt[3]{w}}=\sqrt[5]{w^{1/3}}=\left(w^{1/3}\right)^{1/5}=w^{(1/3)(1/5)}=w^{1/15}=\sqrt[15]{w}$

109. $9^{1/2}=3$

111. $50^{-1/4}\approx 0.3761$

113. $\sqrt[3]{5^2}\approx 2.9240$

115. $\sqrt{10^3}\approx 31.6228$

Section 6.3 Practice Exercises

1. **a.** $\sqrt[n]{a}$; $\sqrt[n]{b}$

b. The exponent within the radicand is greater than the index

d. 3

e. t^{12}

c. is not

f. No. $\sqrt{2}$ is an irrational number and the decimal form is a nonterminating, nonrepeating decimal

3. $$\left(\frac{p^4}{q^{-6}}\right)^{-1/2}\left(p^3q^{-2}\right)=\frac{\left(p^4\right)^{-1/2}}{\left(q^{-6}\right)^{-1/2}}\left(p^3q^{-2}\right)=\frac{p^{-4/2}}{q^{6/2}}\left(p^3q^{-2}\right)$$
$$=\frac{p^{-2}}{q^3}\cdot p^3q^{-2}=p^{-2+3}q^{-2-3}$$
$$=pq^{-5}=\frac{p}{q^5}$$

5. $x^{4/7}=\sqrt[7]{x^4}$

7. $\sqrt{y^9}=y^{9/2}$

9. $\sqrt{x^5}=\sqrt{x^4\cdot x}=\sqrt{x^4}\cdot\sqrt{x}=x^2\sqrt{x}$

11. $\sqrt[3]{q^7}=\sqrt[3]{q^6\cdot q}=\sqrt[3]{q^6}\cdot\sqrt[3]{q}=q^2\sqrt[3]{q}$

13. $$\sqrt{a^5b^4}=\sqrt{a^4b^4\cdot a}=\sqrt{a^4b^4}\cdot\sqrt{a}$$
$$=a^2b^2\sqrt{a}$$

15. $$-\sqrt[4]{x^8y^{13}}=-\sqrt[4]{x^8y^{12}\cdot y}=-\sqrt[4]{x^8y^{12}}\cdot\sqrt[4]{y}$$
$$=-x^2y^3\sqrt[4]{y}$$

17. $\sqrt{28}=\sqrt{4\cdot 7}=\sqrt{4}\cdot\sqrt{7}=2\sqrt{7}$

19. $\sqrt{20}=\sqrt{4\cdot 5}=\sqrt{4}\cdot\sqrt{5}=2\sqrt{5}$

21. $$5\sqrt{18}=5\sqrt{9\cdot 2}=5\sqrt{9}\cdot\sqrt{2}=5\cdot 3\sqrt{2}$$
$$=15\sqrt{2}$$

23. $\sqrt[3]{54}=\sqrt[3]{27\cdot 2}=\sqrt[3]{27}\cdot\sqrt[3]{2}=3\sqrt[3]{2}$

25. $$\sqrt{25ab^3}=\sqrt{25b^2\cdot ab}$$
$$=\sqrt{25b^2}\cdot\sqrt{ab}=5b\sqrt{ab}$$

27. $$\sqrt[3]{40x^7}=\sqrt[3]{8\cdot 5\cdot x^6\cdot x}=\sqrt[3]{8x^6}\cdot\sqrt[3]{5x}$$
$$=2x^2\sqrt[3]{5x}$$

29. $$\sqrt[3]{-16x^6yz^3}=\sqrt[3]{-8x^6z^3\cdot 2y}$$
$$=\sqrt[3]{-8x^6z^3}\cdot\sqrt[3]{2y}$$
$$=-2x^2z\sqrt[3]{2y}$$

31. $$\sqrt[4]{80w^4z^7}=\sqrt[4]{16w^4z^4\cdot 5z^3}$$
$$=\sqrt[4]{16w^4z^4}\cdot\sqrt[4]{5z^3}$$
$$=2wz\sqrt[4]{5z^3}$$

33. $\sqrt{\dfrac{x^3}{x}}=\sqrt{x^2}=x$

35. $\sqrt{\dfrac{p^7}{p^3}}=\sqrt{p^4}=p^2$

37. $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$

39. $\sqrt[3]{\frac{3}{24}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

41. $\frac{5\sqrt[3]{16}}{6} = \frac{5\sqrt[3]{8\cdot 2}}{6} = \frac{5\sqrt[3]{8}\cdot\sqrt[3]{2}}{6}$
$= \frac{5\cdot 2\sqrt[3]{2}}{6} = \frac{5\sqrt[3]{2}}{3}$

43. $\frac{5\sqrt[3]{72}}{12} = \frac{5\sqrt[3]{8\cdot 9}}{12} = \frac{5\sqrt[3]{8}\cdot\sqrt[3]{9}}{12}$
$= \frac{5\cdot 2\sqrt[3]{9}}{12} = \frac{5\sqrt[3]{9}}{6}$

45. $\sqrt{80} = \sqrt{16\cdot 5} = \sqrt{16}\cdot\sqrt{5}$
$= 4\sqrt{5}$

47. $-6\sqrt{75} = -6\sqrt{25\cdot 3} = -6\sqrt{25}\cdot\sqrt{3}$
$= -6\cdot 5\sqrt{3} = -30\sqrt{3}$

49. $\sqrt{25x^4y^3} = \sqrt{25x^4y^2\cdot y}$
$= \sqrt{25x^4y^2}\cdot\sqrt{y}$
$= 5x^2y\sqrt{y}$

51. $\sqrt[3]{27x^2y^3z^4} = \sqrt[3]{27y^3z^3\cdot x^2z}$
$= \sqrt[3]{27y^3z^3}\cdot\sqrt[3]{x^2z}$
$= 3yz\sqrt[3]{x^2z}$

53. $\sqrt{\frac{12w^5}{3w}} = \sqrt{4w^4} = 2w^2$

55. $\sqrt{\frac{3y^3}{300y^{15}}} = \sqrt{\frac{1}{100y^{12}}} = \frac{1}{10y^6}$

57. $\sqrt[3]{\frac{16a^2b}{2a^2b^4}} = \sqrt[3]{\frac{8}{b^3}}$
$= \frac{2}{b}$

59. $\sqrt{2^3a^{14}b^8c^{31}d^{22}} = \sqrt{2^2a^{14}b^8c^{30}d^{22}\cdot 2c}$
$= \sqrt{2^2a^{14}b^8c^{30}d^{22}}\cdot\sqrt{2c}$
$= 2a^7b^4c^{15}d^{11}\sqrt{2c}$

61. $\sqrt[3]{54a^6b^4} = \sqrt[3]{27a^6b^3\cdot 2b}$
$= \sqrt[3]{27a^6b^3}\cdot\sqrt[3]{2b}$
$= 3a^2b\sqrt[3]{2b}$

63. $-5a\sqrt{12a^3b^4c} = -5a\sqrt{4\cdot 3\cdot a^2\cdot a\cdot b^4\cdot c}$
$= -5a\cdot 2ab^2\sqrt{3ac}$
$= -10a^2b^2\sqrt{3ac}$

65. $\sqrt[4]{7x^5y} = \sqrt[4]{x^4\cdot 7xy}$
$= \sqrt[4]{x^4}\cdot\sqrt[4]{7xy}$
$= x\sqrt[4]{7xy}$

67. $\sqrt{54a^4b^2} = \sqrt{6\cdot 9a^4b^2}$
$= 3a^2b\sqrt{6}$

69. $\frac{2\sqrt{27}}{3}=\frac{2\sqrt{9\cdot 3}}{3}=\frac{2\cdot 3\sqrt{3}}{3}=2\sqrt{3}$

71. $\frac{3\sqrt{125}}{20}=\frac{3\sqrt{25\cdot 5}}{20}=\frac{3\cdot 5\sqrt{5}}{20}=\frac{3\sqrt{5}}{4}$

73. $\frac{1}{\sqrt[3]{w^6}}=\frac{1}{w^2}$

75. $\sqrt{k^3}=\sqrt{k^2\cdot k}=k\sqrt{k}$

77.
$$a^2+b^2=c^2$$
$$8^2+10^2=c^2$$
$$64+100=c^2$$
$$164=c^2$$
$$c=\sqrt{164}$$
$$=\sqrt{4\cdot 41}=2\sqrt{41}\text{ ft}$$

79.
$$a^2+b^2=c^2$$
$$a^2+12^2=18^2$$
$$a^2+144=324$$
$$a^2=180$$
$$a=\sqrt{180}$$
$$=\sqrt{36\cdot 5}=6\sqrt{5}\text{ m}$$

81.
$$a^2+b^2=c^2$$
$$90^2+90^2=c^2$$
$$8100+8100=c^2$$
$$16200=c^2$$
$$c=\sqrt{16200}$$
$$=\sqrt{8100\cdot 2}$$
$$=90\sqrt{2}\text{ ft}\approx 127.3\text{ ft}$$
The distance is $90\sqrt{2}$ ft or approximately 127.3 ft.

83. Let b = the distance from B to C
$$40^2+b^2=50^2$$
$$1600+b^2=2500$$
$$b^2=900$$
$$b=\sqrt{900}=30\text{ mi}$$
The distance along the four lane highway is 40 + 30 = 70 mi. The time from A to C via B is
$$t=\frac{70}{55}=\frac{14}{11}\approx 1.27\text{ hr}$$
The route from A to C via B is the faster.

Section 6.4 Practice Exercises

1. **a.** index; radicand

b. $2\sqrt{3x}$

c. ca **c.** cannot; can

d. $4\sqrt{2}$

3.
$$-\sqrt[4]{x^7y^4}=-\sqrt[4]{x^4y^4\cdot x^3}$$
$$=-\sqrt[4]{x^4y^4}\cdot\sqrt[4]{x^3}=-xy\sqrt[4]{x^3}$$

5. $\frac{\sqrt[3]{7b^8}}{\sqrt[3]{56b^2}}=\sqrt[3]{\frac{7b^8}{56b^2}}=\sqrt[3]{\frac{b^6}{8}}=\frac{b^2}{2}$

7. $\sqrt[4]{x^3y} = \left(x^3y\right)^{1/4} = \left(x^3\right)^{1/4} y^{1/4}$
$= x^{3/4}y^{1/4}$

9. $y^{2/3}y^{1/4} = y^{(2/3)+(1/4)} = y^{11/12}$

11. **a.** $\sqrt{2}$ and $\sqrt[3]{2}$ are not like radicals. The indices are different.

b. $\sqrt{2}$ and $3\sqrt{2}$ are like radicals.

c. $\sqrt{2}$ and $\sqrt{5}$ are not like radicals. The radicands are different.

13. **a.** $7\sqrt{5}+4\sqrt{5}$ and $7x+4x$
Both expressions can be simplified by using the distributive property.

b. $-2\sqrt{6}-9\sqrt{3}$ and $-2x-9y$
Neither expression can be simplified because they do not contain like radicals or like terms.

15. $3\sqrt{5}+6\sqrt{5} = (3+6)\sqrt{5}$
$= 9\sqrt{5}$

17. $3\sqrt[3]{tw}-2\sqrt[3]{tw}+\sqrt[3]{tw} = (3-2+1)\sqrt[3]{tw}$
$= 2\sqrt[3]{tw}$

19. $6\sqrt{10}-\sqrt{10} = (6-1)\sqrt{10} = 5\sqrt{10}$

21. $\sqrt[4]{3}+7\sqrt[4]{3}-\sqrt[4]{14} = (1+7)\sqrt[4]{3}-\sqrt[4]{14}$
$= 8\sqrt[4]{3}-\sqrt[4]{14}$

23. $8\sqrt{x}+2\sqrt{y}-6\sqrt{x} = (8-6)\sqrt{x}+2\sqrt{y}$
$= 2\sqrt{x}+2\sqrt{y}$

25. $\sqrt[3]{ab}+a\sqrt[3]{b}$ cannot be simplified further.

27. $\sqrt{2t}+\sqrt[3]{2t}$ cannot be simplified further.

29. $\frac{5}{6}z\sqrt[3]{6}+\frac{7}{9}z\sqrt[3]{6} = \left(\frac{5}{6}+\frac{7}{9}\right)z\sqrt[3]{6}$
$= \left(\frac{15}{18}+\frac{14}{18}\right)z\sqrt[3]{6}$
$= \frac{29}{18}z\sqrt[3]{6}$

31. $0.81x\sqrt{y}-0.11x\sqrt{y} = (0.81-0.11)x\sqrt{y}$
$= 0.70x\sqrt{y}$

33. Simplify each radical. Then add like radicals.

35. $\sqrt{36}+\sqrt{81}$
$= 6+9$
$= 15$

$$\begin{aligned}3\sqrt{2}+7\sqrt{50} &= 3\sqrt{2}+7\sqrt{25\cdot 2}\\ &= 3\sqrt{2}+7\cdot 5\sqrt{2}\\ &= (3+35)\sqrt{2} = 38\sqrt{2}\end{aligned}$$

37.
$$\begin{aligned}2\sqrt{12}+\sqrt{48} &= 2\sqrt{4\cdot 3}+\sqrt{16\cdot 3}\\ &= 2\cdot 2\sqrt{3}+4\sqrt{3}\\ &= 4\sqrt{3}+4\sqrt{3}\\ &= (4+4)\sqrt{3}\\ &= 8\sqrt{3}\end{aligned}$$

39.
$$\begin{aligned}4\sqrt{7}+\sqrt{63}-2\sqrt{28} &= 4\sqrt{7}+\sqrt{9\cdot 7}-2\sqrt{4\cdot 7}\\ &= 4\sqrt{7}+3\sqrt{7}-2\cdot 2\sqrt{7}\\ &= 4\sqrt{7}+3\sqrt{7}-4\sqrt{7}\\ &= (4+3-4)\sqrt{7}\\ &= 3\sqrt{7}\end{aligned}$$

41.
$$\begin{aligned}&5\sqrt{18}+\sqrt{27}-4\sqrt{50}\\ &= 5\sqrt{9\cdot 2}+\sqrt{9\cdot 3}-4\sqrt{25\cdot 2}\\ &= 5\cdot 3\sqrt{2}+3\sqrt{3}-4\cdot 5\sqrt{2}\\ &= 15\sqrt{2}+3\sqrt{3}-20\sqrt{2}\\ &= (15-20)\sqrt{2}+3\sqrt{3}\\ &= -5\sqrt{2}+3\sqrt{3}\end{aligned}$$

43.
$$\begin{aligned}\sqrt[3]{81}-\sqrt[3]{24} &= \sqrt[3]{27\cdot 3}-\sqrt[3]{8\cdot 3}\\ &= 3\sqrt[3]{3}-2\sqrt[3]{3}\\ &= (3-2)\sqrt[3]{3}\\ &= \sqrt[3]{3}\end{aligned}$$

45.
$$\begin{aligned}&3\sqrt{2a}-\sqrt{8a}-\sqrt{72a}\\ &= 3\sqrt{2a}-\sqrt{4\cdot 2a}-\sqrt{36\cdot 2a}\\ &= 3\sqrt{2a}-2\sqrt{2a}-6\sqrt{2a}\\ &= (3-2-6)\sqrt{2a}\\ &= -5\sqrt{2a}\end{aligned}$$

47.
$$\begin{aligned}&2s^2\sqrt[3]{s^2t^6}+3t^2\sqrt[3]{8s^8}\\ &= 2s^2\sqrt[3]{t^6\cdot s^2}+3t^2\sqrt[3]{8s^6\cdot s^2}\\ &= 2s^2\cdot t^2\sqrt[3]{s^2}+3t^2\cdot 2s^2\sqrt[3]{s^2}\\ &= 2s^2t^2\sqrt[3]{s^2}+6s^2t^2\sqrt[3]{s^2}\\ &= \left(2s^2t^2+6s^2t^2\right)\sqrt[3]{s^2}\\ &= 8s^2t^2\sqrt[3]{s^2}\end{aligned}$$

49.
$$\begin{aligned}7\sqrt[3]{x^4}-x\sqrt[3]{x} &= 7\sqrt[3]{x^3\cdot x}-x\sqrt[3]{x}\\ &= 7x\sqrt[3]{x}-x\sqrt[3]{x}\\ &= (7x-x)\sqrt[3]{x}\\ &= 6x\sqrt[3]{x}\end{aligned}$$

51.
$$\begin{aligned}&5p\sqrt{20p^2}+p^2\sqrt{80}\\ &= 5p\sqrt{4p^2\cdot 5}+p^2\sqrt{16\cdot 5}\\ &= 5p\cdot 2p\sqrt{5}+p^2\cdot 4\sqrt{5}\\ &= 10p^2\sqrt{5}+4p^2\sqrt{5}\\ &= \left(10p^2+4p^2\right)\sqrt{5}\\ &= 14p^2\sqrt{5}\end{aligned}$$

53. $\sqrt[3]{a^2b}-\sqrt[3]{8a^2b}=\sqrt[3]{a^2b}-2\sqrt[3]{a^2b}$
$=(1-2)\sqrt[3]{a^2b}$
$=-\sqrt[3]{a^2b}$

55. $5x\sqrt{x}+6\sqrt{x}=(5x+6)\sqrt{x}$

57. $\sqrt{50x^2}-3\sqrt{8}=\sqrt{25x^2\cdot 2}-3\sqrt{4\cdot 2}$
$=5x\sqrt{2}-3\cdot 2\sqrt{2}$
$=5x\sqrt{2}-6\sqrt{2}$
$=(5x-6)\sqrt{2}$

59. $11\sqrt[3]{54cd^3}-2\sqrt[3]{2cd^3}+d\sqrt[3]{16c}$
$=11\sqrt[3]{27\cdot 2cd^3}-2\sqrt[3]{2cd^3}+d\sqrt[3]{8\cdot 2c}$
$=11\cdot 3d\sqrt[3]{2c}-2\cdot d\sqrt[3]{2c}+d\cdot 2\sqrt[3]{2c}$
$=33d\sqrt[3]{2c}-2d\sqrt[3]{2c}+2d\sqrt[3]{2c}$
$=(33-2+2)d\sqrt[3]{2c}=33d\sqrt[3]{2c}$

61. $\frac{3}{2}ab\sqrt{24a^3}+\frac{4}{3}\sqrt{54a^5b^2}-a^2b\sqrt{150a}=\frac{3}{2}ab\sqrt{4a^2\cdot 6a}+\frac{4}{3}\sqrt{9a^4b^2\cdot 6a}-a^2b\sqrt{25\cdot 6a}$
$=\frac{3}{2}ab\cdot 2a\sqrt{6a}+\frac{4}{3}\cdot 3a^2b\sqrt{6a}-a^2b\cdot 5\sqrt{6a}=3a^2b\sqrt{6a}+4a^2b\sqrt{6a}-5a^2b\sqrt{6a}$
$=(3+4-5)a^2b\sqrt{6a}$
$=2a^2b\sqrt{6a}$

63. $x\sqrt[3]{16}-2\sqrt[3]{27x}+\sqrt[3]{54x^3}$
$=x\sqrt[3]{8\cdot 2}-2\sqrt[3]{27x}+\sqrt[3]{27x^3\cdot 2}$
$=x\cdot 2\sqrt[3]{2}-2\cdot 3\sqrt[3]{x}+3x\sqrt[3]{2}$
$=2x\sqrt[3]{2}-6\sqrt[3]{x}+3x\sqrt[3]{2}$
$=(2+3)x\sqrt[3]{2}-6\sqrt[3]{x}$
$=5x\sqrt[3]{2}-6\sqrt[3]{x}$

65. $\sqrt{x}+\sqrt{y}=\sqrt{x+y}$ False.
$\sqrt{9}+\sqrt{16}\neq\sqrt{9+16}$
$3+4\neq\sqrt{25}$
$7\neq 5$

67. $5\sqrt[3]{x}+2\sqrt[3]{x}=7\sqrt[3]{x}$ True.

69. $\sqrt{y}+\sqrt{y}=\sqrt{2y}$ False.
$\sqrt{y}+\sqrt{y}=2\sqrt{y}\neq\sqrt{2y}$

71. $2w\sqrt{5}+4w\sqrt{5}=6w^2\sqrt{5}$
False: $2w\sqrt{5}+4w\sqrt{5}=6w\sqrt{5}\neq 6w^2\sqrt{5}$

73. $\sqrt{48}+\sqrt{12}=\sqrt{16\cdot 3}+\sqrt{4\cdot 3}$
$=4\sqrt{3}+2\sqrt{3}$
$=(4+2)\sqrt{3}=6\sqrt{3}$

75. $5\sqrt[3]{x^6} - x^2 = 5x^2 - x^2$
$= 4x^2$

77. $\sqrt{18} - 5^2$
The difference of the principal square root of 18 and the square of 5.

79. $\sqrt[4]{x} + y^3$
The sum of the principal fourth root of *x* and the cube of *y*.

81.
$$\begin{aligned} P &= 2\sqrt{6} + 2\sqrt{24} + \sqrt{54} \\ &= 2\sqrt{6} + 2\sqrt{4\cdot 6} + \sqrt{9\cdot 6} \\ &= 2\sqrt{6} + 2\cdot 2\sqrt{6} + 3\sqrt{6} \\ &= 2\sqrt{6} + 4\sqrt{6} + 3\sqrt{6} \\ &= (2+4+3)\sqrt{6} \\ &= 9\sqrt{6} \text{ cm} \\ &\approx 22.0 \text{ cm} \end{aligned}$$

83.
$$\begin{aligned} 2\sqrt{50} + 2x &= 14\sqrt{2} \\ 2\sqrt{25\cdot 2} + 2x &= 14\sqrt{2} \\ 2\cdot 5\sqrt{2} + 2x &= 14\sqrt{2} \\ 10\sqrt{2} + 2x &= 14\sqrt{2} \\ 2x &= 4\sqrt{2} \\ x &= 2\sqrt{2} \text{ ft} \end{aligned}$$

85. **a.** Side from (0, 6) to (6, 9):
$c^2 = 3^2 + 6^2 = 9 + 36 = 45$
$c = \sqrt{45} = \sqrt{9\cdot 5} = 3\sqrt{5}$
Side from (6, 9) to (7, 7):
$c^2 = 1^2 + 2^2 = 1 + 4 = 5$
$c = \sqrt{5}$
Side from (7, 7) to (4, 1):
$c^2 = 3^2 + 6^2 = 9 + 36 = 45$
$c = \sqrt{45} = \sqrt{9\cdot 5} = 3\sqrt{5}$
Side from (4, 1) to (2, 2):
$c^2 = 1^2 + 2^2 = 1 + 4 = 5$
$c = \sqrt{5}$
Side from (2, 2) to (0, 6):
$c^2 = 2^2 + 4^2 = 4 + 16 = 20$
$c = \sqrt{20} = \sqrt{4\cdot 5} = 2\sqrt{5}$
$P = 3\sqrt{5} + \sqrt{5} + 3\sqrt{5} + \sqrt{5} + 2\sqrt{5}$
$= (3+1+3+1+2)\sqrt{5} = 10\sqrt{5}$ yd

b. $10\sqrt{5} \approx 22.36$ yd

c. $C = 1.49(22.36)(3) + 0.06(1.49(22.36)(3))$
$= 99.95 + 6.00 = \$105.95$

Section 6.5 Practice Exercises

1. **a.** $\sqrt[n]{ab}$
b. x
c. a
d. conjugates
e. $m - n$
f. $c + 8\sqrt{c} + 16$

3 $-\sqrt{20a^2b^3c} = -\sqrt{4a^2b^2 \cdot 5bc}$
$= -2ab\sqrt{5bc}$

5. $x^{1/3}y^{1/4}x^{-1/6}y^{1/3} = x^{(1/3)+(-1/6)}y^{(1/4)+(1/3)}$
$= x^{1/6}y^{7/12}$

7. $-2\sqrt[3]{7} + 4\sqrt[3]{7} = (-2+4)\sqrt[3]{7}$
$= 2\sqrt[3]{7}$

9. $\sqrt[3]{7} \cdot \sqrt[3]{3} = \sqrt[3]{21}$

11. $\sqrt{2} \cdot \sqrt{10} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

13. $\sqrt[4]{16} \cdot \sqrt[4]{64} = \sqrt[4]{2^4} \cdot \sqrt[4]{2^6} = 2\sqrt[4]{2^4 \cdot 2^2}$
$= 2 \cdot 2\sqrt[4]{2^2} = 4\sqrt[4]{4}$

15. $\left(4\sqrt[3]{4}\right)\left(2\sqrt[3]{5}\right) = (4 \cdot 2)\left(\sqrt[3]{4} \cdot \sqrt[3]{5}\right)$
$= 8\sqrt[3]{20}$

17. $\left(8a\sqrt{b}\right)\left(-3\sqrt{ab}\right) = (8a)(-3)\left(\sqrt{b} \cdot \sqrt{ab}\right)$
$= -24a\sqrt{ab^2} = -24ab\sqrt{a}$

19. $\sqrt{30} \cdot \sqrt{12} = \sqrt{360} = \sqrt{36 \cdot 10}$
$= 6\sqrt{10}$

21. $\sqrt{6x}\sqrt{12x} = \sqrt{72x^2}$
$= \sqrt{36x^2 \cdot 2} = 6x\sqrt{2}$

23. $\sqrt{5a^3b^2}\sqrt{20a^3b^3} = \sqrt{100a^6b^5}$
$= \sqrt{100a^6b^4 \cdot b}$
$= 10a^3b^2\sqrt{b}$

25. $\left(4\sqrt{3xy^3}\right)\left(-2\sqrt{6x^3y^2}\right) = -8\sqrt{18x^4y^5}$
$= -8\sqrt{9x^4y^4 \cdot 2y}$
$= -8 \cdot 3x^2y^2\sqrt{2y}$
$= -24x^2y^2\sqrt{2y}$

27. $\left(\sqrt[3]{4a^2b}\right)\left(\sqrt[3]{2ab^3}\right)\left(\sqrt[3]{54a^2b}\right)$
$= \sqrt[3]{8a^3b^3 \cdot b} \cdot \sqrt[3]{27 \cdot 2a^2b}$
$= 2ab\sqrt[3]{b} \cdot 3\sqrt[3]{2a^2b}$
$= 6ab\sqrt[3]{2a^2b^2}$

29. $\sqrt{3}\left(4\sqrt{3} - 6\right) = \sqrt{3} \cdot 4\sqrt{3} - \sqrt{3} \cdot (6)$
$= 4\sqrt{9} - 6\sqrt{3}$
$= 4 \cdot 3 - 6\sqrt{3} = 12 - 6\sqrt{3}$

31. $\sqrt{2}\left(\sqrt{6} - \sqrt{3}\right) = \sqrt{2} \cdot \sqrt{6} - \sqrt{2} \cdot \sqrt{3}$
$= \sqrt{12} - \sqrt{6} = \sqrt{4 \cdot 3} - \sqrt{6}$
$= 2\sqrt{3} - \sqrt{6}$

33. $-\frac{1}{3}\sqrt{x}\left(6\sqrt{x} + 7\right) = -\frac{1}{3}\sqrt{x} \cdot 6\sqrt{x} - \frac{1}{3}\sqrt{x} \cdot (7)$
$= -2\sqrt{x^2} - \frac{7}{3}\sqrt{x}$
$= -2x - \frac{7}{3}\sqrt{x}$

35. $\left(\sqrt{3}+2\sqrt{10}\right)\left(4\sqrt{3}-\sqrt{10}\right)=\sqrt{3}\cdot 4\sqrt{3}-\sqrt{3}\cdot\sqrt{10}+2\sqrt{10}\cdot 4\sqrt{3}-2\sqrt{10}\cdot\sqrt{10}$
$=4\sqrt{9}-\sqrt{30}+8\sqrt{30}-2\sqrt{100}=4\cdot 3+7\sqrt{30}-2\cdot 10=12+7\sqrt{30}-20=-8+7\sqrt{30}$

37. $\left(\sqrt{x}+4\right)\left(\sqrt{x}-9\right)=\sqrt{x}\cdot\sqrt{x}-\sqrt{x}\cdot 9+4\cdot\sqrt{x}-4\cdot 9=\sqrt{x^2}-9\sqrt{x}+4\sqrt{x}-36=x-5\sqrt{x}-36$

39. $\left(\sqrt[3]{y}+2\right)\left(\sqrt[3]{y}-3\right)=\sqrt[3]{y}\cdot\sqrt[3]{y}-\sqrt[3]{y}\cdot 3+2\cdot\sqrt[3]{y}-2\cdot 3=\sqrt[3]{y^2}-3\sqrt[3]{y}+2\sqrt[3]{y}-6=\sqrt[3]{y^2}-\sqrt[3]{y}-6$

41. $\left(\sqrt{a}-3\sqrt{b}\right)\left(9\sqrt{a}-\sqrt{b}\right)=\sqrt{a}\cdot 9\sqrt{a}-\sqrt{a}\cdot\sqrt{b}-3\sqrt{b}\cdot 9\sqrt{a}+3\sqrt{b}\cdot\sqrt{b}$
$=9\sqrt{a^2}-\sqrt{ab}-27\sqrt{ab}+3\sqrt{b^2}=9a-28\sqrt{ab}+3b$

43. $\left(\sqrt{p}+2\sqrt{q}\right)\left(8+3\sqrt{p}-\sqrt{q}\right)=\sqrt{p}\cdot 8+\sqrt{p}\cdot 3\sqrt{p}-\sqrt{p}\cdot\sqrt{q}+2\sqrt{q}\cdot 8+2\sqrt{q}\cdot 3\sqrt{p}-2\sqrt{q}\cdot\sqrt{q}$
$=8\sqrt{p}+3\sqrt{p^2}-\sqrt{pq}+16\sqrt{q}+6\sqrt{pq}-2\sqrt{q^2}=8\sqrt{p}+3p+5\sqrt{pq}+16\sqrt{q}-2q$

45. $\left(\sqrt{15}\right)^2=15$

47. $\left(\sqrt{3y}\right)^2=3y$

49. $\left(\sqrt[3]{6}\right)^3=6$

51. $\sqrt{709}\cdot\sqrt{709}=\left(\sqrt{709}\right)^2=709$

53. **(a)** $(x+y)(x-y)=x^2-y^2$
(b) $(x+5)(x-5)=x^2-5^2$
$=x^2-25$

55. $\left(\sqrt{13}+4\right)^2=\left(\sqrt{13}\right)^2+2\left(\sqrt{13}\right)(4)+4^2$
$=13+8\sqrt{13}+16$
$=29+8\sqrt{13}$

57. $\left(\sqrt{p}-\sqrt{7}\right)^2=\left(\sqrt{p}\right)^2-2\left(\sqrt{p}\right)\left(\sqrt{7}\right)+\left(\sqrt{7}\right)^2$
$=p-2\sqrt{7p}+7$

59. $\left(\sqrt{2a}-3\sqrt{b}\right)^2$
$=\left(\sqrt{2a}\right)^2-2\left(\sqrt{2a}\right)\left(3\sqrt{b}\right)+\left(3\sqrt{b}\right)^2$
$=2a-6\sqrt{2ab}+9b$

61. $\left(\sqrt{3}+x\right)\left(\sqrt{3}-x\right)=\left(\sqrt{3}\right)^2-x^2=3-x^2$

63. $\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)=\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2$
$=6-2=4$

65. $\left(\frac{2}{3}\sqrt{x}+\frac{1}{2}\sqrt{y}\right)\left(\frac{2}{3}\sqrt{x}-\frac{1}{2}\sqrt{y}\right)$

$$=\left(\frac{2}{3}\sqrt{x}\right)^2-\left(\frac{1}{2}\sqrt{y}\right)^2=\frac{4}{9}x-\frac{1}{4}y$$

67. a. $\left(\sqrt{3}+\sqrt{x}\right)\left(\sqrt{3}-\sqrt{x}\right)=\sqrt{3}^2-\sqrt{x}^2$

$$=3-x$$

b. $\left(\sqrt{3}+\sqrt{x}\right)\left(\sqrt{3}+\sqrt{x}\right)$

$$=\sqrt{3}^2+2\sqrt{3}\sqrt{x}+\sqrt{x}^2$$
$$=3+2\sqrt{3x}+x$$

c. $\left(\sqrt{3}-\sqrt{x}\right)\left(\sqrt{3}-\sqrt{x}\right)$

$$=\sqrt{3}^2-2\sqrt{3}\sqrt{x}+\sqrt{x}^2$$
$$=3-2\sqrt{3x}+x$$

69. $\sqrt{3}\cdot\sqrt{2}=\sqrt{6}$ True.

71. $\left(x-\sqrt{5}\right)^2=x-5$ False.

$$\left(x-\sqrt{5}\right)^2=x^2-2x\sqrt{5}+\left(\sqrt{5}\right)^2$$
$$=x^2-2x\sqrt{5}+5$$

73. $5\left(3\sqrt{4x}\right)=15\sqrt{20x}$ False.

5 is multiplied by 3 only.

75. $\frac{3\sqrt{x}}{3}=\sqrt{x}$ True.

77. $\left(-\sqrt{6x}\right)^2=6x$

79. $\left(\sqrt{3x+1}\right)^2=3x+1$

81. $\left(\sqrt{x+3}-4\right)^2$

$$=\left(\sqrt{x+3}\right)^2-2\left(\sqrt{x+3}\right)(4)+(4)^2$$
$$=x+3-8\sqrt{x+3}+16$$
$$=x+19-8\sqrt{x+3}$$

83. $\left(\sqrt{2t-3}+5\right)^2$

$$=\left(\sqrt{2t-3}\right)^2+2\left(\sqrt{2t-3}\right)(5)+(5)^2$$
$$=2t-3+10\sqrt{2t-3}+25$$
$$=2t+22+10\sqrt{2t-3}$$

85. $A=\sqrt{40}\cdot 3\sqrt{2}=3\sqrt{80}=3\sqrt{16\cdot 5}$

$$=3\cdot 4\sqrt{5}=12\sqrt{5}\text{ ft}^2$$

87. $A=\frac{1}{2}\cdot 3\sqrt{5}\cdot 6\sqrt{12}=9\sqrt{60}=9\sqrt{4\cdot 15}$

$$=9\cdot 2\sqrt{15}=18\sqrt{15}\text{ in.}^2$$

89. $\sqrt{x} \cdot \sqrt[4]{x} = x^{1/2} \cdot x^{1/4} = x^{(1/2)+(1/4)}$
$= x^{3/4} = \sqrt[4]{x^3}$

91. $\sqrt[5]{2z} \cdot \sqrt[3]{2z} = (2z)^{1/5} \cdot (2z)^{1/3}$
$= (2z)^{(1/5)+(1/3)}$
$= (2z)^{8/15} = \sqrt[15]{(2z)^8}$

93. $\sqrt[3]{p^2} \cdot \sqrt{p^3} = p^{2/3} \cdot p^{3/2}$
$= p^{(2/3)+(3/2)} = p^{13/6}$
$= \sqrt[6]{p^{13}} = \sqrt[6]{p^{12} \cdot p} = p^2 \sqrt[6]{p}$

95. $\dfrac{\sqrt{u^3}}{\sqrt[3]{u}} = \dfrac{u^{3/2}}{u^{1/3}} = u^{(3/2)-(1/3)}$
$= u^{7/6} = \sqrt[6]{u^7}$
$= \sqrt[6]{u^6 \cdot u} = u\sqrt[6]{u}$

97. $\sqrt[3]{x} \cdot \sqrt[6]{y} = x^{1/3} \cdot y^{1/6} = x^{2/6} \cdot y^{1/6}$
$= \left(x^2 y\right)^{1/6}$
$= \sqrt[6]{x^2 y}$

99. $\sqrt[4]{8} \cdot \sqrt{3} = \sqrt[4]{2^3} \cdot \sqrt{3}$
$= 2^{3/4} \cdot 3^{1/2} = 2^{3/4} \cdot 3^{2/4}$
$= \left(2^3 \cdot 3^2\right)^{1/4} = \sqrt[4]{2^3 \cdot 3^2}$
$= \sqrt[4]{8 \cdot 9} = \sqrt[4]{72}$

101. $\sqrt[3]{2xy} \cdot \sqrt[4]{5xy} = (2xy)^{1/3}(5xy)^{1/4} = 2^{1/3}x^{1/3}y^{1/3}5^{1/4}x^{1/4}y^{1/4} = 2^{4/12}x^{4/12}y^{4/12}5^{3/12}x^{3/12}y^{3/12}$
$= 2^{4/12} \cdot 5^{3/12}x^{7/12}y^{7/12}$
$= \left(2^4 \cdot 5^3 x^7 y^7\right)^{1/12}$
$= \sqrt[12]{2^4 5^3 x^7 y^7}$

103. $\sqrt[3]{4m^2n} \cdot \sqrt{6mn} = \left(4m^2n\right)^{1/3}(6mn)^{1/2} = 4^{1/3}m^{2/3}n^{1/3}6^{1/2}m^{1/2}n^{1/2} = 4^{2/6}m^{4/6}n^{2/6}6^{3/6}m^{3/6}n^{3/6}$
$= 4^{2/6} \cdot 6^{3/6}m^{7/6}n^{5/6} = \left(4^2 \cdot 6^3 m^7 n^5\right)^{1/6}$
$= \sqrt[6]{2^4 2^3 3^3 m^7 n^5} = \sqrt[6]{2^6 \cdot 2 \cdot 3^3 m^6 mn^5}$
$= 2m\sqrt[6]{2 \cdot 3^3 mn^5}$

105. $\left(\sqrt[3]{a} + \sqrt[3]{b}\right)\left(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}\right) = \sqrt[3]{a} \cdot \sqrt[3]{a^2} - \sqrt[3]{a} \cdot \sqrt[3]{ab} + \sqrt[3]{a} \cdot \sqrt[3]{b^2} + \sqrt[3]{b} \cdot \sqrt[3]{a^2} - \sqrt[3]{b} \cdot \sqrt[3]{ab} + \sqrt[3]{b} \cdot \sqrt[3]{b^2}$
$= \sqrt[3]{a^3} - \sqrt[3]{a^2b} + \sqrt[3]{ab^2} + \sqrt[3]{a^2b} - \sqrt[3]{ab^2} + \sqrt[3]{b^3}$
$= a + b$

Problem Recognition Exercises

1. **a.** $\sqrt{24}=\sqrt{4\cdot 6}=2\sqrt{6}$

b. $\sqrt[3]{24}=\sqrt[3]{8\cdot 3}=2\sqrt[3]{3}$

3. **a.** $\sqrt{200y^6}=\sqrt{100\cdot 2y^6}=10y^3\sqrt{2}$

b. $\sqrt[3]{200y^6}=\sqrt[3]{8\cdot 25y^6}=2y^2\sqrt[3]{25}$

5. **a.** $\sqrt{80}=\sqrt{16\cdot 5}=4\sqrt{5}$

b. $\sqrt[3]{80}=\sqrt[3]{8\cdot 10}=2\sqrt[3]{10}$

c. $\sqrt[4]{80}=\sqrt[4]{16\cdot 5}=2\sqrt[4]{5}$

7. **a.** $\sqrt{x^5y^6}=\sqrt{x^4\cdot x\cdot y^6}=x^2y^3\sqrt{x}$

b. $\sqrt[3]{x^5y^6}=\sqrt[3]{x^3\cdot x^2\cdot y^6}=xy^2\sqrt[3]{x^2}$

c. $\sqrt[4]{x^5y^6}=\sqrt[4]{x^4\cdot x\cdot y^4\cdot y^2}=xy\sqrt[4]{xy^2}$

9. **a.** $\sqrt[3]{32s^5t^6}=\sqrt[3]{8\cdot 4\cdot s^3\cdot s^2\cdot t^6}$

$=2st^2\sqrt[3]{4s^2}$

b. $\sqrt[4]{32s^5t^6}=\sqrt[4]{16\cdot 2\cdot s^4\cdot s\cdot t^4\cdot t^2}$

$=2st\sqrt[4]{2st^2}$

c. $\sqrt[5]{32s^5t^6}=\sqrt[5]{32\cdot s^5\cdot t^5\cdot t}$

$=2st\sqrt[5]{t}$

11. **a.** $\sqrt{5}+\sqrt{5}=2\sqrt{5}$

b. $\sqrt{5}\cdot\sqrt{5}=\sqrt{25}=5$

13. **a.** $2\sqrt{6}-5\sqrt{6}=-3\sqrt{6}$

b. $2\sqrt{6}\cdot 5\sqrt{6}=10\sqrt{36}=10\cdot 6=60$

15. **a.** $\sqrt{8}+\sqrt{2}=\sqrt{4\cdot 2}+\sqrt{2}=2\sqrt{2}+\sqrt{2}$

$=3\sqrt{2}$

b. $\sqrt{8}\cdot\sqrt{2}=\sqrt{16}=4$

17. **a.** $5\sqrt{18}-4\sqrt{8}=5\sqrt{9\cdot 2}-4\sqrt{4\cdot 2}$

$=5\cdot 3\sqrt{2}-4\cdot 2\sqrt{2}$

$=15\sqrt{2}-8\sqrt{2}=7\sqrt{2}$

b. $5\sqrt{18}\cdot 4\sqrt{8}=20\sqrt{144}=20\cdot 12=240$

19. **a.** $4\sqrt[3]{24}+6\sqrt[3]{3}=4\sqrt[3]{8\cdot 3}+6\sqrt[3]{3}$

$=4\cdot 2\sqrt[3]{3}+6\sqrt[3]{3}$

$=8\sqrt[3]{3}+6\sqrt[3]{3}$

$=14\sqrt[3]{3}$

b. $4\sqrt[3]{24}\cdot 6\sqrt[3]{3}=24\sqrt[3]{72}$

$=24\sqrt[3]{8\cdot 9}$

$=24\cdot 2\sqrt[3]{9}$

$=48\sqrt[3]{9}$

Section 6.6 Practice Exercises

1. **a** radical

d. Rationalizing.

b. $\frac{\sqrt[n]{a}}{\sqrt[b]{b}}$

c. c $\frac{4}{x^2}$

e. is; is not

f. denominator

3. $2y\sqrt{45}+3\sqrt{20y^2} = 2y\sqrt{9\cdot 5}+3\sqrt{4y^2\cdot 5}$
$= 2y\cdot 3\sqrt{5}+3\cdot 2y\sqrt{5}$
$= 6y\sqrt{5}+6y\sqrt{5}$
$= 12y\sqrt{5}$

5. $\left(-6\sqrt{y}+3\right)\left(3\sqrt{y}+1\right)$
$= -6\sqrt{y}\cdot 3\sqrt{y}-6\sqrt{y}\cdot(1)+3\cdot 3\sqrt{y}+3\cdot 1$
$= -18\sqrt{y^2}-6\sqrt{y}+9\sqrt{y}+3$
$= -18y+3\sqrt{y}+3$

7. $\left(8-\sqrt{t}\right)^2 = 8^2-2\cdot 8\cdot\sqrt{t}+\left(\sqrt{t}\right)^2$
$= 64-16\sqrt{t}+t$

9. $\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{2}-\sqrt{7}\right) = \left(\sqrt{2}\right)^2-\left(\sqrt{7}\right)^2$
$= 2-7$
$= -5$

11. $\sqrt{\frac{49x^4}{y^6}} = \frac{\sqrt{49x^4}}{\sqrt{y^6}} = \frac{7x^2}{y^3}$

13. $\sqrt{\frac{8a^2}{x^6}} = \frac{\sqrt{2\cdot 4a^2}}{\sqrt{x^6}} = \frac{2a\sqrt{2}}{x^3}$

15. $\sqrt[3]{\frac{-16j^3}{k^3}} = \frac{\sqrt[3]{-8j^3\cdot 2}}{\sqrt[3]{k^3}} = \frac{-2j\sqrt[3]{2}}{k}$

17. $\frac{\sqrt{72ab^5}}{\sqrt{8ab}} = \sqrt{\frac{72ab^5}{8ab}} = \sqrt{9b^4}$
$= 3b^2$

19. $\frac{\sqrt[4]{3b^3}}{\sqrt[4]{48b^{11}}} = \sqrt[4]{\frac{3b^3}{48b^{11}}} = \sqrt[4]{\frac{1}{16b^8}} = \frac{\sqrt[4]{1}}{\sqrt[4]{16b^8}}$
$= \frac{1}{2b^2}$

21. $\frac{\sqrt{3yz^2}}{\sqrt{w^4}} = \frac{z\sqrt{3y}}{w^2}$

23. $\frac{x}{\sqrt{5}} = \frac{x}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$

25. $\frac{7}{\sqrt[3]{x}} = \frac{7}{\sqrt[3]{x}}\cdot\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

27. $\frac{8}{\sqrt{3z}} = \frac{8}{\sqrt{3z}}\cdot\frac{\sqrt{3z}}{\sqrt{3z}}$

29. $\frac{1}{\sqrt[4]{8a^2}} = \frac{1}{\sqrt[4]{8a^2}}\cdot\frac{\sqrt[4]{2a^2}}{\sqrt[4]{2a^2}}$

31. $\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{1\sqrt{3}}{\sqrt{3^2}}=\frac{\sqrt{3}}{3}$

33. $\sqrt{\frac{1}{x}}=\frac{\sqrt{1}}{\sqrt{x}}=\frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{1\sqrt{x}}{\sqrt{x^2}}=\frac{\sqrt{x}}{x}$

35. $\frac{6}{\sqrt{2y}}=\frac{6}{\sqrt{2y}}\cdot\frac{\sqrt{2y}}{\sqrt{2y}}=\frac{6\sqrt{2y}}{\sqrt{(2y)^2}}$
$=\frac{6\sqrt{2y}}{2y}=\frac{3\sqrt{2y}}{y}$

37. $\sqrt{\frac{a^3}{2}}=\frac{\sqrt{a^3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2a\cdot a^2}}{\sqrt{4}}$
$=\frac{a\sqrt{2a}}{2}$

39. $\frac{6}{\sqrt{8}}=\frac{6}{\sqrt{4\cdot 2}}=\frac{6}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$
$=\frac{3\sqrt{2}}{\sqrt{(2)^2}}=\frac{3\sqrt{2}}{2}$

41. $\frac{3}{\sqrt[3]{2}}=\frac{3}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{3\sqrt[3]{2^2}}{\sqrt[3]{2^3}}$
$=\frac{3\sqrt[3]{4}}{2}$

43. $\frac{-6}{\sqrt[4]{x}}=\frac{-6}{\sqrt[4]{x}}\cdot\frac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}}=\frac{-6\sqrt[4]{x^3}}{\sqrt[4]{x^4}}$
$=\frac{-6\sqrt[4]{x^3}}{x}$

45. $\frac{7}{\sqrt[3]{4}}=\frac{7}{\sqrt[3]{2^2}}=\frac{7}{\sqrt[3]{2^2}}\cdot\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$
$=\frac{7\sqrt[3]{2}}{\sqrt[3]{2^3}}=\frac{7\sqrt[3]{2}}{2}$

47. $\sqrt[3]{\frac{4}{w^2}}=\frac{\sqrt[3]{4}}{\sqrt[3]{w^2}}=\frac{\sqrt[3]{4}}{\sqrt[3]{w^2}}\cdot\frac{\sqrt[3]{w}}{\sqrt[3]{w}}$
$=\frac{\sqrt[3]{4}\sqrt[3]{w}}{\sqrt[3]{w^3}}=\frac{\sqrt[3]{4w}}{w}$

49. $\sqrt[4]{\frac{16}{3}}=\frac{\sqrt[4]{16}}{\sqrt[4]{3}}=\frac{2}{\sqrt[4]{3}}\cdot\frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}}=\frac{2\sqrt[4]{3^3}}{\sqrt[4]{3^4}}$
$=\frac{2\sqrt[4]{27}}{3}$

51. $\frac{2}{\sqrt[3]{4x^2}}=\frac{2}{\sqrt[3]{2^2x^2}}\cdot\frac{\sqrt[3]{2x}}{\sqrt[3]{2x}}=\frac{2\sqrt[3]{2x}}{\sqrt[3]{2^3x^3}}$
$=\frac{2\sqrt[3]{2x}}{2x}$
$=\frac{\sqrt[3]{2x}}{x}$

53. $\frac{8}{7\sqrt{24}}=\frac{8}{7\sqrt{4\cdot 6}}=\frac{8}{7\cdot 2\sqrt{6}}=\frac{4}{7\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$
$=\frac{4\sqrt{6}}{7\sqrt{6^2}}=\frac{4\sqrt{6}}{7\cdot 6}$
$=\frac{2\sqrt{6}}{21}$

55. $\dfrac{1}{\sqrt{x^7}}=\dfrac{1}{\sqrt{x^6\cdot x}}=\dfrac{1}{x^3\sqrt{x}}$

$=\dfrac{1}{x^3\sqrt{x}}\cdot\dfrac{\sqrt{x}}{\sqrt{x}}$

$=\dfrac{\sqrt{x}}{x^3\sqrt{x^2}}=\dfrac{\sqrt{x}}{x^3\cdot x}=\dfrac{\sqrt{x}}{x^4}$

57. $\dfrac{2}{\sqrt{8x^5}}=\dfrac{2}{\sqrt{4x^4\cdot 2x}}=\dfrac{2}{2x^2\sqrt{2x}}$

$=\dfrac{1}{x^2\sqrt{2x}}\cdot\dfrac{\sqrt{2x}}{\sqrt{2x}}=\dfrac{\sqrt{2x}}{x^2\sqrt{2^2x^2}}$

$=\dfrac{\sqrt{2x}}{x^2\cdot 2x}=\dfrac{\sqrt{2x}}{2x^3}$

59. $\sqrt{2}+\sqrt{6}$

61. $\sqrt{x}-23$

63. $\dfrac{4}{\sqrt{2}+3}=\dfrac{4}{\sqrt{2}+3}\cdot\dfrac{\sqrt{2}-3}{\sqrt{2}-3}$

$=\dfrac{4\left(\sqrt{2}-3\right)}{\left(\sqrt{2}\right)^2-3^2}$

$=\dfrac{4\sqrt{2}-12}{2-9}$

$=\dfrac{4\sqrt{2}-12}{-7}$

65. $\dfrac{8}{\sqrt{6}-2}=\dfrac{8}{\sqrt{6}-2}\cdot\dfrac{\sqrt{6}+2}{\sqrt{6}+2}$

$=\dfrac{8\left(\sqrt{6}+2\right)}{\left(\sqrt{6}\right)^2-2^2}$

$=\dfrac{8\left(\sqrt{6}+2\right)}{6-4}=\dfrac{8\left(\sqrt{6}+2\right)}{2}$

$=4\left(\sqrt{6}+2\right)=4\sqrt{6}+8$

67. $\dfrac{\sqrt{7}}{\sqrt{3}+2}=\dfrac{\sqrt{7}}{\sqrt{3}+2}\cdot\dfrac{\sqrt{3}-2}{\sqrt{3}-2}$

$=\dfrac{\sqrt{7}\left(\sqrt{3}-2\right)}{\left(\sqrt{3}\right)^2-2^2}$

$=\dfrac{\sqrt{7}\cdot\sqrt{3}-\sqrt{7}\cdot(2)}{3-4}$

$=\dfrac{\sqrt{21}-2\sqrt{7}}{-1}$

$=-\sqrt{21}+2\sqrt{7}$

69. $\dfrac{-1}{\sqrt{p}+\sqrt{q}}=\dfrac{-1}{\sqrt{p}+\sqrt{q}}\cdot\dfrac{\sqrt{p}-\sqrt{q}}{\sqrt{p}-\sqrt{q}}$

$=\dfrac{-1\left(\sqrt{p}-\sqrt{q}\right)}{\left(\sqrt{p}\right)^2-\left(\sqrt{q}\right)^2}$

$=\dfrac{-\sqrt{p}+\sqrt{q}}{p-q}$

71. $$\frac{x-5}{\sqrt{x}+\sqrt{5}}=\frac{x-5}{\sqrt{x}+\sqrt{5}}\cdot\frac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}}$$

$$=\frac{(x-5)\left(\sqrt{x}-\sqrt{5}\right)}{\left(\sqrt{x}\right)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{\cancel{(x-5)}\left(\sqrt{x}-\sqrt{5}\right)}{\cancel{x-5}}=\sqrt{x}-\sqrt{5}$$

73. $$\frac{\sqrt{w}+2}{9-\sqrt{w}}=\frac{\sqrt{w}+2}{9-\sqrt{w}}\cdot\frac{9+\sqrt{w}}{9+\sqrt{w}}$$

$$=\frac{\left(\sqrt{w}+2\right)\left(9+\sqrt{w}\right)}{(9)^2-\left(\sqrt{w}\right)^2}$$

$$=\frac{w+9\sqrt{w}+2\sqrt{w}+18}{81-w}$$

$$=\frac{w+11\sqrt{w}+18}{81-w}$$

75. $$\frac{3\sqrt{x}-\sqrt{y}}{\sqrt{y}+\sqrt{x}}=\frac{3\sqrt{x}-\sqrt{y}}{\sqrt{y}+\sqrt{x}}\cdot\frac{\sqrt{y}-\sqrt{x}}{\sqrt{y}-\sqrt{x}}$$

$$=\frac{\left(3\sqrt{x}-\sqrt{y}\right)\left(\sqrt{y}-\sqrt{x}\right)}{\left(\sqrt{y}\right)^2-\left(\sqrt{x}\right)^2}$$

$$=\frac{3\sqrt{xy}-3\sqrt{x^2}-\sqrt{y^2}+\sqrt{xy}}{y-x}$$

$$=\frac{4\sqrt{xy}-3x-y}{y-x}$$

77. $$\frac{3\sqrt{10}}{2+\sqrt{10}}=\frac{3\sqrt{10}}{2+\sqrt{10}}\cdot\frac{2-\sqrt{10}}{2-\sqrt{10}}$$

$$=\frac{3\sqrt{10}\left(2-\sqrt{10}\right)}{(2)^2-\left(\sqrt{10}\right)^2}=\frac{6\sqrt{10}-3\sqrt{100}}{4-10}$$

$$=\frac{6\sqrt{10}-3\cdot 10}{-6}=\frac{6\sqrt{10}-30}{-6}$$

$$=\frac{\cancel{-6}\left(5-\sqrt{10}\right)}{\cancel{-6}}=5-\sqrt{10}$$

79. $$\frac{2\sqrt{3}+\sqrt{7}}{3\sqrt{3}-\sqrt{7}}=\frac{2\sqrt{3}+\sqrt{7}}{3\sqrt{3}-\sqrt{7}}\cdot\frac{3\sqrt{3}+\sqrt{7}}{3\sqrt{3}+\sqrt{7}}$$

$$=\frac{\left(2\sqrt{3}+\sqrt{7}\right)\left(3\sqrt{3}+\sqrt{7}\right)}{\left(3\sqrt{3}\right)^2-\left(\sqrt{7}\right)^2}$$

$$=\frac{6\sqrt{9}+2\sqrt{21}+3\sqrt{21}+\sqrt{49}}{9\cdot 3-7}$$

$$=\frac{6\cdot 3+5\sqrt{21}+7}{27-7}$$

81. $$\frac{\sqrt{5}+4}{2-\sqrt{5}}=\frac{\sqrt{5}+4}{2-\sqrt{5}}\cdot\frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$=\frac{\left(\sqrt{5}+4\right)\left(2+\sqrt{5}\right)}{(2)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{2\sqrt{5}+\sqrt{25}+8+4\sqrt{5}}{4-5}$$

$$= \frac{18 + 5\sqrt{21} + 7}{20}$$

$$= \frac{25 + 5\sqrt{21}}{20} = \frac{\not{5}\left(5 + \sqrt{21}\right)}{\not{5} \cdot 4}$$

$$= \frac{5 + \sqrt{21}}{4}$$

$$= \frac{6\sqrt{5} + 5 + 8}{-1} = \frac{6\sqrt{5} + 13}{-1}$$

$$= -6\sqrt{5} - 13$$

83. $$\frac{16}{\sqrt[3]{4}} = \frac{16}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{16\sqrt[3]{2}}{\sqrt[3]{2^3}} = \frac{16\sqrt[3]{2}}{2} = 8\sqrt[3]{2}$$

85. $$\frac{4}{x - \sqrt{2}} = \frac{4}{x - \sqrt{2}} \cdot \frac{x + \sqrt{2}}{x + \sqrt{2}} = \frac{4\left(x + \sqrt{2}\right)}{x^2 - \left(\sqrt{2}\right)^2} = \frac{4x + 4\sqrt{2}}{x^2 - 2}$$

87.
$$T\left(x\right) = 2\pi\sqrt{\frac{x}{32}}$$

$$T\left(1\right) = 2\pi\sqrt{\frac{1}{32}} = 2\pi\frac{1}{\sqrt{16 \cdot 2}} = \frac{2\pi}{4\sqrt{2}} = \frac{2\pi}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\pi\sqrt{2}}{4 \cdot 2} = \frac{\pi\sqrt{2}}{4} \text{ sec} \approx 1.11 \text{ sec}$$

89. **a.** $$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2^2}} = \frac{\sqrt{2}}{2}$$

b. $$\frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \frac{\sqrt[3]{4}}{2}$$

91. **a.** $$\frac{1}{\sqrt{5a}} = \frac{1}{\sqrt{5a}} \cdot \frac{\sqrt{5a}}{\sqrt{5a}} = \frac{\sqrt{5a}}{\sqrt{\left(5a\right)^2}} = \frac{\sqrt{5a}}{5a}$$

b. $$\frac{1}{\sqrt{5} + a} = \frac{1}{\sqrt{5} + a} \cdot \frac{\sqrt{5} - a}{\sqrt{5} - a} = \frac{\sqrt{5} - a}{\left(\sqrt{5}\right)^2 - a^2} = \frac{\sqrt{5} - a}{5 - a^2}$$

93. $$\frac{\sqrt{6}}{2} + \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{2} + \frac{1}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{6} = \frac{\sqrt{6}}{2} \cdot \frac{3}{3} + \frac{\sqrt{6}}{6} = \frac{3\sqrt{6} + \sqrt{6}}{6} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$$

95. $$\sqrt{15} - \sqrt{\frac{3}{5}} + \sqrt{\frac{5}{3}} = \sqrt{15} - \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{15}}{1} - \frac{\sqrt{15}}{5} + \frac{\sqrt{15}}{3}$$

$$= \frac{\sqrt{15}}{1} \cdot \frac{15}{15} - \frac{\sqrt{15}}{5} \cdot \frac{3}{3} + \frac{\sqrt{15}}{3} \cdot \frac{5}{5}$$

$$= \frac{15\sqrt{15} - 3\sqrt{15} + 5\sqrt{15}}{15} = \frac{17\sqrt{15}}{15}$$

97. $$\sqrt[3]{25}+\frac{3}{\sqrt[3]{5}}=\sqrt[3]{5^2}+\frac{3}{\sqrt[3]{5}}\cdot\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}=\frac{\sqrt[3]{5^2}}{1}+\frac{3\sqrt[3]{5^2}}{\sqrt[3]{5^3}}$$
$$=\frac{\sqrt[3]{5^2}}{1}+\frac{3\sqrt[3]{5^2}}{5}=\frac{\sqrt[3]{5^2}}{1}\cdot\frac{5}{5}+\frac{3\sqrt[3]{5^2}}{5}$$
$$=\frac{5\sqrt[3]{5^2}+3\sqrt[3]{5^2}}{5}=\frac{8\sqrt[3]{5^2}}{5}=\frac{8\sqrt[3]{25}}{5}$$

99. $$\frac{\sqrt{3}+6}{2}=\frac{\sqrt{3}+6}{2}\cdot\frac{\sqrt{3}-6}{\sqrt{3}-6}=\frac{\left(\sqrt{3}\right)^2-6^2}{2\left(\sqrt{3}-6\right)}$$
$$=\frac{3-36}{2\sqrt{3}-12}=\frac{-33}{2\sqrt{3}-12}$$

101. $$\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$
$$=\frac{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}{\left(\sqrt{a}+\sqrt{b}\right)^2}$$
$$=\frac{a-b}{\left(\sqrt{a}\right)^2+2\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}$$
$$=\frac{a-b}{a+2\sqrt{ab}+b}$$

103. $$\frac{\sqrt{5+3h}-\sqrt{5}}{h}=\frac{\sqrt{5+3h}-\sqrt{5}}{h}\cdot\frac{\sqrt{5+3h}+\sqrt{5}}{\sqrt{5+3h}+\sqrt{5}}$$
$$=\frac{\left(\sqrt{5+3h}\right)^2-\left(\sqrt{5}\right)^2}{h\left(\sqrt{5+3h}+\sqrt{5}\right)}$$
$$=\frac{5+3h-5}{h\left(\sqrt{5+3h}+\sqrt{5}\right)}$$
$$=\frac{3h}{h\left(\sqrt{5+3h}+\sqrt{5}\right)}=\frac{3}{\sqrt{5+3h}+\sqrt{5}}$$

105. $$\frac{\sqrt{4+5h}-2}{h}=\frac{\sqrt{4+5h}-2}{h}\cdot\frac{\sqrt{4+5h}+2}{\sqrt{4+5h}+2}$$
$$=\frac{\left(\sqrt{4+5h}\right)^2-(2)^2}{h\left(\sqrt{4+5h}+2\right)}$$
$$=\frac{4+5h-4}{h\left(\sqrt{4+5h}+2\right)}$$
$$=\frac{5h}{h\left(\sqrt{4+5h}+2\right)}$$
$$=\frac{5}{\sqrt{4+5h}+2}$$

Section 6.7 Practice Exercises

1. a. radical

b. isolate; 7

c. extraneous

d. third

3. $$\sqrt{\frac{9w^3}{16}}=\sqrt{\frac{9w^2\cdot w}{16}}=\frac{3w\sqrt{w}}{4}$$

5. $$\sqrt[3]{54c^4}=\sqrt[3]{27c^3\cdot 2c}$$
$$=3c\sqrt[3]{2c}$$

7. $\left(\sqrt{4x-6}\right)^2 = 4x-6$

9. $\left(\sqrt[3]{9p+7}\right)^3 = 9p+7$

11.
$$\sqrt{x} = 10$$
$$\left(\sqrt{x}\right)^2 = (10)^2$$
$$x = 100$$

Check:
$$\sqrt{100} = 10$$
$$10 = 10$$

The solution is $\{100\}$.

13.
$$\sqrt{x} + 4 = 6$$
$$\sqrt{x} = 2$$
$$\left(\sqrt{x}\right)^2 = 2^2$$
$$x = 4$$

Check:
$$\sqrt{4} + 4 = 6$$
$$2 + 4 = 6$$
$$6 = 6$$

The solution is $\{4\}$.

15.
$$\sqrt{5y+1} = 4$$
$$\left(\sqrt{5y+1}\right)^2 = 4^2$$
$$5y+1 = 16$$
$$5y = 15$$
$$y = 3$$

Check:
$$\sqrt{5(3)+1} = 4$$
$$\sqrt{15+1} = 4$$
$$\sqrt{16} = 4$$
$$4 = 4$$

The solution is $\{3\}$.

17.
$$6 = \sqrt{2z-3} - 3$$
$$9 = \sqrt{2z-3}$$
$$9^2 = \left(\sqrt{2z-3}\right)^2$$
$$81 = 2z - 3$$
$$2z = 84$$
$$z = 42$$

Check:
$$6 = \left(2(42)-3\right)^{1/2} - 3$$
$$6 = (84-3)^{1/2} - 3$$
$$6 = 81^{1/2} - 3$$
$$6 = 9 - 3$$
$$6 = 6$$

The solution is $\{42\}$.

19.
$$\sqrt{x^2+5} = x+1$$
$$\left(\sqrt{x^2+5}\right)^2 = (x+1)^2$$
$$x^2 + 5 = x^2 + 2x + 1$$
$$-2x = -4$$
$$x = 2$$

Check:
$$\sqrt{(2)^2+5} = 2+1$$
$$\sqrt{4+5} = 3$$
$$\sqrt{9} = 43$$
$$3 = 3$$

The solution is $\{2\}$.

21.
$$\sqrt[3]{x-2} - 1 = 2$$
$$\sqrt[3]{x-2} = 3$$
$$\left(\sqrt[3]{x-2}\right)^3 = 3^3$$
$$x - 2 = 27$$
$$x = 29$$

Check:
$$\sqrt[3]{29-2} - 1 = 2$$
$$\sqrt[3]{27} - 1 = 2$$
$$3 - 1 = 2$$
$$2 = 2$$

The solution is $\{29\}$.

23.
$$(15-w)^{1/3}+7=2$$
$$(15-w)^{1/3}=-5$$
$$\left(\sqrt[3]{15-w}\right)^3=(-5)^3$$
$$15-w=-125$$
$$15+125=w$$
$$140=w$$
Check:
$$(15-w)^{1/3}+7=2$$
$$(15-140)^{1/3}+7=2$$
$$(-125)^{1/3}+7=2$$
$$-5+7=2$$
$$2=2$$
The solution is $\{140\}$.

25.
$$3+\sqrt{x-16}=0$$
$$\sqrt{x-16}=-3$$
$$\left(\sqrt{x-16}\right)^2=(-3)^2$$
$$x-16=9$$
$$x=25$$
Check:
$$3+\sqrt{25-16}=0$$
$$3+\sqrt{9}=0$$
$$3+3=0$$
$$6\neq 0$$
{ } (x =25 does not check).

27.
$$2\sqrt{6a+7}-2a=0$$
$$2\sqrt{6a+7}=2a$$
$$\sqrt{6a+7}=a$$
$$\left(\sqrt{6a+7}\right)^2=(a)^2$$
$$6a+7=a^2$$
$$a^2-6a-7=0$$
$$(a-7)(a+1)=0$$
$$a-7=0 \text{ or } a+1=0$$
$$a=7 \text{ or } \quad a=-1$$

Check $a=7$:
$$2\sqrt{6(7)+7}-2(7)=0$$
$$2\sqrt{42+7}-14=0$$
$$2\sqrt{49}-14=0$$
$$2\cdot 7-14=0$$
$$14-14=0$$
$$0=0$$

Check $a=-1$:
$$2\sqrt{6(-1)+7}-2(-1)=0$$
$$2\sqrt{-6+7}+2=0$$
$$2\sqrt{1}+2=0$$
$$2\cdot 1+2=0$$
$$2+2=0$$
$$4\neq 0$$

The solution is {7}. ($a=-1$ does not check).

29.
$$(2x-5)^{1/4}=-1$$
$$\sqrt[4]{2x-5}=-1$$
$$\left(\sqrt[4]{2x-5}\right)^4=(-1)^4$$
$$2x-5=1$$
$$2x=6$$
$$x=3$$

Check:
$$\sqrt[4]{2(3)-5}=-1$$
$$\sqrt[4]{6-5}=-1$$
$$\sqrt[4]{1}=-1$$
$$1\neq -1$$

{ } ($x=3$ does not check).

31.

$$r = \sqrt[3]{\frac{3V}{4\pi}} \quad \text{for } V$$
$$r^3 = \left(\sqrt[3]{\frac{3V}{4\pi}}\right)^3$$
$$r^3 = \frac{3V}{4\pi}$$
$$4\pi r^3 = 3V$$
$$\frac{4\pi r^3}{3} = V$$

33.

$$r = \pi\sqrt{r^2 + h^2} \quad \text{for } h^2$$
$$\frac{r}{\pi} = \sqrt{r^2 + h^2}$$
$$\left(\frac{r}{\pi}\right)^2 = \left(\sqrt{r^2 + h^2}\right)^2$$
$$\frac{r^2}{\pi^2} = r^2 + h^2$$
$$\frac{r^2}{\pi^2} - r^2 = h^2$$

35.

$$(a+5)^2 = a^2 + 2 \cdot a \cdot 5 + 5^2$$
$$= a^2 + 10a + 25$$

37.

$$\left(\sqrt{5a} - 3\right)^2 = \left(\sqrt{5a}\right)^2 - 2 \cdot \sqrt{5a} \cdot 3 + 3^2$$
$$= 5a - 6\sqrt{5a} + 9$$

39.

$$\left(\sqrt{r-3} + 5\right)^2 = \left(\sqrt{r-3}\right)^2 + 2 \cdot \sqrt{r-3} \cdot 5 + 5^2$$
$$= r - 3 + 10\sqrt{r-3} + 25$$
$$= r + 22 + 10\sqrt{r-3}$$

41.

$$\sqrt{a^2 + 2a + 1} = a + 5$$
$$\left(\sqrt{a^2 + 2a + 1}\right)^2 = (a+5)^2$$
$$a^2 + 2a + 1 = a^2 + 10a + 25$$
$$-8a = 24$$
$$a = \frac{24}{-8}$$
$$a = -3$$

Check:

$$\sqrt{(-3)^2 + 2(-3) + 1} = -3 + 5$$
$$\sqrt{9 - 6 + 1} = 2$$
$$\sqrt{4} = 2$$
$$2 = 2$$

The solution is {–3}.

43.

$$\sqrt{25w^2-2w-3}=5w-4$$
$$\left(\sqrt{25w^2-2w-3}\right)^2=(5w-4)^2$$
$$25w^2-2w-3=25w^2-40w+16$$
$$38w=19$$
$$w=\frac{1}{2}$$

Check:

$$\sqrt{25\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-3}=5\left(\frac{1}{2}\right)-4$$
$$\sqrt{\frac{25}{4}-1-3}=\frac{5}{2}-4$$
$$\sqrt{\frac{9}{4}}=-\frac{3}{2}$$
$$\frac{3}{2}\neq-\frac{3}{2}$$

{ } ($w=\frac{1}{2}$ does not check.)

45.

$$4\sqrt{p-2}-2=-p$$
$$4\sqrt{p-2}=-p+2$$
$$\left(4\sqrt{p-2}\right)^2=(-p+2)^2$$
$$16(p-2)=p^2-4p+4$$
$$16p-32=p^2-4p+4$$
$$0=p^2-20p+36$$
$$0=(p-18)(p-2)$$
$$p-18=0 \quad \text{or } p-2=0$$
$$p=18 \text{ or } \quad p=2$$

Check $p=18$:

$$4\sqrt{18-2}-2=-18$$
$$4\sqrt{16}-2=-18$$
$$4(4)-2=-18$$
$$16-2=-18$$
$$14\neq-18$$

Check $p=2$:

$$4\sqrt{2-2}-2=-2$$
$$4\sqrt{0}-2=-2$$
$$4(0)-2=-2$$
$$0-2=-2$$
$$-2=-2$$

The solution is {2}. ($p=18$ does not check.)

47.

$$\sqrt[4]{h+4}=\sqrt[4]{2h-5}$$
$$\left(\sqrt[4]{h+4}\right)^4=\left(\sqrt[4]{2h-5}\right)^4$$
$$h+4=2h-5$$
$$4+5=2h-h$$
$$9=h$$

Check:

$$\sqrt[4]{9+4}=\sqrt[4]{2(9)-5}$$
$$\sqrt[4]{13}=\sqrt[4]{18-5}$$
$$\sqrt[4]{13}=\sqrt[4]{13}$$

The solution is {9}.

49. $\sqrt[3]{5a+3}-\sqrt[3]{a-13}=0$

$$\sqrt[3]{5a+3}=\sqrt[3]{a-13}$$
$$\left(\sqrt[3]{5a+3}\right)^3=\left(\sqrt[3]{a-13}\right)^3$$
$$5a+3=a-13$$
$$4a=-16$$
$$a=-4$$

Check:

$$\sqrt[3]{5(-4)+3}-\sqrt[3]{-4-13}=0$$
$$\sqrt[3]{-20+3}-\sqrt[3]{-17}=0$$
$$\sqrt[3]{-17}-\sqrt[3]{-17}=0$$
$$0=0$$

The solution is {–4}.

51. $\sqrt{5a-9}=\sqrt{5a}-3$

$$\left(\sqrt{5a-9}\right)^2=\left(\sqrt{5a}-3\right)^2$$
$$5a-9=5a-6\sqrt{5a}+9$$
$$6\sqrt{5a}=18$$
$$\sqrt{5a}=3$$
$$\left(\sqrt{5a}\right)^2=3^2$$
$$5a=9$$
$$a=\frac{9}{5}$$

Check:

$$\sqrt{5\left(\frac{9}{5}\right)-9}=\sqrt{5\left(\frac{9}{5}\right)}-3$$
$$\sqrt{9-9}=\sqrt{9}-3$$
$$\sqrt{0}=3-3$$
$$0=0$$

The solution is $\left\{\frac{9}{5}\right\}$.

53. $\sqrt{2h+5}-\sqrt{2h}=1$

$$\sqrt{2h+5}=\sqrt{2h}+1$$
$$\left(\sqrt{2h+5}\right)^2=\left(\sqrt{2h}+1\right)^2$$
$$2h+5=2h+2\sqrt{2h}+1$$
$$4=2\sqrt{2h}$$
$$\sqrt{2h}=2$$
$$\left(\sqrt{2h}\right)^2=2^2$$
$$2h=4$$
$$h=2$$

Check:

$$\sqrt{2(2)+5}-\sqrt{2(2)}=1$$
$$\sqrt{4+5}-\sqrt{4}=1$$
$$\sqrt{9}-\sqrt{4}=1$$
$$3-2=1$$
$$1=1$$

The solution is {2}.

55. $(t-9)^{1/2}-t^{1/2}=3$

$$\sqrt{t-9}-\sqrt{t}=3$$
$$\sqrt{t-9}=\sqrt{t}+3$$
$$\left(\sqrt{t-9}\right)^2=\left(\sqrt{t}+3\right)^2$$
$$t-9=t+6\sqrt{t}+9$$
$$-18=6\sqrt{t}$$
$$\sqrt{t}=-3$$
$$\left(\sqrt{t}\right)^2=(-3)^2$$
$$t=9$$

Check:

$$(9-9)^{1/2}-9^{1/2}=3$$
$$\sqrt{9-9}-\sqrt{9}=3$$
$$\sqrt{0}-\sqrt{9}=3$$
$$0-3=3$$
$$-3\neq 3$$

{ } (The value 9 does not check.)

57. $6=\sqrt{x^2+3}-x\{\ \}$

$$x+6=\sqrt{x^2+3}$$
$$(x+6)^2=\left(\sqrt{x^2+3}\right)^2$$
$$x^2+12x+36=x^2+3$$
$$12x=x^2-x^2+3-36$$
$$12x=0-33$$
$$12x=-33$$
$$x=-\frac{33}{12}$$
$$=-\frac{11}{4}$$

Check:

$$6=\sqrt{\left(-\frac{11}{4}\right)^2+3}-\left(-\frac{11}{4}\right)$$
$$6=\sqrt{\frac{121}{16}+3}+\frac{11}{4}$$
$$6=\sqrt{\frac{169}{16}}+\frac{11}{4}$$
$$6=\frac{13}{4}+\frac{11}{4}$$
$$6=\frac{24}{4}$$
$$6=6$$

The solution is $\left\{-\frac{11}{4}\right\}$.

59. $\sqrt{3t-7}=2-\sqrt{3t+1}$

$$\left(\sqrt{3t-7}\right)^2=\left(2-\sqrt{3t+1}\right)^2$$

$$3t-7=4-4\sqrt{3t+1}+3t+1$$

$$-12=-4\sqrt{3t+1}$$

$$\sqrt{3t+1}=3$$

$$\left(\sqrt{3t+1}\right)^2=(3)^2$$

$$3t+1=9$$

$$3t=8$$

$$t=\frac{8}{3}$$

Check:

$$\sqrt{3\left(\frac{8}{3}\right)-7}=2-\sqrt{3\left(\frac{8}{3}\right)+1}$$

$$\sqrt{8-7}=2-\sqrt{8+1}$$

$$\sqrt{1}=2-\sqrt{9}$$

$$1=2-3$$

$$1\neq-1$$

{ } ($t=\frac{8}{3}$ does not check.)

61. $\sqrt{z+1}+\sqrt{2z+3}=1$

$$\sqrt{2z+3}=1-\sqrt{z+1}$$

$$\left(\sqrt{2z+3}\right)^2=\left(1-\sqrt{z+1}\right)^2$$

$$2z+3=1-2\sqrt{z+1}+z+1$$

$$z+1=-2\sqrt{z+1}$$

$$(z+1)^2=\left(-2\sqrt{z+1}\right)^2$$

$$z^2+2z+1=4(z+1)$$

$$z^2+2z+1=4z+4$$

$$z^2-2z-3=0$$

$$(z-3)(z+1)=0$$

$$z-3=0 \text{ or } z+1=0$$

$$z=3 \text{ or } z=-1$$

The solution is $\{-1\}$. ($z=3$ does not check.)

Check $z=3$:

$$\sqrt{3+1}+\sqrt{2(3)+3}=1$$

$$\sqrt{4}+\sqrt{6+3}=1$$

$$\sqrt{4}+\sqrt{9}=1$$

$$2+3=1$$

$$5\neq1$$

Check $z=-1$:

$$\sqrt{-1+1}+\sqrt{2(-1)+3}=1$$

$$\sqrt{0}+\sqrt{-2+3}=1$$

$$\sqrt{0}+\sqrt{1}=1$$

$$0+1=1$$

$$1=1$$

63.

$$\sqrt{6m+7}-\sqrt{3m+3}=1$$
$$\sqrt{6m+7}=1+\sqrt{3m+3}$$
$$\left(\sqrt{6m+7}\right)^2=\left(1+\sqrt{3m+3}\right)^2$$
$$6m+7=1+2\sqrt{3m+3}+3m+3$$
$$3m+3=2\sqrt{3m+3}$$
$$(3m+3)^2=\left(2\sqrt{3m+3}\right)^2$$
$$9m^2+18m+9=4(3m+3)$$
$$9m^2+18m+9=12m+12$$
$$9m^2+6m-3=0$$
$$3\left(3m^2+2m-1\right)=0$$
$$3(3m-1)(m+1)=0$$
$$3m-1=0 \text{ or } m+1=0$$
$$3m=1 \text{ or } \quad m=-1$$
$$m=\frac{1}{3} \text{ or } \quad m=-1$$

Check $m=\frac{1}{3}$:

$$\sqrt{6\left(\frac{1}{3}\right)+7}-\sqrt{3\left(\frac{1}{3}\right)+3}=1$$
$$\sqrt{2+7}-\sqrt{1+3}=1$$
$$\sqrt{9}-\sqrt{4}=1$$
$$3-2=1$$
$$1=1$$

Check $m=-1$:

$$\sqrt{6(-1)+7}-\sqrt{3(-1)+3}=1$$
$$\sqrt{-6+7}-\sqrt{-3+3}=1$$
$$\sqrt{1}-\sqrt{0}=1$$
$$1-0=1$$
$$1=1$$

The solution is $\left\{\frac{1}{3},-1\right\}$.

65.

$$2+2\sqrt{2t+3}+2\sqrt{3t-5}=0$$
$$2+2\sqrt{2t+3}=-2\sqrt{3t-5}$$
$$1+\sqrt{2t+3}=-\sqrt{3t-5}$$
$$\left(1+\sqrt{2t+3}\right)^2=\left(-\sqrt{3t-5}\right)^2$$
$$1+2\sqrt{2t+3}+2t+3=3t-5$$
$$2\sqrt{2t+3}=t-9$$
$$\left(2\sqrt{2t+3}\right)^2=(t-9)^2$$
$$4(2t+3)=t^2-18t+81$$
$$8t+12=t^2-18t+81$$
$$t^2-18t-8t+81-12=0$$

Check $t=3$:

$$2+2\sqrt{2(3)+3}+2\sqrt{3(3)-5}=0$$
$$2+2\sqrt{6+3}+2\sqrt{9-5}=0$$
$$2+2\sqrt{9}+2\sqrt{4}=0$$
$$2+2\cdot 3+2\cdot 2=0$$
$$2+6+4=0$$
$$12\neq 0$$

$$t^2-26t+69=0$$
$$(t-3)(t-23)=0$$
$$t-3=0 \text{ or } t-23=0$$
$$t=3 \text{ or } \quad t=23$$

Check $t=23$:
$$2+2\sqrt{2(23)+3}+2\sqrt{3(23)-5}=0$$
$$2+2\sqrt{46+3}+2\sqrt{69-5}=0$$
$$2+2\sqrt{49}+2\sqrt{64}=0$$
$$2+2\cdot 7+2\cdot 8=0$$
$$2+14+16=0$$
$$32\neq 0$$

{ } ($t=3$ and $t=23$ do not check.)

67.
$$3\sqrt{y-3}=\sqrt{4y+3}$$
$$\left(3\sqrt{y-3}\right)^2=\left(\sqrt{4y+3}\right)^2$$
$$9(y-3)=4y+3$$
$$9y-27=4y+3$$
$$5y=30$$
$$y=6$$

Check:
$$3\sqrt{6-3}=\sqrt{4(6)+3}$$
$$3\sqrt{3}=\sqrt{24+3}$$
$$3\sqrt{3}=\sqrt{27}$$
$$3\sqrt{3}=3\sqrt{3}$$

The solution is {6}.

69.
$$\sqrt{p+7}=\sqrt{2p}+1$$
$$\left(\sqrt{p+7}\right)^2=\left(\sqrt{2p}+1\right)^2$$
$$p+7=2p+2\sqrt{2p}+1$$
$$-p+6=2\sqrt{2p}$$
$$(-p+6)^2=\left(2\sqrt{2p}\right)^2$$
$$p^2-12p+36=4(2p)$$
$$p^2-12p+36=8p$$
$$p^2-20p+36=0$$
$$(p-18)(p-2)=0$$
$$(p-18)=0 \quad \text{or } (p-2)=0$$
$$p-18=0 \quad \text{or } p-2=0$$
$$p=18 \text{ or } \quad p=2$$

Check $p=18$:
$$\sqrt{18+7}=\sqrt{2\cdot 18}+1$$
$$\sqrt{25}=\sqrt{36}+1$$
$$5=6+1$$
$$5\neq 7$$

Check $p=2$:
$$\sqrt{2+7}=\sqrt{2\cdot 2}+1$$
$$\sqrt{9}=\sqrt{4}+1$$
$$3=2+1$$
$$3=3$$

The solution is {2}. (p = 18 does not check.)

71. $v=\sqrt{2gh}$

a.
$$44=\sqrt{2(32)h}$$
$$44=\sqrt{64h}$$
$$44=8\sqrt{h}$$
$$11=2\sqrt{h}$$
$$11^2=\left(2\sqrt{h}\right)^2$$
$$121=4h$$
$$h=\frac{121}{4}=30.25\text{ ft}$$

b.
$$26=\sqrt{2(9.8)h}$$
$$26=\sqrt{19.6h}$$
$$26^2=\left(\sqrt{19.6h}\right)^2$$
$$676=19.6h$$
$$h=\frac{676}{19.6}$$
$$\approx 34.5\text{ m}$$

73. $C(x)=\sqrt{0.3x+1}$

a.
$$C(x)=\sqrt{0.3(10)+1}=\sqrt{3+1}=\sqrt{4}$$
$$=\$2\text{ million}$$

b.
$$P(x)=R(x)-C(x)$$
$$P(x)=320(10{,}000)-2{,}000{,}000$$
$$=3{,}200{,}000-2{,}000{,}000$$
$$=\$1.2\text{ million}$$

c.
$$4=\sqrt{0.3x+1}$$
$$4^2=\left(\sqrt{0.3x+1}\right)^2$$
$$16=0.3x+1$$
$$15=0.3x$$
$$x=\frac{15}{0.3}$$
$$=50\quad\text{(50,000 passengers)}$$

75. $t(x)=0.90\sqrt[5]{x^3}$

a.
$$4=0.90\sqrt[5]{x^3}$$
$$\frac{4}{0.90}=\sqrt[5]{x^3}$$
$$\left(\frac{4}{0.90}\right)^5=\left(\sqrt[5]{x^3}\right)^5$$
$$1734.15=x^3$$
$$x=\sqrt[3]{1734.15}$$
$$\approx 12\text{ lb}$$

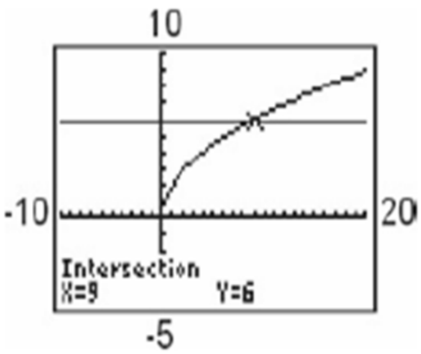
b.
$$t(18)=0.90\sqrt[5]{18^3}$$
$$\approx 5.1\text{ hr}$$

An 18-lb turkey will take about 5.1 hr to cook.

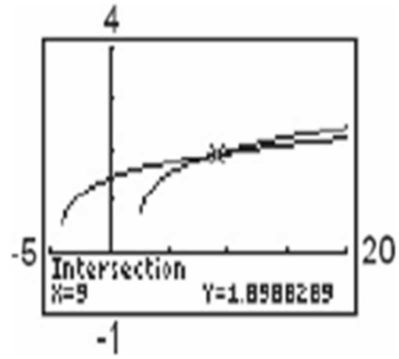
77.
$$a^2+b^2=c^2$$
$$h^2+b^2=5^2$$
$$b^2=5^2-h^2$$
$$b=\sqrt{25-h^2}$$

79.
$$a^2+b^2=c^2$$
$$a^2+14^2=k^2$$
$$a^2=k^2-14^2$$
$$a=\sqrt{k^2-196}$$

81. The x-coordinate of the point of intersection is the solution to the equation.



83.



Section 6.8 Practice Exercises

1. **a.** imaginary

b. $\sqrt{-1}$; -1

c. $i\sqrt{b}$

d. $a+bi$; $\sqrt{-1}$

e. real; b

f. $a+bi$

g. True

h. True

3. $\left(3-\sqrt{x}\right)\left(3+\sqrt{x}\right)=3^2-\left(\sqrt{x}\right)^2=9-x$

5.
$$\sqrt[3]{3p+7}-\sqrt[3]{2p-1}=0$$
$$\sqrt[3]{3p+7}=\sqrt[3]{2p-1}$$
$$\left(\sqrt[3]{3p+7}\right)^3=\left(\sqrt[3]{2p-1}\right)^3$$
$$3p+7=2p-1$$
$$3p-2p=-1-7$$
$$p=-8$$

Check:
$$\sqrt[3]{3(-8)+7}-\sqrt[3]{2(-8)-1}=0$$
$$\sqrt[3]{-24+7}-\sqrt[3]{-16-1}=0$$
$$\sqrt[3]{-17}-\sqrt[3]{-17}=0$$
$$0=0$$

The solution is $\{-8\}$.

7.

$$\sqrt{4a+29} = 2\sqrt{a}+5$$
$$\left(\sqrt{4a+29}\right)^2 = \left(2\sqrt{a}+5\right)^2$$
$$4a+29 = 4a+20\sqrt{a}+25$$
$$4 = 20\sqrt{a}$$
$$5\sqrt{a} = 1$$
$$\left(5\sqrt{a}\right)^2 = 1^2$$
$$25a = 1$$
$$a = \frac{1}{25}$$

The solution is $\left\{\frac{1}{25}\right\}$.

Check:

$$\sqrt{4\left(\frac{1}{25}\right)+29} = 2\sqrt{\frac{1}{25}}+5$$
$$\sqrt{\frac{4}{25}+29} = 2\cdot\frac{1}{5}+5$$
$$\sqrt{\frac{729}{25}} = \frac{2}{5}+5$$
$$\frac{27}{5} = \frac{27}{5}$$

9. $\sqrt{-1} = i$ and $-\sqrt{1} = -1$

11. $\sqrt{-144} = i\sqrt{144} = 12i$

13. $\sqrt{-3} = i\sqrt{3}$

15. $-\sqrt{-20} = -i\sqrt{20} = -i\sqrt{4\cdot 5}$
$= -2i\sqrt{5}$

17.
$$2\sqrt{-25}\cdot 3\sqrt{-4} = 2i\sqrt{25}\cdot 3i\sqrt{4}$$
$$= 5\cdot 2i\cdot 2\cdot 3i$$
$$= 10i\cdot 6i = 60i^2$$
$$= 60(-1) = -60$$

19.
$$7\sqrt{-63}-4\sqrt{-28} = 7\sqrt{-9\cdot 7}-4\sqrt{-4\cdot 7}$$
$$= 7\cdot 3i\sqrt{7}-4\cdot 2i\sqrt{7}$$
$$= 21i\sqrt{7}-8i\sqrt{7}$$
$$= 13i\sqrt{7}$$

21.
$$\sqrt{-7}\cdot\sqrt{-7} = i\sqrt{7}\cdot i\sqrt{7} = i^2\sqrt{49}$$
$$= -1\cdot 7 = -7$$

23.
$$\sqrt{-9}\cdot\sqrt{-16} = 3i\cdot 4i = 12i^2$$
$$= 12(-1) = -12$$

25.
$$\sqrt{-15}\cdot\sqrt{-6} = i\sqrt{15}\cdot i\sqrt{6} = i^2\sqrt{90}$$
$$= -1\sqrt{9\cdot 10}$$
$$= -3\sqrt{10}$$

27.
$$\frac{\sqrt{-50}}{\sqrt{25}} = \frac{\sqrt{-25\cdot 2}}{5} = \frac{5i\sqrt{2}}{5}$$
$$= i\sqrt{2}$$

29.
$$\frac{\sqrt{-90}}{\sqrt{-10}} = \frac{\sqrt{-9\cdot 10}}{\sqrt{-1\cdot 10}}$$
$$= \frac{3i\sqrt{10}}{i\sqrt{10}} = 3$$

31. $i^7 = i^4\cdot i^3 = 1(-i) = -i$

33. $i^{64} = \left(i^4\right)^{16} = 1^{64}$
$= 1$

35. $i^{41} = i^{40} \cdot i = \left(i^4\right)^{10} \cdot i = 1^{10} \cdot i$
$= 1 \cdot i = i$

37. $i^{52} = \left(i^4\right)^{13} = 1^{13} = 1$

39. $i^{23} = i^{20} \cdot i^3 = \left(i^4\right)^5 \cdot i^3 = 1^5\left(-i\right)$
$= 1\left(-i\right) = -i$

41. $i^6 = i^4 \cdot i^2 = 1\left(-1\right) = -1$

43. $a - bi$

45. $-5 + 12i$
Real part: –5; Imaginary part: 12

47. $-6i = 0 - 6i$
Real part: 0; Imaginary part: –6

49. $35 = 35 + 0i$
Real part: 35; Imaginary part: 0

51. $\frac{3}{5} + i$
Real part: $\frac{3}{5}$; Imaginary part: 1

53. $\left(2 - i\right) + \left(5 + 7i\right) = \left(2 + 5\right) + \left(-1 + 7\right)i = 7 + 6i$

55.
$$\begin{aligned}\left(\frac{1}{2} + \frac{2}{3}i\right) - \left(\frac{1}{5} - \frac{5}{6}i\right) &= \frac{1}{2} + \frac{2}{3}i - \frac{1}{5} + \frac{5}{6}i \\ &= \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{2}{3} + \frac{5}{6}\right)i \\ &= \left(\frac{5}{10} - \frac{2}{10}\right) + \left(\frac{4}{6} + \frac{5}{6}\right)i \\ &= \frac{3}{10} + \frac{9}{6}i \\ &= \frac{3}{10} + \frac{3}{2}i\end{aligned}$$

57.
$$\begin{aligned}\sqrt{-98} - \sqrt{-8} &= i\sqrt{98} - i\sqrt{8} \\ &= i\sqrt{7^2 \cdot 2} - i\sqrt{2^2 \cdot 2} \\ &= 7i\sqrt{2} - 2i\sqrt{2} \\ &= 5i\sqrt{2}\end{aligned}$$

59.
$$\begin{aligned}\left(2 + 3i\right) - \left(1 - 4i\right) + \left(-2 + 3i\right) &= 2 + 3i - 1 + 4i - 2 + 3i \\ &= \left(2 - 1 - 2\right) + \left(3 + 4 + 3\right)i \\ &= -1 + 10i\end{aligned}$$

61.
$$\begin{aligned}\left(8i\right)\left(3i\right) &= 24i^2 \\ &= 24\left(-1\right) \\ &= -24 + 0i\end{aligned}$$

63. $6i(1-3i) = 6i(1) - 6i(3i)$
$= 6i - 18i^2$
$= 6i - 18(-1)$
$= 18 + 6i$

65. $(2-10i)(3+2i)$
$= 2(3) + 2(2i) - 10i(3) - 10i(2i)$
$= 6 + 4i - 30i - 20i^2$
$= 6 - 26i - 20(-1)$
$= 6 - 26i + 20 = 26 - 26i$

67. $(-5+2i)(5+2i)$
$= -5(5) - 5(2i) + 2i(5) + (2i)(2i)$
$= -25 - 10i + 10i + 4i^2$
$= -25 + 4(-1) = -25 - 4$
$= -29 + 0i$

69. $(4+5i)^2 = 4^2 + 2 \cdot 4 \cdot 5i + (5i)^2$
$= 16 + 40i + 25i^2 = 16 + 40i + 25(-1)$
$= 16 + 40i - 25$
$= -9 + 40i$

71. $(2+i)(3-2i)(4+3i)$
$= \left[2 \cdot 3 - 2 \cdot 2i + i \cdot 3 - i(2i)\right](4+3i)$
$= \left(6 - 4i + 3i - 2i^2\right)(4+3i)$
$= \left(6 - i - 2(-1)\right)(4+3i)$
$= (6 - i + 2)(4+3i) = (8-i)(4+3i)$
$= 8 \cdot 4 + 8 \cdot 3i - i \cdot 4 - i(3i) = 32 + 24i - 4i - 3i^2$
$= 32 + 20i - 3(-1) = 32 + 20i + 3 = 35 + 20i$

73. $(-4-6i)^2 = (-4)^2 + 2 \cdot 4 \cdot 6i + (6i)^2$
$= 16 + 48i + 36i^2$
$= 16 + 48i + 36(-1)$
$= 16 + 48i - 36$
$= -20 + 48i$

75. $\left(-\frac{1}{2} - \frac{3}{4}i\right)\left(-\frac{1}{2} + \frac{3}{4}i\right) = \left(-\frac{1}{2}\right)^2 - \left(\frac{3}{4}i\right)^2$
$= \frac{1}{4} - \frac{9}{16}i^2$
$= \frac{1}{4} - \frac{9}{16}(-1)$
$= \frac{1}{4} + \frac{9}{16} = \frac{4}{16} + \frac{9}{16} = \frac{13}{16} + 0i$

77. $\frac{2}{1+3i} = \frac{2}{1+3i} \cdot \frac{1-3i}{1-3i}$
$= \frac{2(1-3i)}{1^2 - (3i)^2} = \frac{2(1-3i)}{1-9i^2}$
$= \frac{2(1-3i)}{1-9(-1)} = \frac{2(1-3i)}{1+9}$

79. $\frac{-i}{4-3i} = \frac{-i}{4-3i} \cdot \frac{4+3i}{4+3i}$
$= \frac{-i(4+3i)}{4^2 - (3i)^2}$
$= \frac{-i \cdot 4 - i(3i)}{16 - 9i^2} = \frac{-4i - 3i^2}{16 - 9(-1)}$

$$= \frac{2(1-3i)}{10} = \frac{1-3i}{5}$$

$$= \frac{1}{5} - \frac{3}{5}i$$

$$= \frac{-4i-3(-1)}{16+9} = \frac{3-4i}{25}$$

$$= \frac{3}{25} - \frac{4}{25}i$$

81. $\frac{5+2i}{5-2i} = \frac{5+2i}{5-2i} \cdot \frac{5+2i}{5+2i}$

$$= \frac{5 \cdot 5 + 5 \cdot 2i + 2i \cdot 5 + 2i \cdot 2i}{5^2 - (2i)^2}$$

$$= \frac{25+10i+10i+4i^2}{25-4i^2}$$

$$= \frac{25+20i+4(-1)}{25-4(-1)}$$

$$= \frac{25+20i-4}{25+4}$$

$$= \frac{21+20i}{29}$$

$$= \frac{21}{29} + \frac{20}{29}i$$

83. $\frac{3+7i}{-2-4i} = \frac{3+7i}{-2-4i} \cdot \frac{-2+4i}{-2+4i}$

$$= \frac{3(-2)+3 \cdot 4i - 7i \cdot 2 + 7i \cdot 4i}{(-2)^2 - (4i)^2}$$

$$= \frac{-6+12i-14i+28i^2}{4-16i^2}$$

$$= \frac{-6-2i+28(-1)}{4-16(-1)}$$

$$= \frac{-6-2i-28}{4+16}$$

$$= \frac{-34-2i}{20} = \frac{2(-17-i)}{20}$$

$$= \frac{-17-i}{10} = -\frac{17}{10} - \frac{1}{10}i$$

85. $\frac{13i}{-5-i} = \frac{13i}{-5-i} \cdot \frac{-5+i}{-5+i}$

$$= \frac{13i(-5)+13i \cdot i}{(-5)^2 - i^2}$$

$$= \frac{-65i+13i^2}{25-(-1)} = \frac{-65i+13(-1)}{25+1}$$

$$= \frac{-13-65i}{26} = \frac{13(-1-5i)}{26}$$

$$= \frac{-1-5i}{2} = -\frac{1}{2} - \frac{5}{2}i$$

87. $\frac{2+3i}{6i} = \frac{2+3i}{6i} \cdot \frac{-6i}{-6i}$

$$= \frac{-12i-18i^2}{-36i^2}$$

$$= \frac{-12i-18(-1)}{-36(-1)} = \frac{18-12i}{36}$$

$$= \frac{6(3-2i)}{36} = \frac{3-2i}{6}$$

$$= \frac{1}{2} - \frac{1}{3}i$$

89. $\dfrac{-10+i}{i} = \dfrac{-10+i}{i} \cdot \dfrac{-i}{-i}$

$= \dfrac{10i - i^2}{-i^2}$

$= \dfrac{10i-(-1)}{-(-1)} = \dfrac{1+10i}{1}$

$= 1+10i$

91. $\dfrac{2+\sqrt{-16}}{8} = \dfrac{2+4i}{8} = \dfrac{2(1+2i)}{8}$

$= \dfrac{1+2i}{4}$

$= \dfrac{1}{4} + \dfrac{1}{2}i$

93. $\dfrac{-6+\sqrt{-72}}{6} = \dfrac{-6+\sqrt{-36\cdot 2}}{6}$

$= \dfrac{-6+6i\sqrt{2}}{6}$

$= \dfrac{6\left(-1+i\sqrt{2}\right)}{6}$

$= -1+i\sqrt{2}$

95. $\dfrac{-8-\sqrt{-48}}{4} = \dfrac{-8-\sqrt{-16\cdot 3}}{4}$

$= \dfrac{-8-4i\sqrt{3}}{4}$

$= \dfrac{4\left(-2-i\sqrt{3}\right)}{4}$

$= -2-i\sqrt{3}$

97. $\dfrac{-5+\sqrt{-50}}{10} = \dfrac{-5+\sqrt{-25\cdot 2}}{10}$

$= \dfrac{-5+5i\sqrt{2}}{10} = \dfrac{5\left(-1+i\sqrt{2}\right)}{5\cdot 2} = -\dfrac{1}{2} + \dfrac{\sqrt{2}}{2}i$

99. $x^2 - 4x + 5 = 0 \qquad x = 2+i$

$(2+i)^2 - 4(2+i) + 5 = 0$

$4 + 4i + i^2 - 8 - 4i + 5 = 0$

$4 + 4i - 1 - 8 - 4i + 5 = 0$

$0 = 0$

$2 + i$ is a solution.

101. $x^2 + 12 = 0 \qquad x = -2i\sqrt{3}$

$\left(-2i\sqrt{3}\right)^2 + 12 = 0$

$4i^2 \cdot 3 + 12 = 0$

$-12 + 12 = 0$

$0 = 0$

$-2i\sqrt{3}$ is a solution.

Section 6.9 Practice Exercises

1. **a.** $\sqrt{(x_2 - x_1)^2 + (y_2 - xy_1)^2}$

b. circle; center

c. radius

d. $(x-h)^2 + (y-k)^2 = r^2$

e. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

3. $(x_1, y_1) = (1, 10), \quad (x_2, y_2) = (-2, 4)$

$$d = \sqrt{[(-2)-(1)]^2 + [(4)-(10)]^2}$$
$$= \sqrt{(-3)^2 + (-6)^2} = \sqrt{9+36}$$
$$= \sqrt{45} = 3\sqrt{5}$$

5. $(x_1, y_1) = (6, 7), \quad (x_2, y_2) = (3, 2)$

$$d = \sqrt{[(3)-(6)]^2 + [(2)-(7)]^2}$$
$$= \sqrt{(-3)^2 + (-5)^2} = \sqrt{9+25}$$
$$= \sqrt{34}$$

7. $(x_1, y_1) = \left(-\frac{1}{2}, \frac{5}{8}\right), \quad (x_2, y_2) = \left(-\frac{3}{2}, \frac{1}{4}\right)$

$$d = \sqrt{\left[\left(-\frac{3}{2}\right) - \left(-\frac{1}{2}\right)\right]^2 + \left[\left(\frac{1}{4}\right) - \left(\frac{5}{8}\right)\right]^2}$$
$$= \sqrt{(-1)^2 + \left(-\frac{3}{8}\right)^2}$$
$$= \sqrt{1 + \frac{9}{64}}$$
$$= \sqrt{\frac{73}{64}} = \frac{\sqrt{73}}{8}$$

9. $(x_1, y_1) = (-2, 5), \quad (x_2, y_2) = (-2, 9)$

$$d = \sqrt{[(-2)-(-2)]^2 + [(9)-(5)]^2}$$
$$= \sqrt{(0)^2 + (4)^2}$$
$$= \sqrt{0+16}$$
$$= \sqrt{16}$$
$$= 4$$

11. $(x_1, y_1) = (7, 2), \quad (x_2, y_2) = (15, 2)$

$$d = \sqrt{[(15)-(7)]^2 + [(2)-(2)]^2}$$
$$= \sqrt{(8)^2 + (0)^2} = \sqrt{64+0} = \sqrt{64} = 8$$

13. $(x_1, y_1) = (-1, -5), \quad (x_2, y_2) = (-5, -9)$

$$d = \sqrt{[(-5)-(-1)]^2 + [(-9)-(-5)]^2}$$
$$= \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$$
$$= 4\sqrt{2}$$

15. $(x_1, y_1) = \left(4\sqrt{6}, -2\sqrt{2}\right),\ (x_2, y_2)$
$= \left(2\sqrt{6}, \sqrt{2}\right)$

$$d = \sqrt{\left[2\sqrt{6} - 4\sqrt{6}\right]^2 + \left[\left(\sqrt{2}\right) - \left(-2\sqrt{2}\right)\right]^2}$$
$$= \sqrt{\left(-2\sqrt{6}\right)^2 + \left(3\sqrt{2}\right)^2} = \sqrt{24 + 18} = \sqrt{42}$$

17. Subtract 5 and –7.
This becomes $5 - (-7) = 12$.

19. $(4, 7),\ (-4, y)$

$$10 = \sqrt{[-4-4]^2 + [y-7]^2}$$
$$10 = \sqrt{(-8)^2 + y^2 - 14y + 49}$$
$$10 = \sqrt{y^2 - 14y + 49 + 64}$$
$$100 = y^2 - 14y + 113$$
$$0 = y^2 - 14y + 13$$
$$0 = (y - 13)(y - 1)$$
$$y - 13 = 0 \quad \text{or} \quad y - 1 = 0$$
$$y = 13 \quad \text{or} \quad y = 1$$

21. $(x, 2),\ (4, -1)$

$$5 = \sqrt{[4-x]^2 + [-1-2]^2}$$
$$5 = \sqrt{(4-x)^2 + (-3)^2}$$
$$5 = \sqrt{16 - 8x + x^2 + 9}$$
$$25 = x^2 - 8x + 25$$
$$0 = x^2 - 8x$$
$$0 = x(x - 8)$$
$$x = 0 \quad \text{or} \quad x - 8 = 0$$
$$x = 0 \quad \text{or} \quad x = 8$$

23. yes
The three points define a right triangle.

25. $A: (-3, -2), \quad B: (4, -3), \quad C: (1, 5)$

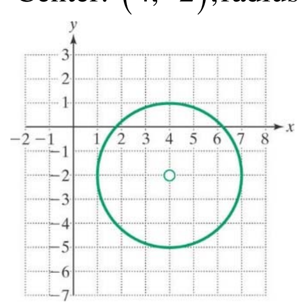
$$d_{AB} = \sqrt{[4 - (-3)]^2 + [-3 - (-2)]^2}$$
$$= \sqrt{(7)^2 + (-1)^2} = \sqrt{49 + 1} = \sqrt{50}$$
$$d_{BC} = \sqrt{[1-4]^2 + [5 - (-3)]^2}$$
$$= \sqrt{(-3)^2 + (8)^2} = \sqrt{9 + 64} = \sqrt{73}$$
$$d_{AC} = \sqrt{[1 - (-3)]^2 + [5 - (-2)]^2}$$
$$= \sqrt{(4)^2 + (7)^2} = \sqrt{16 + 49} = \sqrt{65}$$
$$\left(\sqrt{50}\right)^2 + \left(\sqrt{65}\right)^2 = \left(\sqrt{73}\right)^2$$
$$50 + 65 = 73$$
$$115 \neq 73$$

The three points do not define a right triangle

27. $(x-4)^2 + (y+2)^2 = 9$

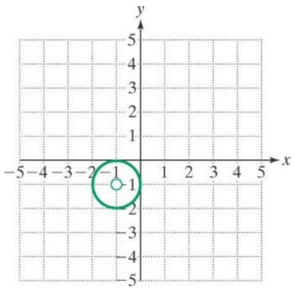
$$[x-4]^2 + [y - (-2)]^2 = 3^2$$
$$h = 4, k = -2, r = 3$$

Center: $(4, -2)$; radius: 3



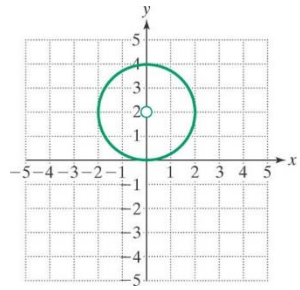
29.

$$(x+1)^2+(y+1)^2=1$$
$$[x-(-1)]^2+[y-(-1)]^2=1^2$$
$$h=-1, k=-1, r=1$$
Center: $(-1,-1)$; radius: 1



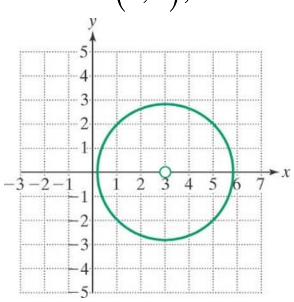
31.

$$x^2+(y-2)^2=4$$
$$[x-0]^2+[y-2]^2=2^2$$
$$h=0, k=2, r=2$$
Center: $(0,2)$; radius: 2



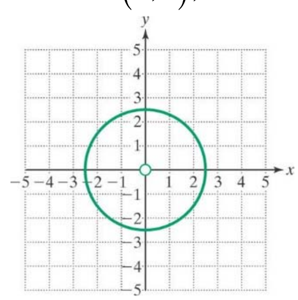
33.

$$(x-3)^2+y^2=8$$
$$[x-3]^2+[y-0]^2=(\sqrt{8})^2$$
$$h=3, k=0, r=2\sqrt{2}$$
Center: $(3,0)$; radius: $2\sqrt{2}$



35.

$$x^2+y^2=6$$
$$[x-0]^2+[y-0]^2=(\sqrt{6})^2$$
$$h=0, k=0, r=\sqrt{6}$$
Center: $(0,0)$; radius: $\sqrt{6}$

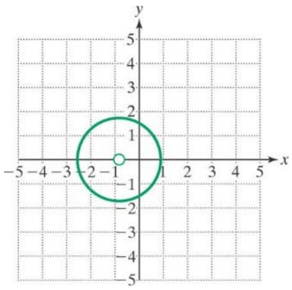


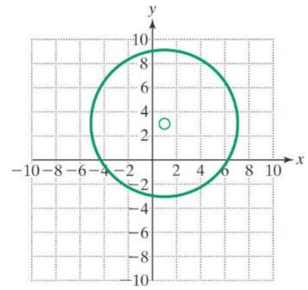
37.

$$\left(x+\frac{4}{5}\right)^2+y^2=\frac{64}{25}$$
$$\left[x-\left(-\frac{4}{5}\right)\right]^2+[y-0]^2=\left(\frac{8}{5}\right)^2$$
$$h=-\frac{4}{5}, k=0, r=\frac{8}{5}$$
Center: $\left(-\frac{4}{5},0\right)$; radius: $\frac{8}{5}$

39.

$$x^2+y^2-2x-6y-26=0$$
$$(x^2-2x\quad)+(y^2-6y\quad)=26$$
$$(x^2-2x+1)+(y^2-6y+9)=26+1+9$$
$$(x-1)^2+(y-3)^2=36$$
$$[x-1]^2+[y-3]^2=6^2$$
Center: $(1,3)$; radius: 6

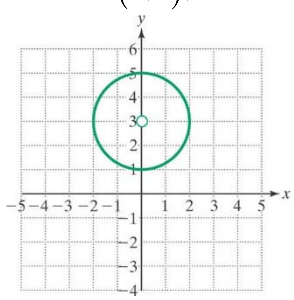




41.

$$x^2+y^2-6y+5=0$$
$$x^2+\left(y^2-6y\quad\right)=-5$$
$$x^2+\left(y^2-6y+9\right)=-5+9$$
$$\left(x-0\right)^2+\left(y-3\right)^2=4$$
$$\left[x-0\right]^2+\left[y-3\right]^2=2^2$$

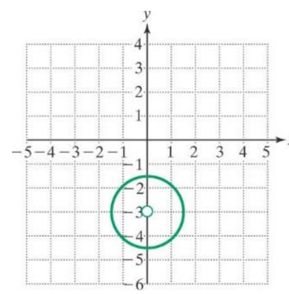
Center: $(0,3)$; radius: 2



43.

$$x^2+y^2+6y+\frac{65}{9}=0$$
$$x^2+\left(y^2+6y\quad\right)=-\frac{65}{9}$$
$$x^2+\left(y^2+6y+9\right)=-\frac{65}{9}+9$$
$$\left(x-0\right)^2+\left(y+3\right)^2=\frac{16}{9}$$
$$\left[x-0\right]^2+\left[y-(-3)\right]^2=\left(\frac{4}{3}\right)^2$$

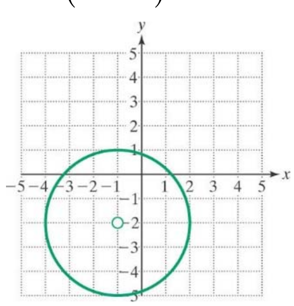
Center: $(0,-3)$; radius: $\frac{4}{3}$



45.

$$x^2+y^2+2x+4y-4=0$$
$$\left(x^2+2x\quad\right)+\left(y^2+4y\right)=4$$
$$\left(x^2+2x+1\right)+\left(y^2+4y+4\right)=4+1+4$$
$$\left(x+1\right)^2+\left(y+2\right)^2=9$$
$$\left[x-(-1)\right]^2+\left[y-(-2)\right]^2=3^2$$

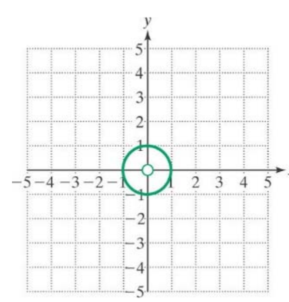
Center: $(-1,-2)$; radius: 3



47.

$$3x^2+3y^2=3$$
$$x^2+y^2=1$$
$$\left[x-0\right]^2+\left[y-0\right]^2=1^2$$

Center: $(0,0)$; radius: 1



49. Center: $(0,0)$; radius: 2

$h=0, k=0, r=2$

$$[x-(0)]^2+[y-(0)]^2=2^2$$
$$x^2+y^2=4$$

51. Center: $(0,2)$; radius: 2

$h=0, k=2, r=2$

$$[x-(0)]^2+[y-(2)]^2=2^2$$
$$x^2+(y-2)^2=4$$

53. Center: $(-2,2)$; radius: 3

$h=-2, k=2, r=3$

$$[x-(-2)]^2+[y-(2)]^2=3^2$$
$$(x+2)^2+(y-2)^2=9$$

55. Center: $(0,0)$; radius: 7

$h=0, k=0, r=7$

$$[x-(0)]^2+[y-(0)]^2=7^2$$
$$x^2+y^2=49$$

57. Center: $(-3,-4)$; diameter: 12

$h=-3, k=-4, r=6$

$$[x-(-3)]^2+[y-(-4)]^2=6^2$$
$$(x+3)^2+(y+4)^2=36$$

59. Center: $(5,3)$; radius: 1.5

$h=5,\ k=3,\ r=1.5$

$$(x-5)^2+(y-3)^2=1.5^2$$
$$(x-5)^2+(y-3)^2=2.25$$

61. $$\left(\frac{4+(-2)}{2},\frac{3+1}{2}\right)$$
$$=\left(\frac{2}{2},\frac{4}{2}\right)$$
$$=(1,2)$$

63. $$\left(\frac{-4+2}{2},\frac{-2+2}{2}\right)=\left(\frac{-2}{2},\frac{0}{2}\right)$$
$$=(-1,0)$$

65. $$\left(\frac{4+(-6)}{2},\frac{0+12}{2}\right)=\left(\frac{-2}{2},\frac{12}{2}\right)$$
$$=(-1,6)$$

67. $$\left(\frac{-3+3}{2},\frac{8+(-2)}{2}\right)=\left(\frac{0}{2},\frac{6}{2}\right)$$
$$=(0,3)$$

69. $\left(\frac{5+(-6)}{2},\frac{2+1}{2}\right)=\left(\frac{-1}{2},\frac{3}{2}\right)=\left(-\frac{1}{2},\frac{3}{2}\right)$

71. $\left(\frac{-2.4+1.6}{2},\frac{-3.1+1.1}{2}\right)=\left(\frac{-0.8}{2},\frac{-2}{2}\right)$
$=\left(-0.4,-1\right)$

73. $\left(x_1,y_1\right)=\left(30,20\right),\ \left(x_2,y_2\right)=\left(50,-5\right)$
$\left(\frac{30+50}{2},\frac{20+(-5)}{2}\right)=\left(\frac{80}{2},\frac{15}{2}\right)=\left(40,7.5\right)$
They should meet 40 miles east and 7.5 miles north of the warehouse.

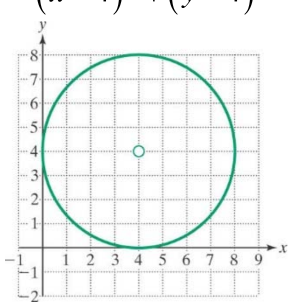
75. a. Midpoint: $\left(\frac{-1+3}{2},\frac{2+4}{2}\right)=\left(1,3\right)$
Center: $\left(1,3\right)$

b. $d=\sqrt{\left[3-(-1)\right]^2+\left(4-2\right)^2}=\sqrt{4^2+2^2}$
$=2\sqrt{5}$
$r=\frac{d}{2}=\sqrt{5}$
$h=1, k=3, r=\sqrt{5}$
$\left(x-1\right)^2+\left(y-3\right)^2=\left(\sqrt{5}\right)^2$
$\left(x-1\right)^2+\left(y-3\right)^2=5$

77. a. Midpoint: $\left(\frac{-2+2}{2},\frac{3+3}{2}\right)=\left(0,3\right)$
Center: $\left(0,3\right)$

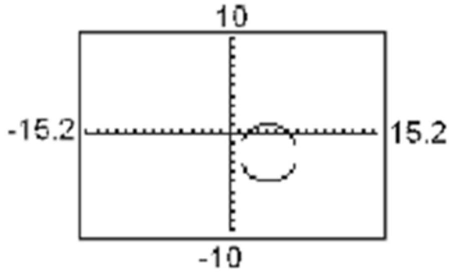
b. $d=\sqrt{\left[2-(-2)\right]^2+\left(3-3\right)^2}=\sqrt{4^2+0^2}$
$=4$
$r=\frac{d}{2}=2$
$h=0, k=3, r=2$
$\left(x-0\right)^2+\left(y-3\right)^2=2^2$
$x^2+\left(y-3\right)^2=4$

79. Center: $\left(4,4\right)$; tangent to x- and y-axes.
$h=4, k=4, r=4$
$\left[x-(4)\right]^2+\left[y-(4)\right]^2=4^2$
$\left(x-4\right)^2+\left(y-4\right)^2=16$

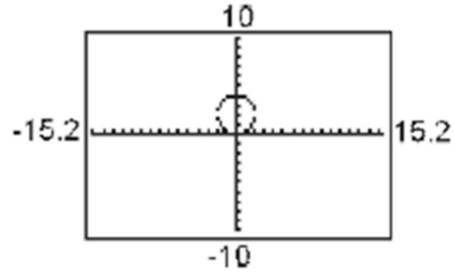


81. $d=\sqrt{\left(-4-1\right)^2+\left(3-1\right)^2}$
$=\sqrt{\left(-5\right)^2+2^2}=\sqrt{25+4}=\sqrt{29}$
Center: $\left(1,1\right)$; radius: $\sqrt{29}$
$h=1, k=1, r=\sqrt{29}$
$\left[x-(1)\right]^2+\left[y-(1)\right]^2=\left(\sqrt{29}\right)^2$
$\left(x-1\right)^2+\left(y-1\right)^2=29$

83.



85.



87.