Section 7.1 Practice Exercises

1. **a.** 0; 0

b. 0

c. $\sqrt{k}\ ;\ -\sqrt{k}$

d. 2; $\{3,-3\}$

e. completing

f. 100

g. 4; 1

h. 8

3. $y^2 = 4$

$y = \pm\sqrt{4}$

$y = \pm 2 \quad \{\pm 2\}$

5. $k^2 - 7 = 0$

$k^2 = 7$

$k = \pm\sqrt{7} \quad \left\{\pm\sqrt{7}\right\}$

7. $36u^2 = 121$

$u^2 = \frac{121}{36} \Rightarrow u = \pm\sqrt{\frac{121}{36}}$

$u = \pm\frac{11}{6} \quad \left\{\frac{11}{6}, -\frac{11}{6}\right\}$

9. $-2m^2 = 50$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm 5i \quad \{\pm 5i\}$

11. $(q+3)^2 = 4$

$q + 3 = \pm\sqrt{4}$

$q + 3 = \pm 2$

$q = -3 \pm 2$

$q = -1$ or $q = -5$

$\{-1,-5\}$

13. $(2y+3)^2 - 7 = 0$

$(2y+3)^2 = 7$

$2y + 3 = \pm\sqrt{7}$

$2y = -3 \pm \sqrt{7}$

$y = \frac{-3 \pm \sqrt{7}}{2} \quad \left\{\frac{-3 \pm \sqrt{7}}{2}\right\}$

15. $(t+5)^2 = -18$

$t + 5 = \pm\sqrt{-18}$

$t + 5 = \pm 3i\sqrt{2}$

$t = -5 \pm 3i\sqrt{2} \quad \{-5 \pm 3i\sqrt{2}\}$

17. $15 = 4 + 3w^2$

$3w^2 = 11 \Rightarrow w^2 = \frac{11}{3}$

$w = \pm\sqrt{\frac{11}{3}} \cdot \sqrt{\frac{3}{3}}$

$w = \pm\frac{\sqrt{33}}{3} \quad \left\{\pm\frac{\sqrt{33}}{3}\right\}$

19. $\left(m+\frac{4}{5}\right)^2+\frac{3}{25}=0$

$\left(m+\frac{4}{5}\right)^2=-\frac{3}{25}$

$m+\frac{4}{5}=\pm\sqrt{-\frac{3}{25}}=\pm\frac{i\sqrt{3}}{5}$

$x=-\frac{4}{5}\pm\frac{\sqrt{3}}{5}i \quad \left\{-\frac{4}{5}\pm\frac{\sqrt{3}}{5}i\right\}$

21. $-y^2-2=14$

$-y^2=14+2$

$-y^2=16$

$y^2=-16$

$y=\sqrt{-16}$

$y=\pm 4i \qquad \{4i,-4i\}$

23. 1. Factoring and applying the zero product rule.

$x^2-36=0$

$(x+6)(x-6)=0$

$x+6=0 \text{ or } x-6=0$

$x=-6 \text{ or } x=6 \quad \{\pm 6\}$

2. Applying the square root property.

$x^2-36=0$

$x^2=36 \Rightarrow x=\pm\sqrt{36}$

$x=\pm 6 \quad \{\pm 6\}$

25. a. $\sqrt{x}=4$

$\left(\sqrt{x}\right)^2=(4)^2$

$x=16 \quad \{16\}$

b. $x^2=4$

$x=\pm\sqrt{4}$

$x=\pm 2 \quad \{2,-2\}$

27. x^2-6x+n

$n=\left(\frac{1}{2}b\right)^2=\left(\frac{1}{2}\cdot(-6)\right)^2=(-3)^2=9$

$x^2-6x+9=(x-3)^2$

29. t^2+8t+n

$n=\left(\frac{1}{2}b\right)^2=\left(\frac{1}{2}\cdot 8\right)^2=(4)^2=16$

$t^2+8t+16=(t+4)^2$

31. c^2-c+n

$n=\left(\frac{1}{2}b\right)^2$

$=\left(\frac{1}{2}\cdot(-1)\right)^2=\left(-\frac{1}{2}\right)^2=\frac{1}{4}$

$c^2-c+\frac{1}{4}=\left(c-\frac{1}{2}\right)^2$

33. y^2+5y+n

$n=\left(\frac{1}{2}b\right)^2$

$=\left(\frac{1}{2}\cdot 5\right)^2=\left(\frac{5}{2}\right)^2=\frac{25}{4}$

$y^2+5y+\frac{25}{4}=\left(y+\frac{5}{2}\right)^2$

35.
$$b^2+\frac{2}{5}b+n$$
$$n=\left(\frac{1}{2}b\right)^2$$
$$=\left(\frac{1}{2}\cdot\frac{2}{5}\right)^2=\left(\frac{1}{5}\right)^2=\frac{1}{25}$$
$$b^2+\frac{2}{5}b+\frac{1}{25}=\left(b+\frac{1}{5}\right)^2$$

37.
$$p^2-\frac{2}{3}p+n$$
$$n=\left(\frac{1}{2}b\right)^2$$
$$=\left(\frac{1}{2}\cdot\left(-\frac{2}{3}\right)\right)^2=\left(-\frac{1}{3}\right)^2=\frac{1}{9}$$
$$p^2-\frac{2}{3}p+\frac{1}{9}=\left(p-\frac{1}{3}\right)^2$$

39. 1. Divide both sides by a to make the leading coefficient 1.
2. Isolate the variable terms on one side of the equation.
3. Complete the square.
4. Apply the square root property and solve for x.

41.
$$t^2+8t+15=0$$
$$t^2+8t=-15$$
$$t^2+8t+16=-15+16$$
$$(t+4)^2=1$$
$$t+4=\pm\sqrt{1}=\pm1$$
$$t=-4\pm1$$
$$t=-3 \text{ or } t=-5 \quad \{-3,-5\}$$

43.
$$x^2+6x=-16$$
$$x^2+6x+9=-16+9$$
$$(x+3)^2=-7$$
$$x+3=\pm\sqrt{-7}$$
$$x+3=\pm i\sqrt{7}$$
$$x=-3\pm i\sqrt{7} \quad \{-3\pm i\sqrt{7}\}$$

45.
$$p^2+4p+6=0$$
$$p^2+4p=-6$$
$$p^2+4p+4=-6+4$$
$$(p+2)^2=-2$$
$$p+2=\pm\sqrt{-2}$$
$$p+2=\pm i\sqrt{2}$$
$$p=-2\pm i\sqrt{2} \quad \{-2\pm i\sqrt{2}\}$$

47.
$$-3y-10=-y^2$$
$$y^2-3y-10=0$$
$$y^2-3y=10$$
$$y^2-3y+\frac{9}{4}=10+\frac{9}{4}$$
$$\left(y-\frac{3}{2}\right)^2=\frac{49}{4}$$

49.
$$2a^2+4a+5=0$$
$$\frac{2a^2}{2}+\frac{4a}{2}+\frac{5}{2}=\frac{0}{2}$$
$$a^2+2a+\frac{5}{2}=0$$
$$a^2+2a=-\frac{5}{2}$$
$$a^2+2a+1=-\frac{5}{2}+1$$

$$y-\frac{3}{2}=\pm\sqrt{\frac{49}{4}}$$
$$y-\frac{3}{2}=\pm\frac{7}{2}$$
$$y=\frac{3}{2}\pm\frac{7}{2}$$
$$y=5 \text{ or } y=-2 \quad \{5,-2\}$$

$$(a+1)^2=-\frac{3}{2}$$
$$a+1=\pm\sqrt{-\frac{3}{2}}\cdot\sqrt{\frac{2}{2}}=\pm\frac{i\sqrt{6}}{2}$$
$$a=-1\pm\frac{\sqrt{6}}{2}i \quad \left\{-1\pm\frac{\sqrt{6}}{2}i\right\}$$

51. $9x^2-36x+40=0$
$$\frac{9x^2}{9}-\frac{36x}{9}+\frac{40}{9}=\frac{0}{9}$$
$$x^2-4x+\frac{40}{9}=0$$
$$x^2-4x=-\frac{40}{9}$$
$$x^2-4x+4=-\frac{40}{9}+4$$
$$(x-2)^2=-\frac{4}{9}$$
$$x-2=\pm\sqrt{-\frac{4}{9}}$$
$$x-2=\pm\frac{2i}{3}$$
$$x=2\pm\frac{2}{3}i \quad \left\{2\pm\frac{2}{3}i\right\}$$

53. $25p^2-10p=2$
$$p^2-\frac{2}{5}p=\frac{2}{25}$$
$$p^2-\frac{2}{5}p+\frac{1}{25}=\frac{2}{25}+\frac{1}{25}$$
$$\left(p-\frac{1}{5}\right)^2=\frac{3}{25}$$
$$p-\frac{1}{5}=\pm\sqrt{\frac{3}{25}}$$
$$p-\frac{1}{5}=\pm\frac{\sqrt{3}}{5}$$
$$p=\frac{1}{5}\pm\frac{\sqrt{3}}{5}$$
$$\left\{\frac{1}{5}\pm\frac{\sqrt{3}}{5}\right\}$$

55. $(2w+5)(w-1)=2$
$$2w^2+3w-5=2$$
$$2w^2+3w-7=0$$
$$\frac{2w^2}{2}+\frac{3w}{2}-\frac{7}{2}=\frac{0}{2}$$
$$w^2+\frac{3}{2}w-\frac{7}{2}=0$$
$$w^2+\frac{3}{2}w=\frac{7}{2}$$
$$w^2+\frac{3}{2}w+\frac{9}{16}=\frac{7}{2}+\frac{9}{16}$$
$$\left(w+\frac{3}{4}\right)^2=\frac{65}{16}$$
$$w+\frac{3}{4}=\pm\sqrt{\frac{65}{16}}=\pm\frac{\sqrt{65}}{4}$$
$$w=-\frac{3}{4}\pm\frac{\sqrt{65}}{4} \quad \left\{-\frac{3}{4}\pm\frac{\sqrt{65}}{4}\right\}$$

57. $n(n-4)=7$
$$n^2-4n=7$$
$$n^2-4n+4=7+4$$
$$(n-2)^2=11\Rightarrow n-2=\pm\sqrt{11}$$
$$n=2\pm\sqrt{11} \quad \left\{2\pm\sqrt{11}\right\}$$

59. $2x(x+6)=14$

$x(x+6)=7$

$x^2+6x=7$

$x^2+6x+9=7+9$

$(x+3)^2=16$

$x+3=\pm\sqrt{16}$

$x=-3\pm 4$

$x=1$ or $x=-7$ $\quad\{1,-7\}$

61. **a.** $d=16t^2$

$t^2=\dfrac{d}{16}$

$t=\dfrac{\sqrt{d}}{4}$

b. $t=\dfrac{\sqrt{1024}}{4}$

$=\dfrac{32}{4}$

$=8$ sec

63. $A=\pi r^2$ for r

$r^2=\dfrac{A}{\pi}$

$r=\sqrt{\dfrac{A}{\pi}}$ or $r=\dfrac{\sqrt{A\pi}}{\pi}$

65. $a^2+b^2+c^2=d^2$ for a

$a^2=d^2-b^2-c^2$

$a=\sqrt{d^2-b^2-c^2}$

67. $V=\dfrac{1}{3}\pi r^2 h$ for r

$3V=\pi r^2 h$

$r^2=\dfrac{3V}{\pi h}$

$r=\sqrt{\dfrac{3V}{\pi h}}$ or $r=\dfrac{\sqrt{3V\pi h}}{\pi h}$

69. $x^2+x^2=6^2$

$2x^2=36$

$x^2=18$

$x=\sqrt{18}$

$=3\sqrt{2}$ ft ≈ 4.2 ft

The shelf extends 4.2 ft.

71. $x^2=50 \Rightarrow x=\sqrt{50}=5\sqrt{2}\approx 7.1$

The sides are 7.1 in.

73. **a.** $P(x)=-\dfrac{1}{8}x^2+5x$

$-\dfrac{1}{8}x^2+5x=20$

$-8\left(-\dfrac{1}{8}x^2+5x\right)=-8(20)$

$x^2-40x=-160$

$x^2-40x+400=-160+400$

b. Profit increases to a point as more books are produced. Beyond that point, the market is "flooded", and profit decreases. Hence there are two points at which the profit is \$20,000. Producing 4.5 thousand books makes the same

$$(x-20)^2 = 240$$
$$x-20 = \pm\sqrt{240} = 20 \pm \sqrt{240}$$
$$x \approx 20 \pm 15.5$$
$$x \approx 4.5 \text{ or } x \approx 35.5$$

4.5 thousand or 35.5 thousand textbooks are sold for a profit of \$20,000.

profit using fewer resources as producing 35.5 thousand books.

Section 7.2 Practice Exercises

1. **a.** quadratic; $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

b. $ax^2+bx+c=0$

c. 8; –42; –27

d. 7; 97

e. b^2-4ac; discriminant

f. imaginary

g. real

h. less than

3. $$\frac{16-\sqrt{320}}{4} = \frac{16-\sqrt{64\cdot 5}}{4} = \frac{16-8\sqrt{5}}{4}$$
$$= \frac{\cancel{4}\left(4-2\sqrt{5}\right)}{\cancel{4}} = 4-2\sqrt{5}$$

5. $$\frac{14-\sqrt{-147}}{7} = \frac{14-\sqrt{-49\cdot 3}}{7} = \frac{14-7i\sqrt{3}}{7}$$
$$= \frac{\cancel{7}\left(2-i\sqrt{3}\right)}{\cancel{7}} = 2-i\sqrt{3}$$

7. $$2(x-5)+x^2 = 3x(x-4)-2x^2$$
$$2x-10+x^2 = 3x^2-12x-2x^2$$
$$x^2-3x^2+2x^2+2x+12x-10=0$$
$$14x-10=0$$

The equation has degree one, so it is linear.

9. $$x^2+11x-12=0$$
$$a=1, b=11, c=-12$$
$$x = \frac{-(11)\pm\sqrt{(11)^2-4(1)(-12)}}{2(1)}$$
$$= \frac{-11\pm\sqrt{121+48}}{2} = \frac{-11\pm\sqrt{169}}{2} = \frac{-11\pm 13}{2}$$
$$x = \frac{2}{2} = 1 \quad \text{or } x = -\frac{24}{2} = -12 \quad \{1,-12\}$$

11. $$9y^2-2y+5=0$$
$$a=9, b=-2, c=5$$

13. $$12p^2-4p+5=0$$
$$a=12, b=-4, c=5$$

$$y=\frac{-(-2)\pm\sqrt{(-2)^2-4(9)(5)}}{2(9)}$$
$$=\frac{2\pm\sqrt{4-180}}{18}$$
$$=\frac{2\pm\sqrt{-176}}{18}$$
$$=\frac{2\pm4i\sqrt{11}}{18}$$
$$=\frac{\cancel{2}\left(1\pm2i\sqrt{11}\right)}{\cancel{2}\cdot9}=\frac{1\pm2i\sqrt{11}}{9}$$
$$=\frac{1}{9}\pm\frac{2\sqrt{11}}{9}i \quad \left\{\frac{1}{9}\pm\frac{2\sqrt{11}}{9}i\right\}$$

$$p=\frac{-(-4)\pm\sqrt{(-4)^2-4(12)(5)}}{2(12)}$$
$$=\frac{4\pm\sqrt{16-240}}{24}$$
$$=\frac{4\pm\sqrt{-224}}{24}$$
$$=\frac{4\pm4i\sqrt{14}}{24}$$
$$=\frac{\cancel{4}\left(1\pm i\sqrt{14}\right)}{\cancel{4}\cdot6}=\frac{1\pm i\sqrt{14}}{6}$$
$$=\frac{1}{6}\pm\frac{\sqrt{14}}{6}i \qquad \left\{\frac{1}{6}\pm\frac{\sqrt{14}}{6}i\right\}$$

15. $-z^2=-2z-35$

$z^2-2z-35=0$

$a=1,b=-2,c=-35$

$$z=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-35)}}{2(1)}$$
$$=\frac{2\pm\sqrt{4+140}}{2}=\frac{2\pm\sqrt{144}}{2}=\frac{2\pm12}{2}$$
$$z=\frac{14}{2}=7 \quad \text{or } z=-\frac{10}{2}=-5 \quad \{7,-5\}$$

17. $y^2+3y=8$

$y^2+3y-8=0$

$a=1,b=3,c=-8$

$$y=\frac{-(3)\pm\sqrt{(3)^2-4(1)(-8)}}{2(1)}$$
$$=\frac{-3\pm\sqrt{9+32}}{2}$$
$$=\frac{-3\pm\sqrt{41}}{2} \qquad \left\{\frac{-3\pm\sqrt{41}}{2}\right\}$$

19. $25x^2-20x+4=0$

$a=25,b=-20,c=4$

$$x=\frac{-(-20)\pm\sqrt{(-20)^2-4(25)(4)}}{2(25)}$$
$$=\frac{20\pm\sqrt{400-400}}{50}$$
$$=\frac{20\pm\sqrt{0}}{50}=\frac{20}{50}$$
$$=\frac{2}{5} \quad \left\{\frac{2}{5}\right\}$$

21. $w(w-6)=-14$

$w^2-6w+14=0$

$a=1,b=-6,c=14$

$$w=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(14)}}{2(1)}$$
$$=\frac{6\pm\sqrt{36-56}}{2}=\frac{6\pm\sqrt{-20}}{2}$$
$$=\frac{6\pm2i\sqrt{5}}{2}=\frac{\cancel{2}\left(3\pm i\sqrt{5}\right)}{\cancel{2}}$$
$$=3\pm i\sqrt{5} \quad \left\{3\pm i\sqrt{5}\right\}$$

23. $(x+2)(x-3)=1$

$x^2-x-6=1$

$x^2-x-7=0$

$a=1, b=-1, c=-7$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-7)}}{2(1)}$$

$$=\frac{1\pm\sqrt{1+28}}{2}$$

$$=\frac{1\pm\sqrt{29}}{2} \quad \left\{\frac{1\pm\sqrt{29}}{2}\right\}$$

25. $-4a^2-2a+3=0$

$a=-4, b=-2, c=3$

$$a=\frac{-(-2)\pm\sqrt{(-2)^2-4(-4)(3)}}{2(-4)}$$

$$=\frac{2\pm\sqrt{4+48}}{-8}=\frac{2\pm\sqrt{52}}{-8}=\frac{2\pm 2\sqrt{13}}{-8}$$

$$=\frac{\cancel{2}\left(1\pm\sqrt{13}\right)}{\cancel{2}(-4)}=\frac{1\pm\sqrt{13}}{-4}$$

$$=\frac{-1\pm\sqrt{13}}{4} \quad \left\{\frac{-1\pm\sqrt{13}}{4}\right\}$$

27. $\frac{1}{2}y^2+\frac{2}{3}=-\frac{2}{3}y$

$6\left(\frac{1}{2}y^2+\frac{2}{3}\right)=6\left(-\frac{2}{3}y\right)$

$3y^2+4=-4y \Rightarrow 3y^2+4y+4=0$

$a=3, b=4, c=4$

$$y=\frac{-(4)\pm\sqrt{(4)^2-4(3)(4)}}{2(3)}$$

$$=\frac{-4\pm\sqrt{16-48}}{6}=\frac{-4\pm\sqrt{-32}}{6}$$

$$=\frac{-4\pm 4i\sqrt{2}}{6}=-\frac{4}{6}\pm\frac{4\sqrt{2}}{6}i$$

$$=-\frac{2}{3}\pm\frac{2\sqrt{2}}{3}i \quad \left\{-\frac{2}{3}\pm\frac{2\sqrt{2}}{3}i\right\}$$

29. $\frac{1}{5}h^2+h+\frac{3}{5}=0$

$5\left(\frac{1}{5}h^2+h+\frac{3}{5}\right)=5(0)$

$h^2+5h+3=0$

$a=1, b=5, c=3$

$$h=\frac{-(5)\pm\sqrt{(5)^2-4(1)(3)}}{2(1)}$$

$$=\frac{-5\pm\sqrt{25-12}}{2}$$

$$=\frac{-5\pm\sqrt{13}}{2} \quad \left\{\frac{-5\pm\sqrt{13}}{2}\right\}$$

31. $0.01x^2+0.06x+0.08=0$

$100(0.01x^2+0.06x+0.08)=100(0)$

$x^2+6x+8=0$

$a=1, b=6, c=8$

33. $0.3t^2+0.7t-0.5=0$

$10(0.3t^2+0.7t-0.5)=10(0)$

$3t^2+7t-5=0$

$a=3, b=7, c=-5$

$$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(8)}}{2(1)}$$
$$=\frac{-6\pm\sqrt{36-32}}{2}=\frac{-6\pm\sqrt{4}}{2}=\frac{-6\pm2}{2}$$
$$x=\frac{-4}{2}=-2 \quad \text{or} \quad x=\frac{-8}{2}=-4 \quad \{-2,-4\}$$

$$t=\frac{-(7)\pm\sqrt{(7)^2-4(3)(-5)}}{2(3)}$$
$$=\frac{-7\pm\sqrt{49+60}}{6}$$
$$=\frac{-7\pm\sqrt{109}}{6} \quad \left\{\frac{-7\pm\sqrt{109}}{6}\right\}$$

35. a. $x^3-27=x^3-3^3$
$$=(x-3)(x^2+3x+9)$$

b. $x^3-27=0$
$$(x-3)(x^2+3x+9)=0$$
$$x-3=0 \text{ or } x^2+3x+9=0$$
$$x=3 \text{ or } a=1,b=3,c=9$$
$$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(9)}}{2(1)}$$
$$=\frac{-3\pm\sqrt{9-36}}{2}=\frac{-3\pm\sqrt{-27}}{2}$$
$$x=3 \text{ or } x=\frac{-3\pm3i\sqrt{3}}{2}$$
$$\left\{3,-\frac{3}{2}\pm\frac{3\sqrt{3}}{2}i\right\}$$

37. a. $3x^3-6x^2+6x$
$$=3x(x^2-2x+2)$$

b. $3x^3-6x^2+6x=0$
$$(3x)(x^2-2x+2)=0$$
$$3x=0 \text{ or } x^2-2x+2=0$$
$$x=0 \text{ or } a=1,b=-2,c=2$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$$
$$=\frac{2\pm\sqrt{4-8}}{2}$$
$$=\frac{2\pm\sqrt{-4}}{2}$$
$$x=0 \text{ or } x=\frac{2\pm2i}{2}=1\pm i \quad \{0,1\pm i\}$$

39. $s^3=27$
$$s=\sqrt[3]{27}=3$$
The length of a side of the cube is 3 ft.

41. Let x = one leg of the triangle
$x+2$ = the other leg of the triangle
$$x^2+(x+2)^2=4^2$$
$$x^2+x^2+4x+4=16$$
$$2x^2+4x-12=0$$
$$a=2,b=4,c=-12$$

43. Let x = one leg of the triangle $x-2.1$ = the other leg of the triangle
$$x^2+(x-2.1)^2=10.2^2$$
$$x^2+x^2-4.2x+4.41=104.04$$
$$2x^2-4.2x-99.63=0$$
$$100(2x^2-4.2x-99.63)=100(0)$$
$$200x^2-420x-9963=0$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-12)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{16 + 96}}{4}$$

$$= \frac{-4 \pm \sqrt{112}}{4}$$

$$= \frac{-4 \pm 10.58}{4}$$

$x \approx 1.6$ or $x \approx 3.6$

The legs are 1.6 in and 3.6 in.

$a = 200, b = -420, c = -9963$

$$x = \frac{-(-420) \pm \sqrt{(-420)^2 - 4(200)(-9963)}}{2(200)}$$

$$x = \frac{420 \pm \sqrt{176,400 + 7,970,400}}{400}$$

$$= \frac{420 \pm \sqrt{8,146,800}}{400} = \frac{420 \pm 2854.259974}{400}$$

$x \approx 8.2$ or $x \approx -6.1$

$x - 2.1 = 8.2 - 2.1 \approx 6.1$

The legs are 8.2 m and 6.1 m.

45. $F(x) = 0.0036x^2 - 0.35x + 9.2$

a. $F(x) = 0.0036(16)^2 - 0.35(16) + 9.2 = 4.5216 \approx 4.5$

The fatality rate is approximately 4.5 fatalities per 100 million miles driven.

b. $F(x) = 0.0036(40)^2 - 0.35(40) + 9.2 = 0.96 \approx 1$

The fatality rate is approximately 1 fatality per 100 million miles driven.

c. $F(x) = 0.0036(80)^2 - 0.35(80) + 9.2 = 4.24 \approx 4.2$

The fatality rate is approximately 4.2 fatalities per 100 million miles driven.

d. $0.0036x^2 - 0.35x + 9.2 = 2.5$

$0.0036x^2 - 0.35x + 6.7 = 0$

$a = 0.0036, b = -0.35, c = 6.7$

$$t = \frac{-(-0.35) \pm \sqrt{(-0.35)^2 - 4(0.0036)(6.7)}}{2(0.0036)} = \frac{0.35 \pm \sqrt{0.02602}}{0.0072} = \frac{0.35 \pm 0.1613}{0.0072}$$

$t \approx 71$ or $t \approx 26$

The fatality rate is 2.5 for drivers 26 years old or 71 years old.

47. $h(t) = -16t^2 + 48t + 48$

$64 = -16t^2 + 48t + 48$

$16t^2 - 48t + 16 = 0$

$t^2 - 3t + 1 = 0$

$a = 1, b = -3, c = 1$

$$t = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2}$$

$t = \dfrac{3+\sqrt{5}}{2} \approx 2.62$ sec or $t = \dfrac{3-\sqrt{5}}{2} \approx 0.38$ sec

49. **a.** $x^2 + 2x = -1$

$x^2 + 2x + 1 = 0$

b. $b^2 - 4ac = 2^2 - 4(1)(1)$

$= 4 - 4 = 0$

c. 1 rational solution

51. **a.** $19m^2 = 8m$

$19m^2 - 8m + 0 = 0$

b. $b^2 - 4ac = (-8)^2 - 4(19)(0)$

$= 64 - 0 = 64$

c. 2 rational solutions

53. **a.** $5p^2 - 21 = 0$

$5p^2 + 0p - 21 = 0$

b. $b^2 - 4ac = (0)^2 - 4(5)(-21)$

$= 0 + 420 = 420$

c. 2 irrational solutions

55. **a.** $4n(n-2) - 5n(n-1) = 4$

$4n^2 - 8n - 5n^2 + 5n = 4$

$-n^2 - 3n - 4 = 0$

$n^2 + 3n + 4 = 0$

b. $b^2 - 4ac = (3)^2 - 4(1)(4)$

$= 9 - 16 = -7$

c. 2 imaginary solutions

57. $f(x) = x^2 - 6x + 5$

$b^2 - 4ac = (-6)^2 - 4(1)(5)$

$= 36 - 20$

$= 16$

two x-intercepts

59. $h(x) = 4x^2 + 12x + 9$

$b^2 - 4ac = (12)^2 - 4(4)(9)$

$= 144 - 144$

$= 0$

one x-intercept

61. $p(x) = 2x^2 + 3x + 6$

$b^2 - 4ac = (3)^2 - 4(2)(6)$

$= 9 - 48$

$= -39$

no x-intercepts

63. $f(x) = x^2 - 5x + 3 = 0$

$a = 1, b = -5, c = 3$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$

$= \dfrac{5 \pm \sqrt{25-12}}{2} \quad = \dfrac{5 \pm \sqrt{13}}{2}$

x-intercepts: $\left(\dfrac{5+\sqrt{13}}{2}, 0\right), \left(\dfrac{5-\sqrt{13}}{2}, 0\right)$

$f(0) = 0^2 - 5(0) + 3 = 3$

y-intercept: $(0,3)$

65. $g(x) = -x^2 + x - 1 = 0 \partial y$

$a = -1, b = 1, c = -1$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(-1)(-1)}}{2(-1)}$$

$$= \frac{-1 \pm \sqrt{1-4}}{-2}$$

$$= \frac{-1 \pm \sqrt{-3}}{-2}$$

$$= \frac{-1 \pm i\sqrt{3}}{-2}$$

x-intercepts: none - solutions imaginary

$$g(0) = -(0)^2 + 0 - 1$$

$$= -1$$

y-intercept: $(0, -1)$

67. $p(x) = 2x^2 + 5x - 2 = 0$

$a = 2, b = 5, c = -2$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-2)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25+16}}{4}$$

$$= \frac{-5 \pm \sqrt{41}}{4}$$

x-intercepts: $\left(\frac{-5+\sqrt{41}}{4}, 0\right), \left(\frac{-5-\sqrt{41}}{4}, 0\right)$

$$p(0) = 2(0)^2 + 5(0) - 2$$

$$= -2$$

y-intercept: $(0, -2)$

69. $a^2 + 3a + 4 = 0$

$a = 1, b = 3, c = 4$

$$a = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9-16}}{2} = \frac{-3 \pm \sqrt{-7}}{2}$$

$$= \frac{-3 \pm i\sqrt{7}}{2} \quad \left\{-\frac{3}{2} \pm \frac{\sqrt{7}}{2}i\right\}$$

71. $(x-2)^2 + 2x^2 - 13x = 10$

$x^2 - 4x + 4 + 2x^2 - 13x - 10 = 0$

$3x^2 - 17x - 6 = 0$

$(3x+1)(x-6) = 0$

$3x + 1 = 0$ or $x - 6 = 0$

$3x = -1$ or $x = 6$

$x = -\frac{1}{3}$ or $x = 6$ $\quad \left\{-\frac{1}{3}, 6\right\}$

73. $4y^2 - 20y + 43 = 0$

$a = 4, b = -20, c = 43$

$$y = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(43)}}{2(4)}$$

$$= \frac{20 \pm \sqrt{400-688}}{8} = \frac{20 \pm \sqrt{-288}}{8}$$

$$= \frac{20 \pm \sqrt{-144(2)}}{8} = \frac{20 \pm 12i\sqrt{2}}{8}$$

$$= \frac{5}{2} \pm \frac{3\sqrt{2}}{2}i \quad \left\{\frac{5}{2} \pm \frac{3\sqrt{2}}{2}i\right\}$$

75. $\left(x + \frac{1}{2}\right)^2 + 4 = 0$

$\left(x + \frac{1}{2}\right)^2 = -4$

$x + \frac{1}{2} = \pm\sqrt{-4}$

$x + \frac{1}{2} = \pm 2i$

$x = -\frac{1}{2} \pm 2i \quad \left\{-\frac{1}{2} \pm 2i\right\}$

77. $2y(y-3) = -1\partial y$

$2y^2 - 6y = -1$

$2y^2 - 6y + 1 = 0$

$a = 2, b = -6, c = 1$

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{36-8}}{4} = \frac{6 \pm \sqrt{28}}{4} = \frac{6 \pm 2\sqrt{7}}{4}$$

$$= \frac{3 \pm \sqrt{7}}{2} \quad \left\{\frac{3 \pm \sqrt{7}}{2}\right\}$$

79. $(2t+5)(t-1) = (t-3)(t+8)$

$2t^2 + 3t - 5 = t^2 + 5t - 24$

$t^2 - 2t + 19 = 0$

$a = 1, b = -2, c = 19$

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(19)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-76}}{2} = \frac{2 \pm \sqrt{-72}}{2} = \frac{2 \pm 6i\sqrt{2}}{2}$$

$= 1 \pm 3i\sqrt{2} \quad \{1 \pm 3i\sqrt{2}\}$

81. $\frac{1}{8}x^2 - \frac{1}{2}x + \frac{1}{4} = 0$

$8\left(\frac{1}{8}x^2 - \frac{1}{2}x + \frac{1}{4}\right) = 8(0)$

$x^2 - 4x + 2 = 0$

$x^2 - 4x = -2$

$x^2 - 4x + 4 = -2 + 4$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2} \quad \{2 \pm \sqrt{2}\}$

83. $32z^2 - 20z - 3 = 0$

$(8z+1)(4z-3) = 0$

$8z + 1 = 0$ or $4z - 3 = 0$

$8z = -1$ or $4z = 3$

$z = -\frac{1}{8}$ or $z = \frac{3}{4} \quad \left\{-\frac{1}{8}, \frac{3}{4}\right\}$

85. $4p^2 - 21 = 0$

$4p^2 = 21$

$p^2 = \frac{21}{4}$

$p = \pm\sqrt{\frac{21}{4}} = \pm\frac{\sqrt{21}}{2} \quad \left\{\pm\frac{\sqrt{21}}{2}\right\}$

87. a. $x^2 + 6x = 5$

$x^2 + 6x + 9 = 5 + 9$

$(x+3)^2 = 14$

$x + 3 = \pm\sqrt{14}$

$x = -3 \pm \sqrt{14}$

$\{-3 \pm \sqrt{14}\}$

b. $x^2 + 6x = 5$

$x^2 + 6x - 5 = 0$

$a = 1, b = 6, c = -5$

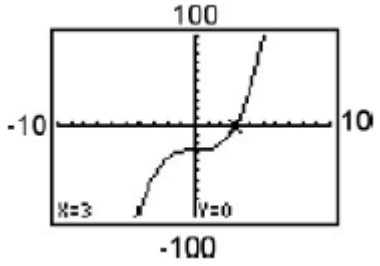
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-5)}}{2(1)} = \frac{-6 \pm \sqrt{36+20}}{2}$$

$$= \frac{-6 \pm \sqrt{56}}{2} = \frac{-6 \pm 2\sqrt{14}}{2} = \frac{2\left(-3 \pm \sqrt{14}\right)}{2}$$

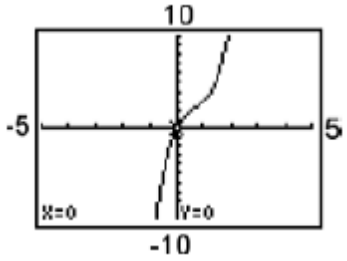
$= -3 \pm \sqrt{14} \quad \{-3 \pm \sqrt{14}\}$

c. Answers will vary.

89.



91.



Section 7.3 Practice Exercises

1. **a** quadratic

b $3x-1$

c $p^{1/3}$

3.
$$\left(x-\frac{3}{2}\right)^2=\frac{7}{4}$$
$$x-\frac{3}{2}=\pm\sqrt{\frac{7}{4}}$$
$$x=\frac{3}{2}\pm\frac{\sqrt{7}}{2} \quad \left\{\frac{3}{2}\pm\frac{\sqrt{7}}{2}\right\}$$

5.
$$x(x+8)=-16$$
$$x^2+8x=-16$$
$$x^2+8x+16=0$$
$$(x+4)^2=0$$
$$x+4=0$$
$$x=-4 \quad \{-4\}$$

7.
$$2x^2-8x-44=0$$
$$a=2, b=-8, c=-44$$
$$k=\frac{-(-8)\pm\sqrt{(-8)^2-4(2)(-44)}}{2(2)}$$
$$=\frac{8\pm\sqrt{64+352}}{4}$$
$$=\frac{8\pm\sqrt{416}}{4}$$
$$=\frac{8\pm4\sqrt{26}}{4}$$
$$=2\pm\sqrt{26} \quad \{2\pm\sqrt{26}\}$$

9. **a.**
$$u^2+10u+24=0$$
$$(u+6)(u+4)=0$$
$$u+6=0 \text{ or } u+4=0$$
$$u=-6 \text{ or } u=-4 \quad \{-6,-4\}$$

b.
$$(y^2+5y)^2+10(y^2+5y)+24=0$$
Let $u=y^2+5y$
$$u^2+10u+24=0 \Rightarrow (u+6)(u+4)=0$$
$$u+6=0 \text{ or } u+4=0$$
$$u=-6 \text{ or } u=-4$$
$$y^2+5y=-6 \text{ or } y^2+5y=-4$$

$y^2+5y+6=0$ or $y^2+5y+4=0$

$(y+3)(y+2)=0$ or $(y+4)(y+1)=0$

$y+3=0$ or $y+2=0$ or $y+4=0$ or $y+1=0$

$y=-3$ or $y=-2$ or $y=-4$ or $y=-1$

$\{-3,-2,-4,-1\}$

11. $(x^2-2x)^2+2(x^2-2x)=3$

Let $u=x^2-2x$

$u^2+2u=3$

$u^2+2u-3=0$

$(u+3)(u-1)=0$

$u+3=0$ or $u-1=0$

$x^2-2x+3=0$ or $x^2-2x-1=0$

$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(3)}}{2(1)}$

or $x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

$x=\dfrac{2\pm\sqrt{-8}}{2}$ or $x=\dfrac{2\pm\sqrt{8}}{2}$

$x=\dfrac{2\pm 2i\sqrt{2}}{2}$ or $x=\dfrac{2\pm 2\sqrt{2}}{2}$

$x=1\pm i\sqrt{2}$ or $x=1\pm\sqrt{2}$ $\{1\pm\sqrt{2},1\pm i\sqrt{2}\}$

13. $(y^2-4y)^2-(y^2-4y)=20$

Let $u=y^2-4y$

$u^2-u=20$

$u^2-u-20=0$

$(u+4)(u-5)=0$

$u+4=0$ or $u-5=0$

$y^2-4y+4=0$ or $y^2-4y-5=0$

$(y-2)^2=0$ or $(y-5)(y+1)=0$

$y-2=0$ or $y-5=0$ or $y+1=0$

$y=2$ or $y=5$ or $y=-1$

$\{2,5,-1\}$

15. $m^{2/3}-m^{1/3}-6=0$

$(m^{1/3})^2-m^{1/3}-6=0$ Let $u=m^{1/3}$

$u^2-u-6=0$

$(u-3)(u+2)=0$

$u-3=0$ or $u+2=0$

$u=3$ or $u=-2$

$m^{1/3}=3$ or $m^{1/3}=-2$

$(m^{1/3})^3=3^3$ or $(m^{1/3})^3=(-2)^3$

$m=27$ or $m=-8$ $\{27,-8\}$

17. $2t^{2/5}+7t^{1/5}+3=0$

$2(t^{1/5})^2+7t^{1/5}+3=0$

Let $u=t^{1/5}$

$2u^2+7u+3=0$

$(2u+1)(u+3)=0$

$2u+1=0$ or $u+3=0$

$2u=-1$ or $u=-3$

$u=-\dfrac{1}{2}$ or $u=-3$

$$t^{1/5} = -\frac{1}{2} \text{ or } t^{1/5} = -3$$

$$\left(t^{1/5}\right)^5 = \left(-\frac{1}{2}\right)^5 \text{ or } \left(t^{1/5}\right)^5 = (-3)^5$$

$$t = -\frac{1}{32} \text{ or } t = -24$$

$$\left\{-\frac{1}{32}, -243\right\}$$

19. $y + 6\sqrt{y} = 16$

$\left(\sqrt{y}\right)^2 + 6\sqrt{y} = 16$

Let $u = \sqrt{y}$

$u^2 + 6u = 16$

$u^2 + 6u - 16 = 0$

$(u+8)(u-2) = 0$

$u + 8 = 0$ or $u - 2 = 0$

$u = -8$ or $u = 2$

$\sqrt{y} = -8$ or $\sqrt{y} = 2$

$y = 64$ or $y = 4$

Check:

$y = 64$:	$y = 4$:
$64 + 6\sqrt{64} = 16$	$4 + 6\sqrt{4} = 16$
$64 + 6(8) = 16$	$4 + 6(2) = 16$
$64 + 48 = 16$	$4 + 12 = 16$
$112 \neq 16$	$16 = 16$

Solution: $\{4\}$ ($y = 64$ does not check.)

21. $2x + 3\sqrt{x} - 2 = 0$

$2\left(\sqrt{x}\right)^2 + 3\sqrt{x} - 2 = 0$

Let $u = \sqrt{x}$

$2u^2 + 3u - 2 = 0$

$(2u-1)(u+2) = 0$

$2u - 1 = 0$ or $u + 2 = 0$

$2u = 1$ or $u = -2$

$u = \frac{1}{2}$ or $u = -2$

$\sqrt{x} = \frac{1}{2}$ or $\sqrt{x} = -2$

$x = \frac{1}{4}$ or $x = 4$

Check:

$x = \frac{1}{4}$:	$x = 4$:
$2\left(\frac{1}{4}\right) + 3\sqrt{\frac{1}{4}} - 2 = 0$	$2(4) + 3\sqrt{4} - 2 = 0$
$\frac{1}{2} + 3\left(\frac{1}{2}\right) - 2 = 0$	$8 + 3(2) - 2 = 0$
$\frac{1}{2} + \frac{3}{2} - \frac{4}{2} = 0$	$8 + 6 - 2 = 0$
$0 = 0$	$12 \neq 0$

Solution: $\left\{\frac{1}{4}\right\}$ ($x = 4$ does not check.)

23. $16\left(\frac{x+6}{4}\right)^2+8\left(\frac{x+6}{4}\right)+1=0$

Let $u=\frac{x+6}{4}$

$16u^2+8u+1=0$

$(4u+1)(4u+1)=0$

$4u+1=0 \quad \text{or} \quad 4u+1=0$

$4u=-1 \quad \text{or} \quad 4u=-1$

$4\left(\frac{x+6}{4}\right)=-1 \quad \text{or} \quad 4\left(\frac{x+6}{4}\right)=-1$

$x+6=-1 \quad \text{or} \quad x+6=-1$

$x=-7 \quad \text{or} \quad x=-7 \quad \{-7\}$

25. $x-\sqrt{x}-12=0$

$x-12=\sqrt{x}$

$(x-12)^2=\left(\sqrt{x}\right)^2$

$x^2-24x+144=x$

$x^2-25x+144=0$

$(x-9)(x-16)=0$

$x-9=0 \text{ or } x-16=0$

$x=9 \text{ or } x=16$

Check:

$9-\sqrt{9}-12=0 \quad 16-\sqrt{16}-12=0$

$9-3-12=0 \quad 16-4-12=0$

$-6\neq 0 \quad 0=0$

The solution is $\{16\}$. ($x=9$ does not check.)

Yes, we obtain the same answers.

27. $w^4+4w^2-45=0$

$(w^2+9)(w^2-5)=0$

$w^2+9=0 \text{ or } w^2-5=0$

$w^2=-9 \text{ or } w^2=5$

$w=\pm\sqrt{-9} \text{ or } w=\pm\sqrt{5}$

$w=\pm 3i \text{ or } w=\pm\sqrt{5}$

$\{\pm 3i,\pm\sqrt{5}\}$

29. $y^2\left(4y^2+17\right)=15$

$4y^4+17y^2-15=0$

$(4y^2-3)(y^2+5)=0$

$4y^2-3=0 \text{ or } y^2+5=0$

$4y^2=3 \text{ or } y^2=-5$

$2y=\pm\sqrt{3} \text{ or } y=\pm\sqrt{-5}$

$y=\pm\frac{\sqrt{3}}{2} \text{ or } y=\pm i\sqrt{5}$

$\left\{\pm\frac{\sqrt{3}}{2},\pm i\sqrt{5}\right\}$

31. $1+\frac{5}{x}=-\frac{3}{x^2}$

$x^2\left(1+\frac{5}{x}\right)=x^2\left(-\frac{3}{x^2}\right)$

$x^2+5x=-3$

$x^2+5x+3=0$

33. $\frac{3x}{x+1}-\frac{2}{x-3}=1$

$(x+1)(x-3)\left(\frac{3x}{x+1}-\frac{2}{x-3}\right)=(x+1)(x-3)\cdot 1$

$3x(x-3)-2(x+1)=x^2-2x-3$

$3x^2-9x-2x-2=x^2-2x-3$

$a=1, b=5, c=3$

$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(3)}}{2(1)}$$

$$x=\frac{-5\pm\sqrt{13}}{2} \qquad \left\{\frac{-5\pm\sqrt{13}}{2}\right\}$$

$$2x^2-9x+1=0$$

$a=2, b=-9, c=1$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(1)}}{2(2)}$$

$$x=\frac{9\pm\sqrt{73}}{4} \qquad \left\{\frac{9\pm\sqrt{73}}{4}\right\}$$

35.

$$\frac{x}{2x-1}=\frac{1}{x-2}$$

$$(2x-1)(x-2)\left(\frac{x}{2x-1}\right)=(2x-1)(x-2)\left(\frac{1}{x-2}\right)$$

$$x(x-2)=1(2x-1)$$

$$x^2-2x=2x-1 \Rightarrow x^2-4x=-1$$

$$x^2-4x+4=-1+4$$

$$(x-2)^2=3 \Rightarrow x-2=\pm\sqrt{3}$$

$$x=2\pm\sqrt{3} \quad \left\{2\pm\sqrt{3}\right\}$$

37.

$$x^4-16=0$$

$$(x^2-4)(x^2+4)=0$$

$$x^2-4=0 \text{ or } x^2+4=0$$

$$x^2=4 \text{ or } x^2=-4$$

$$x=\pm\sqrt{4} \text{ or } x=\pm\sqrt{-4}$$

$$x=\pm 2 \text{ or } x=\pm 2i$$

Solutions: $\{2,-2,2i,-2i\}$

39.

$$(4x+5)^2+3(4x+5)+2=0$$

Let $u=4x+5$

$$u^2+3u+2=0$$

$$(u+2)(u+1)=0$$

$$u=-2 \text{ or } u=-1$$

$$4x+5=-2 \text{ or } 4x+5=-1$$

$$4x=-7 \text{ or } 4x=-6$$

$$x=-\frac{7}{4} \text{ or } x=-\frac{6}{4}=-\frac{3}{2} \quad \left\{-\frac{7}{4},-\frac{3}{2}\right\}$$

41.

$$4m^4-9m^2+2=0$$

$$4(m^2)^2-9m^2+2=0$$

Let $u=m^2$

$$4u^2-9u+2=0$$

$$(4u-1)(u-2)=0$$

$$4u-1=0 \text{ or } u-2=0$$

$$4m^2-1=0 \text{ or } m^2-2=0$$

$$4m^2=1 \text{ or } m^2=2$$

$$m^2=\frac{1}{4} \text{ or } m=\pm\sqrt{2}$$

$$m=\pm\frac{1}{2} \text{ or } m=\pm\sqrt{2}$$

$$\left\{\frac{1}{2},-\frac{1}{2},\sqrt{2},-\sqrt{2}\right\}$$

43. $x^6 - 9x^3 + 8 = 0$

$\left(x^3\right)^2 - 9x^3 + 8 = 0$

Let $u = x^3$

$u^2 - 9u + 8 = 0$

$(u-8)(u-1) = 0$

$u - 8 = 0$ or $u - 1 = 0$

$x^3 - 8 = 0$ or $x^3 - 1 = 0$

$(x-2)\left(x^2+2x+4\right) = 0$ or $(x-1)\left(x^2+x+1\right) = 0$

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$ or $x - 1 = 0$ or $x^2 + x + 1 = 0$

$x = 2$ or $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$ or $x = 1$ or $x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$x = 2$ or $x = \dfrac{-2 \pm \sqrt{-12}}{2}$ or $x = 1$ or $x = \dfrac{-1 \pm \sqrt{-3}}{2}$

$x = 2$ or $x = \dfrac{-2 \pm 2i\sqrt{3}}{2}$ or $x = 1$ or $x = \dfrac{-1 \pm i\sqrt{3}}{2}$

$x = 2$ or $x = -1 \pm i\sqrt{3}$ or $x = 1$ or $x = \dfrac{-1 \pm i\sqrt{3}}{2}$ $\left\{2, 1, -1 \pm i\sqrt{3}, -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i\right\}$

45. $\sqrt{x^2 + 20} = 3\sqrt{x}$

$\left(\sqrt{x^2 + 20}\right)^2 = \left(3\sqrt{x}\right)^2$

$x^2 + 20 = 9x$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x - 5 = 0$ or $x - 4 = 0$

$x = 5$ or $x = 4$

Check:

$x = 5$: $\sqrt{5^2 + 20} = 3\sqrt{5}$; $\sqrt{45} = 3\sqrt{5}$; $3\sqrt{5} = 3\sqrt{5}$

$x = 4$: $\sqrt{4^2 + 20} = 3\sqrt{4}$; $\sqrt{36} = 3(2)$; $6 = 6$

Solutions: {5, 4}

47. $\sqrt{4t + 1} = t + 1$

$\left(\sqrt{4t+1}\right)^2 = (t+1)^2$

$4t + 1 = t^2 + 2t + 1$

$t^2 - 2t = 0$

$t(t-2) = 0$

$t = 0$ or $t - 2 = 0$

$t = 0$ or $t = 2$

Check:

$t = 0$: $\sqrt{4(0)+1} = 0 + 1$; $\sqrt{1} = 1$; $1 = 1$

$t = 2$: $\sqrt{4(2)+1} = 2 + 1$; $\sqrt{9} = 3$; $3 = 3$

Solutions: {0, 2}

49. $2\left(\frac{t-4}{3}\right)^2-\left(\frac{t-4}{3}\right)-3=0$

Let $u=\frac{t-4}{3}$

$2u^2-u-3=0 \Rightarrow (2u-3)(u+1)=0$

$2u-3=0 \text{ or } u+1=0$

$2u=3 \text{ or } u=-1$

$u=\frac{3}{2} \text{ or } u=-1$

$\frac{t-4}{3}=\frac{3}{2} \text{ or } \frac{t-4}{3}=-1$

$t-4=\frac{9}{2} \text{ or } t-4=-3$

$t=\frac{17}{2} \text{ or } t=1 \quad \left\{\frac{17}{2},1\right\}$

51. $x^{2/3}+x^{1/3}=20$

$\left(x^{1/3}\right)^2+x^{1/3}-20=0$

Let $u=x^{1/3}$

$u^2+u-20=0$

$(u+5)(u-4)=0$

$u+5=0 \text{ or } u-4=0$

$u=-5 \text{ or } u=4$

$x^{1/3}=-5 \text{ or } x^{1/3}=4$

$\left(x^{1/3}\right)^3=(-5)^3 \text{ or } \left(x^{1/3}\right)^3=(4)^3$

$x=-125 \text{ or } x=64$

$\{-125,64\}$

53. $m^4+2m^2-8=0$

$(m^2+4)(m^2-2)=0$

$m^2+4=0 \text{ or } m^2-2=0$

$m^2=-4 \text{ or } m^2=2$

$m=\pm\sqrt{-4} \text{ or } m=\pm\sqrt{2}$

$m=\pm 2i \text{ or } m=\pm\sqrt{2} \quad \{\pm 2i,\ \pm\sqrt{2}\}$

55. $a^3+16a-a^2-16=0$

$a(a^2+16)-1(a^2+16)=0$

$(a^2+16)(a-1)=0$

$a^2+16=0 \text{ or } a-1=0$

$a^2=-16 \text{ or } a=1$

$a=\pm 4i \text{ or } a=1 \quad \{\pm 4i,1\}$

57. $x^3+5x-4x^2-20=0$

$x(x^2+5)-4(x^2+5)=0$

$(x^2+5)(x-4)=0$

$x^2+5=0 \text{ or } x-4=0$

$x^2=-5 \text{ or } x=4$

$x=\pm i\sqrt{5} \text{ or } x=4$

$\{\pm i\sqrt{5},4\}$

59. $\left(\frac{2}{x-3}\right)^2+8\left(\frac{2}{x-3}\right)+12=0$

$u^2+8u+12=0 \quad \text{Let } u=\frac{2}{x-3}$

$(u+6)(u+2)=0$

$u+6=0 \text{ or } u+2=0$

$u=-6 \text{ or } u=-2$

$\frac{2}{x-3}=-6 \text{ or } \frac{2}{x-3}=-2$

$2=-6(x-3) \text{ or } 2=-2(x-3)$

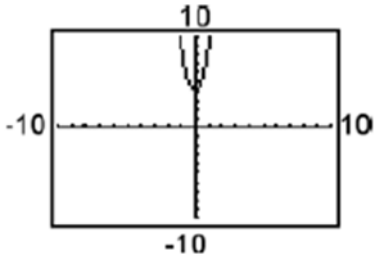
$2=-6x+18 \text{ or } 2=-2x+6$

$$-16 = -6x \qquad \text{or} \quad -4 = -2x$$
$$x = \frac{8}{3} \qquad \text{or} \quad x = 2 \quad \left\{\frac{8}{3}, 2\right\}$$

61. a. $\left\{\pm i\sqrt{2}\right\}$

b. Two imaginary solutions; no real solutions

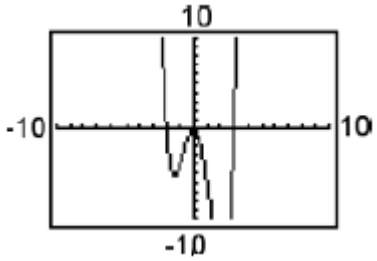
c. No x-intercepts

d. 

63. a. $\{0, 3, -2\}$

b. Three real solutions; no imaginary solutions

c. Three x-intercepts

d. 

Problem Recognition Exercises

1. a.
$$x^2 + 10x + 3 = 0$$
$$x^2 + 10x = -3$$
$$x^2 + 10x + 25 = -3 + 25$$
$$(x+5)^2 = 22$$
$$x + 5 = \pm\sqrt{22}$$
$$x = -5 \pm \sqrt{22} \quad \left\{-5 \pm \sqrt{22}\right\}$$

b.
$$x^2 + 10x + 3 = 0$$
$$a = 1,\ b = 10,\ c = 3$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(3)}}{2(1)}$$
$$= \frac{-10 \pm \sqrt{100 - 12}}{2} = \frac{-10 \pm \sqrt{88}}{2}$$
$$= \frac{-10 \pm 2\sqrt{22}}{2} = -5 \pm \sqrt{22} \quad \left\{-5 \pm \sqrt{22}\right\}$$

3. a.
$$3t^2 + t + 4 = 0$$
$$3t^2 + t = -4$$
$$t^2 + \frac{1}{3}t = -\frac{4}{3}$$
$$t^2 + \frac{1}{3}t + \frac{1}{36} = -\frac{4}{3} + \frac{1}{36}$$
$$\left(t + \frac{1}{6}\right)^2 = -\frac{47}{36}$$

b.
$$3t^2 + t + 4 = 0$$
$$a = 3,\ b = 1,\ c = 4$$
$$t = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(4)}}{2(3)}$$
$$= \frac{-1 \pm \sqrt{1 - 48}}{6}$$
$$= \frac{-1 \pm \sqrt{-47}}{6}$$

$$t+\frac{1}{6}=\pm\frac{i\sqrt{47}}{6}$$

$$t=\frac{-1\pm i\sqrt{47}}{6} \quad \left\{-\frac{1}{6}\pm\frac{\sqrt{47}}{6}i\right\}$$

$$=\frac{-1\pm i\sqrt{47}}{6} \quad \left\{-\frac{1}{6}\pm\frac{\sqrt{47}}{6}i\right\}$$

5. a. Quadratic equation

b. $t^2+5t-14=0$

$(t+7)(t-2)=0$

$t+7=0 \quad \text{or} \quad t-2=0$

$t=-7 \quad \text{or} \quad t=2 \quad \{-7,2\}$

7. a. Quadratic in form

b. $a^4-10a^2+9=0$

Let $t=a^2$

$t^2-10t+9=0$

$(t-9)(t-1)=0$

$t-9=0 \quad \text{or} \quad t-1=0$

$t=9 \quad \text{or} \quad t=1$

$a^2=9 \quad \text{or} \quad a^2=1$

$a=\pm 3 \quad \text{or} \quad a=\pm 1$

$\{-3,3,-1,1\}$

9. a. Quadratic in form

b. $x-3x^{1/2}-4=0$

Let $a=x^{1/2}$

$a^2-3a-4=0$

$(a-4)(a+1)=0$

$a-4=0 \quad \text{or} \quad a+1=0$

$a=4 \quad \text{or} \quad a=-1$

$x^{1/2}=4 \quad \text{or} \quad x^{1/2}=-1$

$x=16 \quad \text{or} \quad x=1$

Check:

$x=16$:

$16-3(16)^{1/2}-4=0$

$16-3(4)-4=0 \Rightarrow 16-12-4=0$

$0=0$

$x=1$:

$1-3(1)^{1/2}-4=0$

$1-3(1)-4=0 \Rightarrow 1-3-4=0$

$-6\neq 0$

Solutions: $\{16\}$ ($x=1$ does not check.)

11. a. Linear equation

b.

13. a. Quadratic equation

b. $5a(a+6)=10(3a-1)$

$5a^2+30a=30a-10$

$$8b(b+1)+2(3b-4)=4b(2b+3)$$
$$8b^2+8b+6b-8=8b^2+12b$$
$$14b-8=12b$$
$$2b=8$$
$$b=4 \quad \{4\}$$

$$5a^2=-10$$
$$a^2=-2$$
$$a=\pm\sqrt{-2}=\pm i\sqrt{2} \quad \left\{\pm i\sqrt{2}\right\}$$

15. a. Rational equation

b. $\dfrac{t}{t+5}+\dfrac{3}{t-4}=\dfrac{17}{t^2+t-20} \Rightarrow \dfrac{t}{t+5}+\dfrac{3}{t-4}=\dfrac{17}{(t+5)(t-4)}$

$$(t+5)(t-4)\left(\frac{t}{t+5}+\frac{3}{t-4}\right)=(t+5)(t-4)\left(\frac{17}{(t+5)(t-4)}\right)$$
$$t(t-4)+3(t+5)=17$$
$$t^2-4t+3t+15=17 \Rightarrow t^2-t-2=0$$
$$(t-2)(t+1)=0$$
$$t-2=0 \text{ or } t+1=0$$
$$t=2 \text{ or } t=-1 \quad \{2,-1\}$$

17. a. Quadratic equation

b. $c^2-20c-1=0$

$a=1,\ b=-20,\ c=-1$

$$c=\frac{-(-20)\pm\sqrt{(-20)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{20\pm\sqrt{400+4}}{2}$$
$$=\frac{20\pm\sqrt{404}}{2}$$
$$=\frac{20\pm 2\sqrt{101}}{2}$$
$$=10\pm\sqrt{101} \quad \left\{10\pm\sqrt{101}\right\}$$

19. a. Quadratic equation

b. $2u(u-3)=4(2-u)$

$$2u^2-6u=8-4u$$
$$2u^2-2u-8=0$$
$$u^2-u-4=0$$

$a=1,\ b=-1,\ c=-4$

$$u=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-4)}}{2(1)}$$
$$=\frac{1\pm\sqrt{1+16}}{2}$$
$$=\frac{1\pm\sqrt{17}}{2} \quad \left\{\frac{1\pm\sqrt{17}}{2}\right\}$$

21. a. Radical equation

b. $\sqrt{2b+3}=b \Rightarrow \left(\sqrt{2b+3}\right)^2=(b)^2$

$$2b+3=b^2 \Rightarrow b^2-2b-3=0$$

23. a. Quadratic in form (or radical)

b. $x^{2/3}+2x^{1/3}-15=0$

Let $t=x^{1/3}$

$$t^2+2t-15=0$$

$(b-3)(b+1)=0$

$b-3=0 \text{ or } b+1=0$

$b=3 \text{ or } b=-1$

Check:

$\sqrt{2(3)+3}=3$

$\sqrt{6+3}=3$

$\sqrt{9}=3 \Rightarrow 3=3$

Check:

$\sqrt{2(-1)+3}=-1 \Rightarrow \sqrt{-2+3}=-1$

$\sqrt{1}=-1 \Rightarrow 1 \neq -1$

$\{3\}$ ($b=-1$ does not check.)

$(t-3)(t+5)=0$

$t-3=0 \text{ or } t+5=0$

$t=3 \text{ or } t=-5$

$x^{1/3}=3 \text{ or } x^{1/3}=-5$

$x=3^3 \text{ or } x=(-5)^3$

$x=27 \text{ or } x=-125$

$\{27,-125\}$

Section 7.4 Practice Exercises

1. **a.** parabola

b. $>$; $<$

c. lowest; highest

d. (h,k); upward; downward

e. vertex; $x=h$

3. $(y-3)^2=-4$

$y-3=\pm\sqrt{-4}$

$y-3=\pm 2i$

$y=3\pm 2i \quad \{3\pm 2i\}$

5. $5t(t-2)=-3$

$5t^2-10t=-3$

$5t^2-10t+3=0$

$a=5, b=-10, c=3$

$t=\dfrac{-(-10)\pm\sqrt{(-10)^2-4(5)(3)}}{2(5)}$

$=\dfrac{10\pm\sqrt{100-60}}{10}=\dfrac{10\pm\sqrt{40}}{10}$

$=\dfrac{10\pm 2\sqrt{10}}{10}=\dfrac{\cancel{2}\left(5\pm\sqrt{10}\right)}{\cancel{2}\cdot 5}$

$=\dfrac{5\pm\sqrt{10}}{5} \quad \left\{\dfrac{5\pm\sqrt{10}}{5}\right\}$

7. $x^{2/3}+5x^{1/3}+6=0$

$\left(x^{1/3}\right)^2+5x^{1/3}+6=0$

Let $u=x^{1/3}$

$u^2+5u+6=0$

$(u+3)(u+2)=0$

$u+3=0 \text{ or } u+2=0$

$u=-3 \text{ or } u=-2$

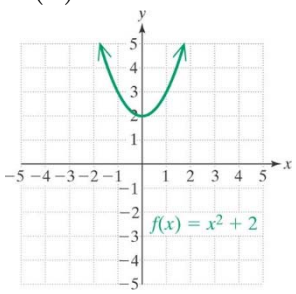
$x^{1/3}=-3 \text{ or } x^{1/3}=-2$

$\left(x^{1/3}\right)^3=(-3)^3 \text{ or } \left(x^{1/3}\right)^3=(-2)^3$

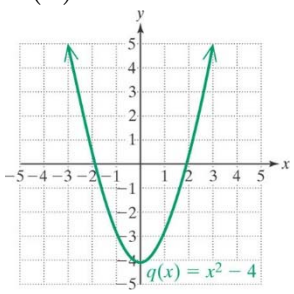
$x=-27 \text{ or } x=-8 \quad \{-27,-8\}$

9. The value of k shifts the graph of $y = x^2$ vertically.

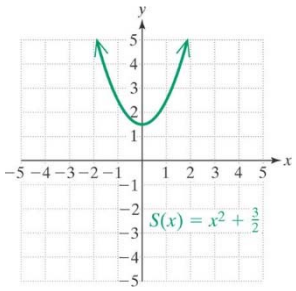
11. $f(x) = x^2 + 2$



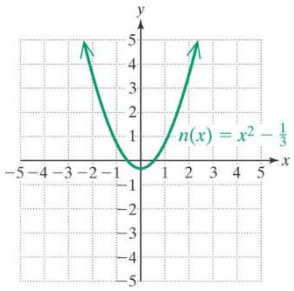
13. $q(x) = x^2 - 4$



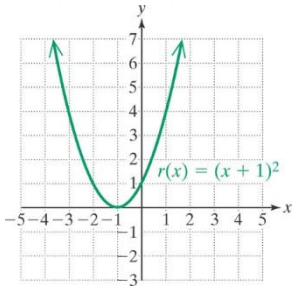
15. $S(x) = x^2 + \frac{3}{2}$



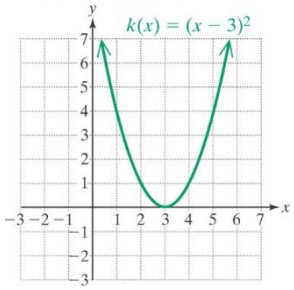
17. $n(x) = x^2 - \frac{1}{3}$



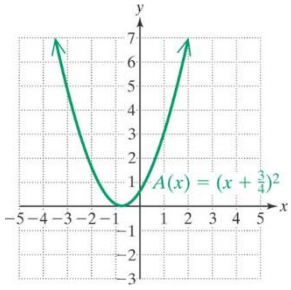
19. $r(x) = (x+1)^2$



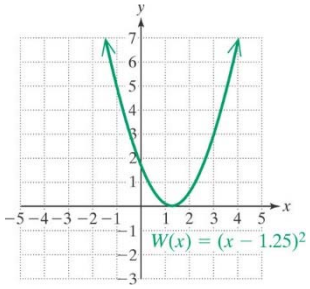
21. $k(x) = (x-3)^2$



23. $A(x) = \left(x + \frac{3}{4}\right)^2$

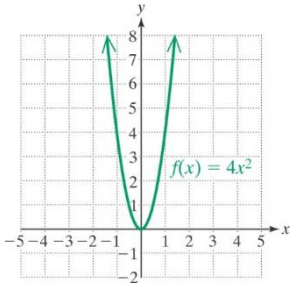


25. $W(x) = (x - 1.25)^2$

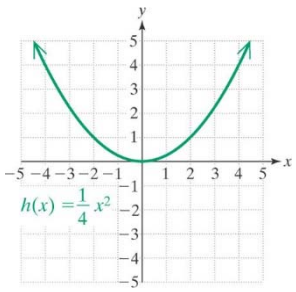


27. The value of a vertically stretches or shrinks the graph of $y = x^2$.

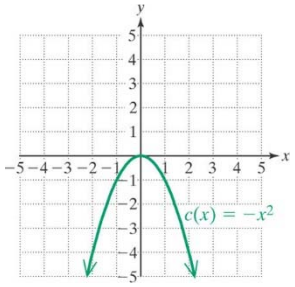
29. $f(x) = 4x^2$



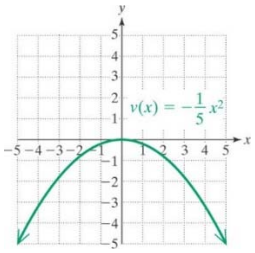
31. $h(x) = \frac{1}{4}x^2$



33. $c(x) = -x^2$



35. $v(x) = -\frac{1}{5}x^2$



37. d

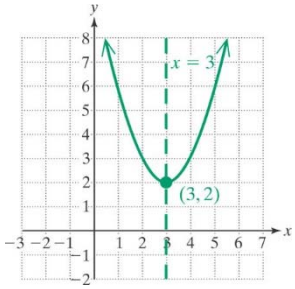
39. g

41. a

43. e

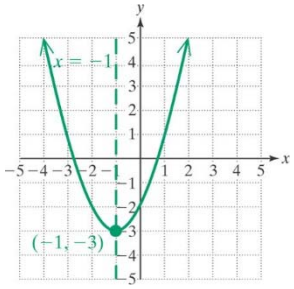
45. $f(x) = (x-3)^2 + 2$

Domain: $(-\infty, \infty)$; range: $[2, \infty)$



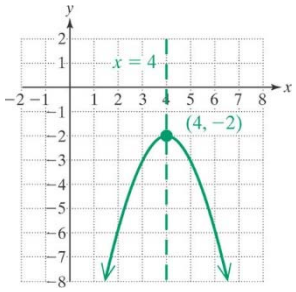
47. $f(x) = (x+1)^2 - 3$

Domain: $(-\infty, \infty)$; range: $[-3, \infty)$



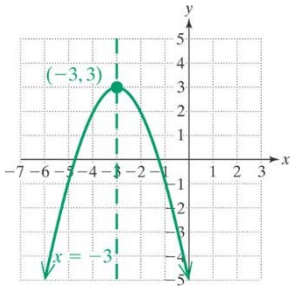
49. $f(x) = -(x-4)^2 - 2$

Domain: $(-\infty, \infty)$; range: $(-\infty, -2]$



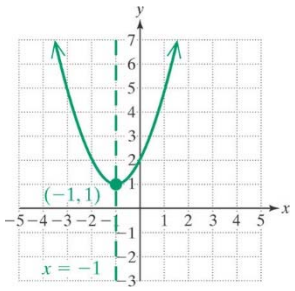
51. $f(x) = -(x+3)^2 + 3$

Domain: $(-\infty, \infty)$; range: $(-\infty, 3]$



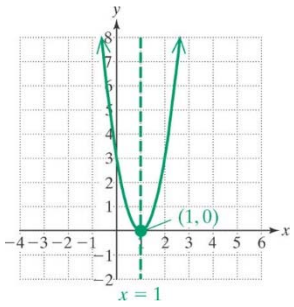
53. $f(x) = (x+1)^2 + 1$

Domain: $(-\infty, \infty)$; range: $[1, \infty)$



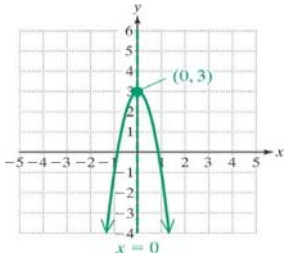
55. $f(x) = 3(x-1)^2$

Domain: $(-\infty, \infty)$; range: $[0, \infty)$



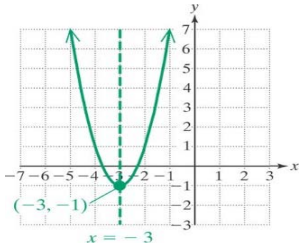
57. $f(x) = -4x^2 + 3$

Domain: $(-\infty, \infty)$; range: $(-\infty, 3]$



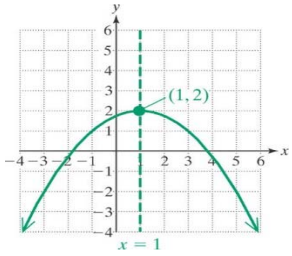
59. $f(x) = 2(x+3)^2 - 1$

Domain: $(-\infty, \infty)$; range: $[-1, \infty)$



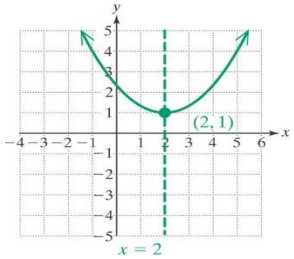
61. $f(x) = -\frac{1}{4}(x-1)^2 + 2$

Domain: $(-\infty, \infty)$; range: $(-\infty, 2]$



63. $f(x) = \frac{1}{3}(x-2)^2 + 1$

Domain: $(-\infty, \infty)$; range: $[1, \infty)$



65. **a.** $y=x^2+3$ is $y=x^2$ shifted up 3 units.

b. $y=(x+3)^2$ is $y=x^2$ shifted left 3 units.

c. $y=3x^2$ is $y=x^2$ with a vertical stretch.

67. $f(x)=4(x-6)^2-9$

Vertex: (6, –9) is a minimum point with minimum value of –9.

69. $p(x)=-\frac{2}{5}(x-2)^2+5$

Vertex: (2, 5) is a maximum point with maximum value of 5.

71. $k(x)=\frac{1}{2}(x+8)^2$

Vertex: (–8, 0) is a minimum point with minimum value of 0.

73. $n(x)=-6x^2+\frac{21}{4}$

Vertex: $\left(0,\frac{21}{4}\right)$ is a maximum point with maximum value of $\frac{21}{4}$.

75. $A(x)=2(x-7)^2-\frac{3}{2}$

Vertex: $\left(7,-\frac{3}{2}\right)$ is a minimum point with minimum value of $-\frac{3}{2}$.

77. $F(x)=7x^2$

Vertex: $(0,0)$ is a minimum point with minimum value of 0.

79. True, since the parabola opens down.

81. False, since the minimum value corresponds to the *y*-value of 8.

83. **a.** $H(x)=\frac{1}{90}(x-60)^2+30$

Vertex: $(60,30)$

b. The minimum height is 30 ft.

c. $H(0)=\frac{1}{90}(0-60)^2+30$

$=\frac{1}{90}(3600)+30=40+30=70$

The towers are 70 ft high.

85. $h(t)=-16t^2+96t+6z$

a. $h(3)=-16(3)^2+96(3)+6$

$=-16(9)+288+6$

$=-144+288+6=150$

b. $h(t)=-16t^2+96t+6$

$=-16(t^2-6t)+6$

$=-16(t^2-6t+9)+6+16(9)$

$=-16(t-3)^2+150$

The fireworks will explode at a height of 150 ft.

Yes, because the ordered pair (3, 150) is the vertex.

Section 7.5 Practice Exercises

1. **a.** $\frac{-b}{2a}; \frac{-b}{2a}$

b. True

c. True

d. True

e. False

3. The graph of p is the graph of $y = x^2$ shrunk vertically by a factor of $\frac{1}{4}$.

5. The graph of r is the graph of $y = x^2$ shifted up 7 units.

7. The graph of t is the graph of $y = x^2$ shifted to the left 10 units.

9. $x^2 - 8x + n$

$$n = \left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot(-8)\right)^2$$
$$= (-4)^2 = 16$$
$$x^2 - 8x + 16 = (x-4)^2$$

11. $y^2 + 7y + n$

$$n = \left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot(7)\right)^2 = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$$
$$y^2 + 7y + \frac{49}{4} = \left(y + \frac{7}{2}\right)^2$$

13. $b^2 + \frac{2}{9}b + n$

$$n = \left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot\left(\frac{2}{9}\right)\right)^2$$
$$= \left(\frac{1}{9}\right)^2 = \frac{1}{81}$$
$$b^2 + \frac{2}{9}b + \frac{1}{81} = \left(b + \frac{1}{9}\right)^2$$

15. $t^2 - \frac{1}{3}t + n$

$$n = \left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot\left(-\frac{1}{3}\right)\right)^2$$
$$= \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$$
$$t^2 - \frac{1}{3}t + \frac{1}{36} = \left(t - \frac{1}{6}\right)^2$$

17. $g(x) = x^2 - 8x + 5$

$$= 1\left(x^2 - 8x\right) + 5$$
$$= 1\left(x^2 - 8x + 16 - 16\right) + 5$$

19. $n(x) = 2x^2 + 12x + 13$

$$= 2\left(x^2 + 6x\right) + 13$$
$$= 2\left(x^2 + 6x + 9 - 9\right) + 13$$

$$= 1\left(x^2 - 8x + 16\right) - 16 + 5$$

$$g(x) = (x-4)^2 - 11$$

Vertex: $(4,-11)$

21. $p(x) = -3x^2 + 6x - 5$

$$= -3\left(x^2 - 2x\right) - 5$$

$$= -3\left(x^2 - 2x + 1 - 1\right) - 5$$

$$= -3\left(x^2 - 2x + 1\right) + 3 - 5$$

$$p(x) = -3(x-1)^2 - 2$$

Vertex: $(1,-2)$

25. $F(x) = 5x^2 + 10x + 1$

$$= 5\left(x^2 + 2x\right) + 1$$

$$= 5\left(x^2 + 2x + 1 - 1\right) + 1$$

$$= 5\left(x^2 + 2x + 1\right) - 5 + 1$$

$$F(x) = 5(x+1)^2 - 4$$

Vertex: $(-1,-4)$

29. $Q(x) = x^2 - 4x + 7$

$a = 1, b = -4, c = 7$

$$\frac{-b}{2a} = \frac{-(-4)}{2(1)}$$

$$= \frac{4}{2} = 2$$

$$Q(2) = 2^2 - 4(2) + 7$$

$$= 2\left(x^2 + 6x + 9\right) - 18 + 13$$

$$n(x) = 2(x+3)^2 - 5$$

Vertex: $(-3,-5)$

23. $k(x) = x^2 + 7x - 10$

$$= 1\left(x^2 + 7x\right) - 10$$

$$= 1\left(x^2 + 7x + \frac{49}{4} - \frac{49}{4}\right) - 10$$

$$= 1\left(x^2 + 7x + \frac{49}{4}\right) - \frac{49}{4} - \frac{40}{4}$$

$$k(x) = \left(x + \frac{7}{2}\right)^2 - \frac{89}{4}$$

Vertex: $\left(-\frac{7}{2}, -\frac{89}{4}\right)$

27. $P(x) = -2x^2 + x$

$$= -2\left(x^2 - \frac{1}{2}x\right)$$

$$= -2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right)$$

$$= -2\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) + \frac{1}{8}$$

$$P(x) = -2\left(x - \frac{1}{4}\right)^2 + \frac{1}{8}$$

Vertex: $\left(\frac{1}{4}, \frac{1}{8}\right)$

31. $r(x) = -3x^2 - 6x - 5$

$a = -3, b = -6, c = -5$

$$\frac{-b}{2a} = \frac{-(-6)}{2(-3)}$$

$$= \frac{6}{-6} = -1$$

$$r(-1) = -3(-1)^2 - 6(-1) - 5$$

$= 4 - 8 + 7$

$= 3$

Vertex: $(2,3)$

$= -3 + 6 - 5$

$= -2$

Vertex: $(-1,-2)$

33. $N(x) = x^2 + 8x + 1$

$a = 1, b = 8, c = 1$

$\frac{-b}{2a} = \frac{-(8)}{2(1)}$

$= \frac{-8}{2} = -4$

$N(-4) = (-4)^2 + 8(-4) + 1$

$= 16 - 32 + 1 = -15$

Vertex: $(-4,-15)$

35. $m(x) = \frac{1}{2}x^2 + x + \frac{5}{2}$

$a = \frac{1}{2}, b = 1, c = \frac{5}{2}$

$\frac{-b}{2a} = \frac{-(1)}{2\left(\frac{1}{2}\right)} = \frac{-1}{1} = -1$

$m(-1) = \frac{1}{2}(-1)^2 + (-1) + \frac{5}{2}$

$= \frac{1}{2} - 1 + \frac{5}{2} = 2$

Vertex: $(-1,2)$

37. $k(x) = -x^2 + 2x + 2$

$a = -1, b = 2, c = 2$

$\frac{-b}{2a} = \frac{-(2)}{2(-1)}$

$= \frac{-2}{-2} = 1$

$k(1) = -(1)^2 + 2(1) + 2$

$= -1 + 2 + 2 = 3$

Vertex: $(1,3)$

39. $A(x) = -\frac{1}{3}x^2 + x$

$a = -\frac{1}{3}, b = 1, c = 0$

$\frac{-b}{2a} = \frac{-(1)}{2\left(-\frac{1}{3}\right)} = \frac{-1}{-\frac{2}{3}} = \frac{3}{2}$

$A\left(\frac{3}{2}\right) = -\frac{1}{3}\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) = -\frac{3}{4} + \frac{3}{2} = \frac{3}{4}$

Vertex: $\left(\frac{3}{2}, \frac{3}{4}\right)$

41. a. $p(x) = x^2 + 8x + 1$

$p(x) = (x^2 + 8x + 16) + 1 - 16$

$p(x) = (x + 4)^2 - 15$

Vertex: $(-4,-15)$

b. $p(x) = x^2 + 8x + 1$

$a = 1, b = 8, c = 1$

43. a. $f(x) = 2x^2 + 4x + 6$

$f(x) = 2(x^2 + 2x + 1) + 6 - 2$

$f(x) = 2(x + 1)^2 + 4$

Vertex: $(-1,4)$

b. $f(x) = 2x^2 + 4x + 6$

$a = 2, b = 4, c = 6$

$$\frac{-b}{2a}=\frac{-(8)}{2(1)}=\frac{-8}{2}=-4$$

$$p(-4)=(-4)^2+8(-4)+1$$
$$=16-32+1=-15$$

Vertex: $(-4,-15)$

$$\frac{-b}{2a}=\frac{-(4)}{2(2)}=\frac{-4}{4}=-1$$

$$f(-1)=2(-1)^2+4(-1)+6$$
$$=2-4+6=4$$

Vertex: $(-1,4)$

45. a. $f(x)=x^2+2x-3$

$a=1, b=2, c=-3$

$$\frac{-b}{2a}=\frac{-(2)}{2(1)}$$
$$=\frac{-2}{2}$$
$$=-1$$

$$f(-1)=(-1)^2+2(-1)-3$$
$$=1-2-3$$
$$=-4$$

Vertex: $(-1,-4)$

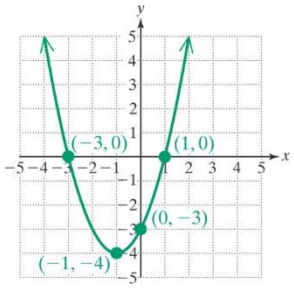
b. $$y=(0)^2+2(0)-3$$
$$=0+0-3$$
$$=-3$$

y-intercept: $(0,-3)$

c. $$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x+3=0 \text{ or } x-1=0$$
$$x=-3 \text{ or } x=1$$

x-intercepts: $(-3,0),(1,0)$

d.



47. a. $f(x)=2x^2-2x+4$

$a=2, b=-2, c=4$

$$\frac{-b}{2a}=\frac{-(-2)}{2(2)}$$
$$=\frac{2}{4}=\frac{1}{2}$$

$$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)+4$$
$$=\frac{1}{2}-1+4=\frac{7}{2}$$

Vertex: $\left(\frac{1}{2},\frac{7}{2}\right)$

b. $$y=2(0)^2-2(0)+4$$
$$=0-0+4$$
$$=4$$

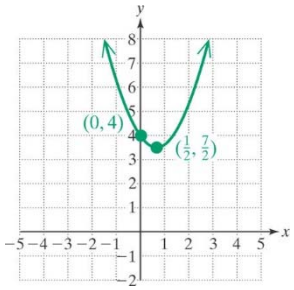
y-intercept: $(0,4)$

c. $$2x^2-2x+4=0$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(2)(4)}}{2(2)}$$
$$=\frac{2\pm\sqrt{4-32}}{4}=\frac{2\pm\sqrt{-28}}{4}$$

No x-intercepts (Complex solutions)

d.



49. **a.** $f(x) = -x^2 + 3x - \frac{9}{4}$

$a = -1, b = 3, c = -\frac{9}{4}$

$\frac{-b}{2a} = \frac{-(3)}{2(-1)} = \frac{-3}{-2} = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) - \frac{9}{4}$

$= -\frac{9}{4} + \frac{9}{2} - \frac{9}{4} = 0$

Vertex: $\left(\frac{3}{2}, 0\right)$

b. $y = -(0)^2 + 3(0) - \frac{9}{4}$

$= -0 + 0 - \frac{9}{4} = -\frac{9}{4}$

y-intercept: $\left(0, -\frac{9}{4}\right)$

c. $-x^2 + 3x - \frac{9}{4} = 0$

$4x^2 - 12x + 9 = 0$

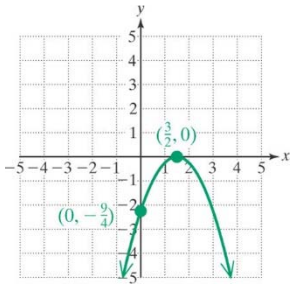
$(2x - 3)^2 = 0$

$2x - 3 = 0$

$2x = 3 \Rightarrow x = \frac{3}{2}$

x-intercept: $\left(\frac{3}{2}, 0\right)$

d.



51. **a.** $f(x) = -x^2 - 2x + 3$

$a = -1, b = -2, c = 3$

$\frac{-b}{2a} = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$

$f(-1) = -(-1)^2 - 2(-1) + 3$

$= -1 + 2 + 3 = 4$

Vertex: $(-1, 4)$

b. $y = -(0)^2 - 2(0) + 3$

$= -0 + 0 + 3 = 3$

y-intercept: $(0, 3)$

c. $-x^2 - 2x + 3 = 0$

$x^2 + 2x - 3 = 0$

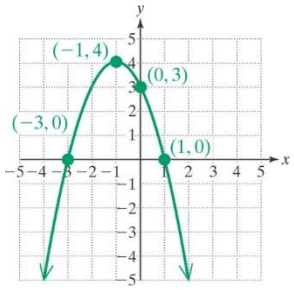
$(x + 3)(x - 1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$x = -3$ or $x = 1$

x-intercept: $(-3, 0), (1, 0)$

d.



53. $C(x) = 2x^2 - 40x + 2200$

$a = 2, b = -40, c = 2200$

55. **a.** $h(t) = -16t^2 + 100t + 8$

$a = -16, b = 100, c = 8$

$$\frac{-b}{2a}=\frac{-(-40)}{2(2)}=\frac{40}{4}=10$$

$$C(10)=2(10)^2-40(10)+2200$$
$$=200-400+2200=2000$$

Vertex: $(10,2000)$

Mia must package 10 MP3 players to minimize her average cost at $2000.

57. a. $m(x)=-0.04x^2+3.6x-49$

$a=-0.04, b=3.6, c=-49$

$$\frac{-b}{2a}=\frac{-(3.6)}{2(-0.04)}=\frac{-3.6}{-0.08}=45$$

The maximum gas mileage will occur at a speed of 45 mph.

b. $m(45)=-0.04(45)^2+3.6(45)-49$
$=-81+162-49=32$

The maximum gas mileage is 32 mpg.

$$\frac{-b}{2a}=\frac{-(100)}{2(-16)}=\frac{-100}{-32}=3.125$$

$h(3.125)$

$=-16(3.125)^2+100(3.125)+8$

$=-156.25+312.5+8=164.25$

b. The maximum height it 164.25 ft.

The fuses should be set for 3.125 sec.

59. a. $b(t)=-\frac{1}{1152}t^2+\frac{1}{12}t$

$a=-\frac{1}{1152}, b=\frac{1}{12}, c=0$

$$\frac{-b}{2a}=\frac{-\left(\frac{1}{12}\right)}{2\left(-\frac{1}{1152}\right)}=\frac{-\frac{1}{12}}{-\frac{1}{576}}=\frac{576}{12}=48$$

The maximum yield occurs at 48 hours.

b. $b(48)$

$=-\frac{1}{1152}(48)^2+\frac{1}{12}(48)$

$=-2+4=2$

The maximum yield is 2 grams.

61. Substitute each ordered pair for x and y into the standard form of a parabola to get three equations in a, b, and c:

$4=a(0)^2+b(0)+c$

$4=c$

$0=a(1)^2+b(1)+c$

$0=a+b+c$

$-10=a(-1)^2+b(-1)+c$

$-10=a-b+c$

Substitute $c=4$ and solve for a:

63. Substitute each ordered pair for x and y into the standard form of a parabola to get three equations in a, b, and c:

$1=a(2)^2+b(2)+c$

$1=4a+2b+c$ (A)

$5=a(-2)^2+b(-2)+c$

$5=4a-2b+c$ (B)

$-4=a(1)^2+b(1)+c$

$-4=a+b+c$ (C)

Subtract (B) from (A) and solve for b:

$a+b+4=0 \quad \rightarrow a+b=-4$

$a-b+4=-10 \rightarrow \underline{a-b=-14}$

$2a=-18$

$a=-9$

Solve for b:

$-9+b=-4$

$b=5$

The equation is: $y=-9x^2+5x+4$.

$4a+2b+c=1$

$-\left(4a-2b+c=5\right)$

$4b \quad =-4$

$b=-1$

Substitute $b=-1$ into (B) and (C), subtract and solve for a:

$5=4a-2\left(-1\right)+c \rightarrow 4a+c=3$

$-4=\ a+\ \left(-1\right)+c \rightarrow \underline{\ a+c=-3}$

$3a=6 \quad \Rightarrow a=2$

Solve for c:

$2+\left(-1\right)+c=-4$

$1+c=-4 \quad \Rightarrow c=-5$

The equation is: $y=2x^2-x-5$.

65. Substitute each ordered pair for x and y into the standard form of a parabola to get three equations in a, b, and c:

$-4=a\left(2\right)^2+b\left(2\right)+c$

$-4=4a+2b+c \qquad$ (A)

$1=a\left(1\right)^2+b\left(1\right)+c$

$1=a+b+c \qquad$ (B)

$-7=a\left(-1\right)^2+b\left(-1\right)+c$

$-7=a-b+c \qquad$ (C)

Subtract (C) from (B) and solve for b:

$a+b+c=\ 1$

$-\left(a-b+c=-7\right)$

$2b \quad =8$

$b=4$

Substitute $b=4$ into (A) and (B), subtract and solve for a:

$4a+2\left(4\right)+c=-4 \rightarrow 4a+c=-12$

$a+\ \left(4\right)+c=\ 1 \rightarrow \underline{\ a+c=-3}$

$3a=-9$

$a=-3$

Solve for c:

$-3+4+c=1 \Rightarrow 1+c=1$

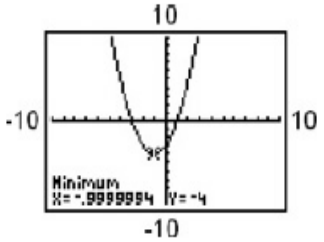
$c=0$

The equation is: $y=-3x^2+4x$.

67. **a.** The sum of the three sides must equal the total amount of fencing.

b. $A=x\left(200-2x\right)$

69.



c. $A = 200x - 2x^2$

$A = -2x^2 + 200x$

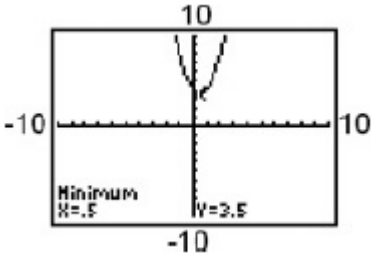
$a = -2, b = 200, c = 0$

$x = \dfrac{-b}{2a} = \dfrac{-(200)}{2(-2)} = \dfrac{-200}{-4} = 50$

$y = 200 - 2(50) = 200 - 100 = 100$

The dimensions of the corral are 50 ft by 100 ft.

71.



73.