Math 1314 Final Exam Review Problems
Spring 2019

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1. Solve the equation by factoring and applying the zero product property.

   (a) $3x^2 + 14x - 49 = 0$

   (b) $24x^2 + 6 = 24x$

   (c) $(n + 5)(n - 7) = 28$

   (d) $10m(m + 3) = 3m - 5$

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2. Use the Square Root Property to solve the equation.

   (a) $16x^2 = 17$

   (b) $(k + 6)^2 = 28$

   (c) $12x^2 + 48 = 0$

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3. Determine the value of n that makes the polynomial a perfect square. Then write the polynomial as the square of a binomial.

   (a) $p^2 + 22p + n$

   (b) $u^2 - 19u + n$

   (c) $x^2 - \frac{2}{3}x + n$

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4. Solve by completing the square.

   (a) $x^2 + 14x - 5 = 0$

   (b) $8x^2 + 3x - 32 = 0$

   (c) $4y^2 + 8y = 11$

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5. Use the quadratic formula to solve the equation

   (a) $2x(x + 3) = -1$

   (b) $(3x - 2)(x - 1) = -3$

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6. Solve the equation using any method

$$\frac{5}{x-4} - \frac{8}{x+1} = \frac{34}{x^2-3x-4}$$

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7. Solve for the indicated variable.

(a) $A = \pi r^2 h$ for $r > 0$.

(b) $kw^2 - cw = r$ for w. Assume $k > 0$.

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8. Solve the equations. For each equation, find the ***sum of the solutions***.

(a) $\sqrt{6x+7} - 2x = 3$

(b) $\sqrt{2x+3} - \sqrt{x-2} = 2$

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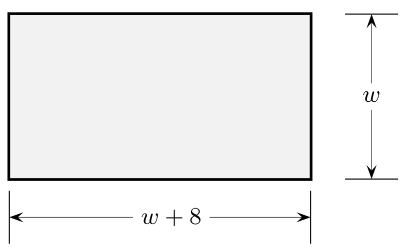
9. A rectangle has an area of 105 yds^2. The length of the rectangle is 8 yds more than its width w. Find the ***perimeter*** P of the rectangle.



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10. The length of the longer leg of a right triangle is 14 ft longer than the length of the shorter leg x. The hypotenuse is 6 ft longer than twice the length of the shorter leg. Find the dimensions of the triangle.

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11. Solve the equation $4x^{2/3} - 9x^{1/3} = 9$. Find the ***product*** of the solutions.

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12. Solve the equation $(x-3)^4 - 5(x-3)^2 + 4 = 0$. Find the ***product*** of the solutions.

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13. Solve the *absolute-value* equations. For each equation, find the ***sum*** S of the solutions.

(a) $|8x-3| - 12 = 4$.

(b) $|5x+4| = |x+9|$.

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14. Solve the following inequalities. Write the solution sets in interval notation.

(a) $(x-4)(5x-8) > 0$

(b) $\dfrac{(x-4)(5x-8)}{x-15} < 0$

(c) $\dfrac{x-4}{x-15} < 2$

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15. Solve the absolute value inequalities. Write the solution sets using interval notation.

    (a) $|5x - 12| - 4 \geq 20$

    (b) $|4x - 9| < 10$

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16. Let $P(3, -8)$ and $Q(-2, 5)$ be given points.

    (a) Use the ***distance formula*** to find the exact distance between P and Q.

    (b) Find the ***midpoint*** of the line segment whose endpoints are the given points P and Q.

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17. Write the given equation in the form $(x - h)^2 + (y - k)^2 = r^2$. Identify the center and radius.

    $2x^2 + 2y^2 + 16x - 20y + 50 = 0$

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18. The endpoints of the diameter of a circle are $(-2, 3)$ and $(-10, 9)$. Write an equation of this circle in standard form and identify its center and radius.

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19. Find an equation for the line with the given property. Write the equations in slope-intercept form.

    (a) Perpendicular to the line $x - 5y = 3$ and containing the point $(5, 3)$.

    (b) Parallel to the line $2x - 5y = 3$ and containing the point $(5, 3)$.

    (c) Containing the points $(5, 3)$ and $(-1, 8)$.

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20. Find the domain of each function. Write the domain using interval notation.

    (a) $f(x) = \dfrac{5x - 4}{x + 3}$ (b) $g(x) = \dfrac{5x}{\sqrt{x + 3}}$ (c) $h(x) = \dfrac{x - 4}{|x + 3|}$

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21. Determine if each function is ***even***, ***odd***, or neither.

    (a) $f(x) = -x^5 + x^3$ (b) $g(x) = x^2 - |x| + 1$ (c) $h(x) = 5x^2 + 3x$

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22. Determine if the graph of each equation is ***symmetric*** with respect to the x-axis, y-axis, origin, or none of these.

    (a) $y = -x^2 + 3$ (b) $x = -|y| + 4$ (c) $y = x^2 + 5x + 1$ (d) $x^2 - y^2 = 5$.

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23. Define the function f by

$$f(x) = \begin{cases} 10x + 2 & \text{if } x \geq 6, \\ 5x - 6 & \text{if } -3 \leq x < 6, \\ x^2 - 2 & \text{if } x < -3. \end{cases}$$

    Evaluate:

    (a) $f(-4) + f(5)$ (b) $f(0)$ (c) $f(6)$

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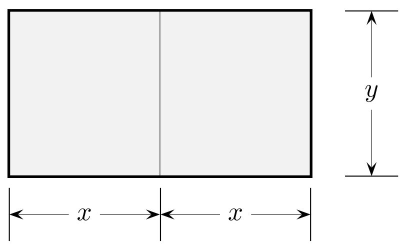
24. Solve the problem.

If $f(x) = \dfrac{3x - B}{x - A}$, $f(3) = 0$, and $f(-6)$ is undefined, what are the values of A and B?

25. Two chicken coops are to be built adjacent to one another from 120 ft of fencing.



(a) What dimensions x and y should be used to maximize the area of an individual coop?

(b) What is the maximum area of an individual coop?

26. Determine the ***end behavior*** of the graph of each polynomial function given below.

(a) $f(x) = -4x^5 + 6x^3 + 2x$ (b) $g(x) = 5x(2x - 3)^3(x + 2)^2$ (c) $h(x) = -8x^4 - 5x^3 - 1$

27. Evaluate the function for the indicated value, then simplify as much as possible.

$f(x) = x^2 - 3x + 5$. Find $f(x + 1)$.

28. Find $\dfrac{f(x+h) - f(x)}{h}$ for the following function f.

$f(x) = x^2 - 3x + 5$.

29. For the given functions f and g, find $(f \cdot g)(x)$.

$f(x) = \dfrac{x - 1}{x^2 - 25}$, $g(x) = \dfrac{x + 5}{1 - x}$

30. For the given functions f and g, find $\left(\dfrac{f}{g}\right)(-2)$.

$f(x) = -6x + 1$, $g(x) = x^2$

31. For the given functions f and g, find and simplify the ***composite*** functions.

(a) $(f \circ g)(x)$.

(b) $(g \circ f)(x)$.

$f(x) = 6x^2 - x + 1$, $g(x) = x - 3$

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32. Describe, with graph transformations, how the graph of $f(x) = (x-2)^2 + 5$ relates to the graph of the parent function $g(x) = x^2$.

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33. Form a polynomial $f(x)$ with real coefficients having the given degree and zeros.

    degree: 4; zeros: $-1, 2$, and $1 - 2i$.

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34. Use the **Rational Zeros Theorem** to find all the real zeros of the polynomial function. Use the zeros to factor f over the real numbers.

    $f(x) = 4x^3 - 11x^2 - 6x + 9$

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35. Use the **Rational Zeros Theorem** to list all the possible rational zeros of the polynomial function. Do not find the actual zeros.

    $f(x) = 6x^4 + 3x^3 - 4x^2 + 2$

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36. Solve the problem.

    Find m so that $x + 4$ is a factor of $5x^3 + 18x^2 + mx + 16$.

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37. Find the ***average rate of change*** of the function f from $x_1 = 1$ to $x_2 = 5$, where $f(x) = 4x^2 - 6x + 1$

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38. Find the ***vertical asymptotes*** (VA) and ***horizontal asymptotes*** (HA), if any, for each function.

    (a) $f(x) = \dfrac{x-1}{x^2-25}$ (b) $g(x) = \dfrac{3x-7}{5x+12}$ (c) $h(x) = \dfrac{x^2-4}{x+1}$

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39. Find the zeros of the polynomial function and state the multiplicity of each zero.

    $f(x) = 10x(x-1)^4(x+3)^2$

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40. If the vertex of the parabola $y = 4x^2 - 5x + 10$ is the point (h, k), what is the value of k?

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41. Find the inverse function of the function f, if it exists, where

    (a) $f(x) = -12x + 5$.

    (b) $f(x) = (x-1)^3 + 4$.

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42. Use the Remainder Theorem and synthetic division to find $f(k)$, where

    $k = \frac{1}{2}$ and $f(x) = 4x^3 - 7x^2 + 5x - 3$

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43. Divide the polynomials using *synthetic division.*

    $\dfrac{3x^4 - 5x^2 + 15x + 2}{x - 2}$

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44. Divide the polynomials using *long division.*

$$\frac{3x^4 - 2x^3 - 5x^2 + 15x + 2}{x^2 - 3}$$

45. Solve the nonlinear system . Provide the product P of the y-values of the solutions and the sum S of the x-values of the solutions.

$$x^2 - xy = 20$$
$$x - 2y = 3$$

46. Solve the system of equations using Gaussian elimination.

$$x - 3y - 2z = 0$$
$$2x - 7y - 6z = 7$$
$$4x + 5y + 2z = 1$$

Then compute the ***sum*** $S = x + y + z$ of the solution (x, y, z) of the system.

47. Write the expression $\log_3\left(\frac{r\sqrt[3]{ab}}{c^5}\right)$ as a sum, difference, or product of logarithms.

Assume that all variables represent positive real numbers.

48. Suppose $\log_x 8 = B$, where B is a positive real number and $x > 0$. Solve for x as a function of B.

Find a value of B such that the solution x of the equation $\log_x 8 = B$ is a positive integer.

49. Let $f(x) = \log_5(x + 3)$.

(a) Write the domain and range of f in interval notation.

(b) Determine the vertical asymptote of f.

50. Solve the logarithmic equation $2 + \log_3(2x + 5) - \log_3(x) = 4$.

If the reciprocal of the solution is written as a reduced fraction $\frac{n}{m}$ (where n and m are integers whose greatest common factor is 1 and where $m > 0$), what is the value of m?

51. Solve the logarithmic equations. For each equation, find the sum S of all solutions. (Note: If there is only one solution $x = a$ for a given equation, then $S = a$ for that equation.)

(a) $\log_3(x + 5) + \log_3(x - 3) = 2$.

(b) $\log_2(x - 4) + \log_2(10 - x) = 3$

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52. Solve the exponential equations.

    (a) $4e^{(3x+2)} = 2.$

    (b) $7^{(3x+2)} - 15 = -3.$

    For each equation, express the solution set in terms of the natural logarithm.

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53. Solve the exponential equation.

    $16^{(3x+2)} = 4^{(5x-8)}$

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54. Solve for x.

    $$\begin{vmatrix} x & 5 \\ -2 & 8 \end{vmatrix} = 12$$

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55. Use the given matrices to compute the given expression.

    Let $\mathrm{M} = \begin{bmatrix} 5 & 6 \\ -2 & 0 \end{bmatrix}$ $\quad \mathrm{N} = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$

    Find 4M − 3N.

Answers to the Math 1314 Final Exam Review

Exercise 1:

(a) $x = -7$ or $x = \dfrac{7}{3}$ (b) $x = \dfrac{1}{2}$ (c) $n = -7$ or $n = 9$ (d) $m = -\dfrac{1}{5}$ or $m = -\dfrac{5}{2}$

Exercise 2:

(a) $x = \pm\sqrt{\dfrac{17}{16}} = \pm\dfrac{\sqrt{17}}{4}$

(b) $k = -6 \pm 2\sqrt{7}$

(c) $x = \pm 2i$

Exercise 3:

(a) $n = 121 \Rightarrow p^2 + 22p + 121 = (p+11)^2$

(b) $n = \dfrac{361}{4} \Rightarrow u^2 - 19u + \dfrac{361}{4} = \left(u - \dfrac{19}{2}\right)^2$

(c) $n = \dfrac{1}{9} \Rightarrow x^2 - \dfrac{2}{3}x + \dfrac{1}{9} = \left(x - \dfrac{1}{3}\right)^2$

Exercise 4:

(a) $x = -7 \pm 3\sqrt{6}$ (b) $x = \dfrac{-3 \pm \sqrt{1033}}{16}$ (c) $y = -1 \pm \dfrac{\sqrt{15}}{2}$

Exercise 5:

(a) $x = \dfrac{-3 \pm \sqrt{7}}{2}$ (b) $x = \dfrac{5 \pm i\sqrt{35}}{6}$

Exercise 6: $x = 1$

Exercise 7:

(a) $r = \dfrac{\sqrt{A\pi h}}{\pi h}$ (b) $w = \dfrac{c + \sqrt{c^2 + 4kr}}{2k}$ or $w = \dfrac{c - \sqrt{c^2 + 4kr}}{2k}$

Exercise 8:

(a) $x = -\dfrac{1}{2}$ or $x = -1$. Therefore, the sum of the solutions is $-\dfrac{3}{2}$.

(b) $x = 3$ or $x = 11$. Therefore, the sum of the solutions is 14.

Exercise 9: $P = 44$ yds.

Exercise 10: Shortest Leg: $x = 10$ Long Leg: $x = 24$ Hypotenuse: $x = 26$

Exercise 11: $x = 27$ or $x = -\frac{27}{64}$. Hence, the product of the solutions is $-\frac{729}{64}$.

Exercise 12: The solution set is $\{1, 2, 4, 5\}$. Hence, the product of the solutions is 40.

Exercise 13:
(a) $x = \frac{19}{8}$ or $x = -\frac{13}{8}$. Hence, the sum of the solutions is $\frac{3}{4}$.
(b) $x = \frac{5}{4}$ or $x = -\frac{13}{6}$. Hence, the sum of the solutions is $-\frac{11}{12}$.

Exercise 14:
(a) $\left(-\infty, \frac{8}{5}\right) \cup (4, \infty)$ (b) $\left(-\infty, \frac{8}{5}\right) \cup (4, 15)$ (c) $(-\infty, 15) \cup (26, \infty)$

Exercise 15:
(a) $\left(-\infty, -\frac{12}{5}\right] \cup \left[\frac{36}{5}, \infty\right)$ (b) $\left(-\frac{1}{4}, \frac{19}{4}\right)$

Exercise 16:
(a) distance $= \sqrt{194}$ (b) midpoint $= \left(\frac{1}{2}, -\frac{3}{2}\right)$

Exercise 17:
Standard Form: $(x + 4)^2 + (y - 5)^2 = 16$
Center $= (-4, 5)$, Radius $= 4$

Exercise 18:
Standard Form: $(x + 6)^2 + (y - 6)^2 = 25$
Center $= (-6, 6)$, Radius $= 5$

Exercise 19:
(a) $y = -5x + 28$ (b) $y = \frac{2}{5}x + 1$ (c) $y = -\frac{5}{6}x + \frac{43}{6}$

Exercise 20:
(a) $(-\infty, -3) \cup (-3, \infty)$ (b) $(-3, \infty)$ (c) $(-\infty, -3) \cup (-3, \infty)$

Exercise 21: (a) odd (b) even (c) neither

Exercise 22:
(a) y-axis (b) x-axis (c) none of these (d) x-axis, y-axis, and the origin

Exercise 23: (a) 33 (b) −6 (c) 62

Exercise 24: $A = -6,\ B = 9$

Exercise 25:
Dimensions: $x = 15\,ft$, $y = 20ft$,
Area = **300** ft^2

Exercise 26:
(a) Left-End Behavior: Up Left, Right-End Behavior: Down Right
(b) Left-End Behavior: Up Left, Right-End Behavior: Up Right
(c) Left-End Behavior: Down Left, Right-End Behavior: Down Right

Exercise 27: $f(x+1) = x^2 - x + 3$

Exercise 28: $2x + h - 3$

Exercise 29: $(fg)(x) = \dfrac{-1}{x-5}$, $x \neq 1$ and $x \neq -5$

Exercise 30: $\dfrac{13}{4}$

Exercise 31: (a) $(f \circ g)(x) = 6x^2 - 37x + 58$ (b) $(g \circ f)(x) = 6x^2 - x - 2$

Exercise 32:
Starting with the graph of g, translate 2 units to the right and then translate 5 units up.
In other words, the graph of f is obtained from the graph of g by the following successive transformations:

1. Shift the graph of $g(x) = x^2$ two units to the right (obtaining $g(x-2) = (x-2)^2$).
2. Shift the graph of $g(x-2) = (x-2)^2$ five units up (obtaining $f(x) = g(x-2) + 5 = (x-2)^2 + 5$).

Exercise 33:
$f(x) = a(x+1)(x-2)(x^2 - 2x + 5)$ where a is any nonzero real number (We may take $a = 1$, if desired.).

Exercise 34:
Possible Rational Zeros: $\pm 9,\ \pm\frac{9}{2},\ \pm\frac{9}{4},\ \pm 3,\ \pm\frac{3}{2},\ \pm\frac{3}{4},\ \pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{4}$

Zeros of f: $-1,\ \frac{3}{4}$, and 3 Factored form: $f(x) = (x+1)(4x-3)(x-3)$

Exercise 35: Possible Rational Zeros: $\pm 2,\ \pm\frac{2}{3},\ \pm\frac{1}{3},\ \pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{6}$

Exercise 36: $m = -4$

Exercise 37: 18

Exercise 38:

(a) Vertical Asymptotes: $x = 5$ and $x = -5$ Horizontal Asymptote: $y = 0$

(b) Vertical Asymptote: $x = -\frac{12}{5}$ Horizontal Asymptote: $y = \frac{3}{5}$

(c) Vertical Asymptote: $x = -1$ Horizontal Asymptote: None

Exercise 39:

$x = 0$, Multiplicity: 1

$x = 1$, Multiplicity: 4

$x = -3$, Multiplicity: 2

Exercise 40: $k = \frac{135}{16}$

Exercise 41:

(a) $f^{-1}(x) = \frac{5-x}{12}$

(b) $f^{-1}(x) = \sqrt[3]{x-4} + 1$

Exercise 42: $-\frac{7}{4}$

Exercise 43: $3x^3 + 6x^2 + 7x + 29 + \frac{60}{x-2}$

Exercise 44: $3x^2 - 2x + 4 + \frac{9x+14}{x^2-3}$

Exercise 45:

Solutions: $\left(-8, -\frac{11}{2}\right)$ and $(5, 1)$

Hence, the *sum* of the x-coordinates of the solutions is -3 and the *product* of the y-coordinates of the solutions is $-\frac{11}{2}$.

Exercise 46:
Solution: $(-1, 3, -5)$
Hence, the sum of the coordinates of the solution is $S = x + y + z = -3$.

Exercise 47: $\log_3(r) + \frac{1}{3}\log_3(a) + \frac{1}{3}\log_3(b) - 5\log_3(c)$

Exercise 48:
$x = 8^{1/B}$
There are many values of B that one may choose, namely $B = 1$, or $B = 3$, or $B = \frac{3}{2}$, or $B = \frac{5}{2}$, ...

Exercise 49:
Domain: $(-3, \infty)$ Range: $(-\infty, \infty)$ Vertical Asymptote: $x = -3$

Exercise 50: $m = 5$

Exercise 51:
(a) Solution Set $= \{4\}$. Sum of solutions $= 4$.
(b) Solution Set $= \{6, 8\}$. Sum of solutions $= 14$.

Exercise 52:
(a) Solution Set $= \left\{\dfrac{-2 - \ln(2)}{3}\right\}$.
(b) Solution Set $= \left\{\dfrac{\ln(12) - 2\ln(7)}{\ln(343)}\right\}$.

Exercise 53: $x = -12$

Exercise 54: $x = \dfrac{1}{4}$

Exercise 55: $\begin{bmatrix} 17 & 30 \\ -23 & -9 \end{bmatrix}$